Savitribai Phule Pune University

(Formerly University of Pune)

Course Work Structure
Ph.D. Mathematics

(Faculty of Science and Technology)

(w. e. f. Academic Year 2025-26)

Prepared by – BOS Mathematics, SPPU

Approved by – Academics Council, SPPU

Preamble

The Board of Studies in Mathematics Savitribai Phule Pune University is pleased to present the Ph.D. Coursework in Mathematics, which forms an essential component of the doctoral program. It aims to bridge the transition from postgraduate studies to independent research by strengthening both the breadth and depth of mathematical knowledge.

The coursework has been carefully designed in accordance with the guidelines of the University Grants Commission (UGC) and the University, with the objective of providing research scholars a strong academic foundation for pursuing advanced studies and research.

The Ph.D. coursework in Mathematics is designed as a foundational stage of the doctoral program, intended to equip research scholars with the necessary theoretical background, methodological tools, and analytical skills which is required for pursuing advanced research. The curriculum provides a balance between core areas of mathematics and emerging research domains. It ensurs that research scholars will gain not only conceptual clarity but also the ability to apply advanced techniques to complex problems.

The course work motivates students by linking their research journey with career growth and future opportunities. It helps in shaping mindset, developing skills, and creating a sense of direction. Students often come from diverse backgrounds. Coursework helps them reach a common baseline of knowledge. This coursework introduces the advanced study of pure and applied mathematics, research methodology, and the latest developments in the field. It also provides knowledge of research-oriented software such as Scilab, KASH, Maxima, SAGE, Mathematica, MATLAB, and other related tools.

By engaging in seminars, workshops, and conferences, students learn communication and presentation skills.The assignments, seminars, and discussions, students learn to question, analyze, and evaluate.This builds curiosity and inspires them to look at problems from new perspectives.The subject-specific courses expose students to multiple areas within their field.

The intention of course work is to nurture critical thinking, encourage innovative approaches, and prepare scholars for conducting high-quality research leading to their doctoral dissertation.

Ph.D. coursework motivates students by giving them confidence, clarity, and competence - transforming their mindset from “student” to “independent researcher”

We hope this coursework will build strong academic foundation and serve as a stepping stone towards high-quality research, fostering academic excellence and innovation.

******

Contents

1 Program Outcome (PO)

After completion of this Program,

1. Students can attain proficiency in academic writing, research paper preparation, and effective communication of research outcomes through publications, presentations, and conferences.
2. Students can enhance the capacity for logical reasoning, critical analysis, synthesis of ideas, and innovative problem-solving.
3. Students can learn professional research ethics, intellectual honesty, and responsibility in conducting and reporting research.
4. Students can develop the research competancy with theoretical and applied aspects of the subject which helps the ability to identify, define, and analyze complex research problems.
5. Students can acquire advanced knowledge of research methodologies, tools, and techniques required for conducting high-quality research.
6. Students can demonstrate intellectual independence, self-directed learning, and the ability to continuously update knowledge and skills.
7. Students can synthesize findings to draw meaningful and reliable conclusions.
8. Scholars will acquire in-depth knowledge of their subject area and related interdisciplinary domains.
9. Student can aquire the ability to think critically and creatively to solve complex real-world problems.
10. Students will contribute their original ideas and solutions that advance knowledge and use the potential for social, technological, or industrial impact.
11. Students will develop the resilience and adaptability needed to engage with emerging challenges and opportunities in academia, industry, and society.
12. Students will contribute to the advancement of knowledge communities and play a proactive role in policy-making, consultancy, and innovation.
13. Students will integrate their knowledge and research findings into practices and solutions that address social, cultural, environmental, and economic challenges.

******

2 General Guidelines

The syllabus for the Ph.D. course work in Mathematics is framed as per the University Circular no. 98/2025 dated April 8, 2025. The Ph.D. Course Work Structure for Mathematics is as follows:

Sr.No.	Course	Name of the Course	Credits	Hours	Responsibility
1	1	Research Methodology	04	60	Research Center & Research Supervisor
2	2	Attending at least one Seminar /Conference /Workshop (National / International) and presenting paper / poster	01	–	Research Center & Research Supervisor
3	3	Two subject-specific advanced level courses	08	120	Research Center
4	4	Research and Publication Ethics	02	30	Research Center shall conduct the UGC-approved 02 credits course as per UGC letter No. D.O.F. 1-1/2018 (Journal/CARE) dated Dec. 2019 and SPPU circular No. 65/2020 dated 3rd March 2020 OR Research Scholar can complete the Research and Publication Ethics course run by the Centre of Publication Ethics, SPPU.
5	5	Pedagogical Training / Industrial Visit Report / Assessment Statement	01	30	Research Center & Research Supervisor
		Total	**16**	**240**	

1. The coursework is for 16 credits.

2. Contents of Course 1 and Course 3 are appended below.

3. The recognized Research Center, based on the recommendations of the Research Advisory Committee, shall prescribe courses for the Ph.D. scholar, including research methodology and two subject-specific advanced courses, to be completed within two years of registration.

4. A course on Publication Ethics must be completed within two years from the date of registration by the Ph.D. scholar.

5. Candidates with an M.Phil. degree from SPPU who are admitted to the Ph.D. program, or those who have completed M.Phil. coursework and are permitted to proceed to a Ph.D., will be exempt from Ph.D. coursework, provided both programs are in the same subject. If the M.Phil. and Ph.D. programs are in different subjects,

the candidate must complete Courses 2, 3, and 5 as specified, as per University Circular No. 07/2025 Reference Number PGS/87 dated 9th January 2025.

6. All other candidates admitted to the Ph.D. program are required to complete the coursework approved by the Academic Council of SPPU, achieving a minimum of 55% marks or its equivalent grade on the UGC 10-point scale, within the first two years of registration. Candidates will be permitted to continue with their Ph.D. studies and submit the thesis only after fulfilling these requirements.

7. Grades in the course work, including research methodology courses, shall be finalized after a combined assessment by the Research Advisory Committee, and the final grades shall be communicated to the P. G. Admission Section of the SPPU.

8. All Ph.D. scholars, irrespective of discipline, shall be required to train in teaching/education /pedagogy/writing related to their chosen Ph.D. subject during their doctoral period. Ph.D. scholars may also be assigned 4-6 hours per week of teaching/research assistantship for conducting tutorial or laboratory work and evaluations.

9. The Research Advisory Committee of the Research Center can also recommend online or offline courses offered by UGC, National Agency/Institute, MOOCs, SWAYAM, NPTEL, AFS, ATM, NBHM, NCM etc., of 4 Credits each as part of the credit requirements for the Ph.D. program. If necessary, course work courses 1, 3 and 5 as specified may be carried out by candidates in sister Departments / Institutes either within or outside the University for which due credit will be given to them. The candidate can opt for such courses upon recommendation of the Research Advisory Committee of the Research Center.

10. If necessary, courses 1, 3 and 5 as specified may be carried out by candidates in sister Departments / Institutes either within or outside the University for which due credit will be given to them. The candidate can opt for such courses upon recommendation of the Research Advisory Committee of the Research Center.

11. University Department/ Research Centre may introduce additional course(s) in respect of course 3 on recommendations of the Research Center. The syllabus of such course(s) will be prepared by the concerned teacher and will be flexible to accommodate new developments in that area.

12. The policies and procedures determined by the University will be followed for the conduct of examinations and declaration of the result of the candidate.

******

3 Ph. D. (Mathematics) Course Work Structure

3.1 Course 1-RM: Research Methodology

Credits: 04 **No. of Hours: 60**

I. History and development of Mathematical thinking.

II. LATEX

III. At least one Mathematical software out of the following: Scilab /KASH/ Maxima/ SAGE/Mathematica/MatLab. Other software suggested by the research guide

IV. Exposure to Math SciNet, JSTORE, Science-Direct, Scopus and other online journal database, writing review of a research paper

V. Scientific writing .

VI. Plagiarism awareness and usage of related software?s.

VII. Paper/ poster presentation in seminars/conferences/workshops.

******

3.2 Course 2-SCW : Attending at least one relevant Seminar / Conference / Workshop etc. not less than two days. (National/International)

Credits: 01

It is highly desired that candidate shall present his/her research work in a Seminar/ Conference/Workshop etc. (National/International)

******

3.3 Course 3-SPAL: Two subject specific advanced level courses

Credits: 08 (Each course of 4 credits) **No. of Hours: 120**

Sr No.	Course Code	Course Title	Number of Credits
1	SPAL : 01	Participation in Workshop	4
2	SPAL : 02	Topics in Algebra	4
3	SPAL : 03	Topics in Analysis	4
4	SPAL : 04	Topics in Topology and Geometry	4
5	SPAL : 05	Topics in Differential Equations	4
6	SPAL : 06	Topics in Discrete Mathematics	4
7	SPAL : 07	Topic in Algebraic Graph Theory	4
8	SPAL : 08	Topics in Computational Mathematics and Data Science	4
9	SPAL : 09	Topics in Integral Transforms	4
10	SPAL : 10	Topics in Mathematics for Machine Learning	4

Course Completion Guidelines :

A student is required to complete any two courses from the approved list. Each course is structured into four units/modules (except SPAL : 01) as outlined in the syllabus.

- To complete a course, a student must complete any two of the four units.
- If a student completes all four units of a particular course, this will be counted as the completion of two separate courses.

Illustration: Suppose a student takes the course Topics in Discrete Mathematics.

- In Semester I, the student completes Unit I and Unit III. This will be treated as completion of one course.
- Later, in the same or a subsequent semester, if the student completes Unit II and Unit IV, this will be treated as the completion of a second distinct course.

In this case, the course titles will be reflected as:

- SPAL : 06 Topics in Discrete Mathematics (Units I & III)
- SPAL : 06 Topics in Discrete Mathematics (Units II & IV)

This flexible structure allows students to design their own learning pathways while ensuring depth and breadth of coverage within the subject.

******

3.4 Course 4-SPAL: Research and Publication ethics

Credits: 02 **No. of Hours: 30**

Research Center shall conduct the UGC approved 02 credits course as per UGC letter No. D.O.F. 1-1/2018 (Journal/CARE) Dated Dec. 2019 and SPPU circular No. 65/2020 dated 3rd March 2020.

OR

Research Scholar can complete the Research and Publication ethics course run by the Centre of Publication Ethics Savitribai Phule Pune University.

Modules	Unit Title	Teaching Hours
	Theory	
RPE 1	Philosophy and Ethics	4
RPE 2	Scientific Conduct	4
RPE 3	Publication Ethics	7
	Practice	
RPE 4	Open Access Publishing	4
RPE 5	Publication Misconduct	4
RPE 6	Database and Research Metrics	7

******

3.5 Course 5-SPAL: Pedagogical Training / Industrial Visit Report / Assessment Statement

Credits: 01 **No. of Hours: 30**

Research Center and Research Supervisor

******

4 Evaluation and Grades

4.1 Table 1: Percentage to grade and grade point

Sr. No.	Grade Letter	Grade Point	Marks
1	O (Outstanding)	10	90 to 100
2	A+ (Excellent)	9	80 to 89
3	A (Very Good)	8	75 to 79
4	B+ (Good)	7	70 to 74
5	B (Above Average)	6	65 to 69
6	C (Average)	5	60 to 64
7	D (Pass)	4	55 to 59
8	F (Fail)	0	<55
9	Ab (Absent)	0	–

4.2 Table 2: Illustration of the Structure of CGPA and Marks Scheme for Ph. D. Course Work (Sample Copy)

Sr No	Course Code	Course Name	Credit	Internal Marks	External Marks	Grade Letter	Grade Point (0–10)	Credit Point
1	RM	Research Methodology	4	30	70	A	8	32
2	SCW	Seminar/ Conference/ Workshop	1	–	–	O	10	10
3	SPAL-1	Subject Specific Advanced Level Course-1	4	30	70	O	10	40
4	SPAL-2	Subject Specific Advanced Level Course-2	4	30	70	A+	9	36
5	RPE	Research and Publication Ethics	2	15	35	B+	7	14
6	PIA	Pedagogical Training / Industrial Visit Report / Assessment Statement	1	07	18	A	8	8
		Total	16	112	263			140

- Course Grade Point Average(CGPA) is the weighted average for the grade points obtained in all the courses for all the semester.
- For example : A student has registered for the program Ph. D. (Mathematics) having courses with credits $C_i, i = 1, 2, \cdots n$ and his /her grade points in those courses are $G_i, \quad i = 1, 2, \cdots n$ respectively.
- Then students

$$CGPA = \frac{\sum_{i=1}^{n} C_i G_i}{\sum_{i=1}^{n} C_i}.$$

- CGPA is calculated up to **TWO** decimal places by rounding off.

4.3 Table 3: CGPA Distribution and Corresponding Grade of the Degree Awarded

Based on the performance of the student in the examination declare the result and issue the Grade sheet. The class shall be awarded to the student on the CGPA calculated as mentioned in the following Table

Sr. No.	CGPA	Overall Grade
01	$CGPA \geq 9.50$	**O** (Outstanding)
02	$8.25 \leq CGPA < 9.50$	**A+** (Excellent)
03	$6.75 \leq CGPA < 8.25$	**A** (Very Good)
04	$5.75 \leq CGPA < 6.75$	**B+** (Good)
05	$5.25 \leq CGPA < 5.75$	**B** (Above Average)
06	$4.75 \leq CGPA < 5.25$	**C** (Average)
07	$4.00 \leq CGPA < 4.75$	**D** (Pass)

Illustrative example for Course Grade Point Average(CGPA) calculation:

Name : Infinity

Examination: Ph. D. (Mathematics)

Academic Year : 2025-26

Course	Credit(C_i)	Grade	Grade Point(G_i)	Credit Point
RM: Research Methodology	4	**B**	06	24
SCW : Seminar/ Conference/ Workshop	1	**A**	08	08
SPAL-1 Subject Specific Advanced Level Course	4	**A+**	09	36
SPAL-2 Subject Specific Advanced Level Course	4	**B+**	07	28
RPE: Research and Publication Ethics	2	**O**	10	20
PIA : Pedagogical Training	1	**D**	04	04
Total	16			120

$$CGPA = \frac{4\times 6+1\times 8+4\times 9+4\times 7+2\times 10+1\times 4}{4+1+4+4+2+1}$$
$$= \frac{24+08+36+28+20+04}{16} = \frac{120}{16} = 7.50$$

- **Course Grade Point Average(CGPA) : 7.50**
- **Final Grade is A**

5 Detail Syllabus of Courses 3-SPAL

5.1 SPAL-1: Participation in Workshop

I. Participation in advance level workshop or Seminar Series (equivalent to 4 credits) in any topic of research interest can be considered as a 4 credit course. However, the workshop should consist of contents and sessions equivalent to a 4-credit course as determined by the reserch center. It is mandatory to obtain the approval of the Department Committee in order to consider the course as equivalent to a Ph.D. course under course 3.

II. Student can do workshop/ seminar / school devised by UGC, MOOCs, SWAYAM, NPTEL, AFS, ATM, NBHM or any other National Agency/Institute etc. to broaden the knowledge of a research in Mathematics and also inculcating problem solving skills for better understanding of the subject. The Research Guide will request the coordinator of the workshop/ seminar / school to give a grade to the student based on the participation of the student in the workshop. In case coordinator of the workshop/ seminar / school is unable to evaluate the student, the evaluation of such students will be conducted by the research center based on the contents of the workshop/ seminar/ school attended by the student.

******

5.2 SPAL-2: Topics in Algebra

Any **TWO** of the following four topics can be covered as part of the course.

I. **Group Theory:** Describing a group through generators and relations, Group actions, semi-direct products, extension problem and group extensions, classifying extensions.

II. **Rings and Modules:** Commutative and Non-commutative rings, Artinian and Noetherian rings and modules, ACC and DCC conditions, Krull-Schmidt theorem.

III. **Galois Theory:** Galois Groups of Field Extensions, Fundamental Theorem of Galois Theory, Solvability by Radicals, Hilbert's theorem.

IV. **Representation Theory:** Definition and Examples, Irreducible and Indecomposable representations, Maschke's theorem, Characters of Groups, Schur?s lemma, First and Second Orthogonality relations, character table, regular representations, Finite Fourier transforms, Applications of representation theory.

Reference Books:

1. D. Dumit and R. Foote, Abstract Algebra, Wiley.

2. Benjamin Steinberg, Representation Theory of Finite Groups, Springer.

3. T. Y. Lam, A First Course in Non-commutative Rings, Springer.

******

5.3 SPAL -3: Topics in Analysis

Any **TWO** of the following four topics can be covered as part of this course.

I. **Measure Theory:** Review of basic measure theory, axioms of probability, random variables and measurable functions, probability distributions, and stochastic processes, Kolmogorov's axioms,Â measurable spaces and probability measures, probability distributions (both discrete and continuous), different modes of convergence, Â almost sure convergence,Â convergence in probability,Â andÂ convergence in distribution,Â weak and strong laws of large numbers, sums of independent random variables, Central Limit Theorem (CLT).

II. **Functional Analysis:** Review of basic Functional Analysis and Operator Theory, compact operators spectral theory of bounded and compact operators, spectral theorem for compact self-adjoint operators, Â using functional analysis to solve optimization problems.

III. **Harmonic Analysis:** Definition of Fourier series, pointwise and uniform convergence, Dirichlet and Fejer kernels, Gibbs phenomenon, applications to problems like Laplace's equation, the Fourier Transform on real line, the Fourier transform in L1 and L2 spaces, convolution, the Plancherel theorem and applications, the convergence of Fourier series, Applications to partial differential equations, probability theory (Central Limit Theorem) and signal processing, Introduction to harmonic analysis on more abstract spaces like locally compact groups, Haar measure.

IV. **Complex Analysis:** Understanding transformations that preserve angles, including Schwarz reflection principle and the Riemann mapping theorem, Taylor and Laurent series, classification of singularities (poles, essential singularities), Picard's theorem, Poisson integral formula.

Reference Books:

1. E. M. Stein and R. Shakarchi, Fourier Analysis, Princeton University Press.

2. H. L. Royden and P. M. Fitzpatrick, Real Analysis (Fourth Edition), Pearson IN.

3. Balmohan Limaye, Functional Analysis, New Age International Publishers.

4. J. B. Conway, Functions of One Complex Variable, Springer Verlag.

******

5.4 SPAL-4: Topics in Topology and Geometry

Any **TWO** of the following four topics can be covered as part of this course.

I. **Algebraic Topology:** Homotopy, covering spaces, fundamental group, mapping cones, mapping cylinders, and suspensions, quotient spaces, Seifert-van Kampen Theorem, simplicial complexes, chain complexes, simplicial homology, singular homology.

II. **Manifolds:** Introduction to the concept of a manifold, smooth manifolds, tangent spaces and vector fields, differential forms, the exterior derivative, calculus of exterior derivatives, Poincare's lemma, generalizing the concept of integration to manifolds, Stokes' theorem.

III. **Differential Geometry:** Curvature and torsion formulas, Serret-Frenet formulas and the Frenet frame, The Fundamental Theorem for space curves, First and second fundamental forms, Gauss map and its properties, Principal curvatures, Gaussian curvature, and mean curvature, Isometries and the Gauss-Bonnet Theorem.

IV. **Lie Groups:** Review of differential geometry, including differentiable manifolds, vector fields, and tangent spaces, Matrix groups, Lie Groups and Lie algebras, Lie algebra concepts and propertie, exponential map, structure of Lie groups, group actions on manifolds and the formation of homogeneous spaces.

Reference Books:

1. James Munkres, Topology (Second Edition), PHI Publications.

2. Allen Hatcher, Algebraic Topology, Cambridge University Press

3. James Munkres, Analysis on Manifolids, Adison Wesley Publishing Company.

4. Brian Hall, Lie Groups, Lie algebras and Representations, Springer.

5. Christian Blatter, Elementary Differential Geometry, Springer.

******

5.5 SPAL-5: Topics in Differential Equations

Any TWO of the following four topics can be covered as part of the course.

I. **Ordinary Differential Equations (ODEs) and Qualitative Theory:** Existence and uniqueness theorems (Picard Lindelf, Peano), Linear and nonlinear systems, Stability theory (Lyapunov methods), Sturm-Liouville theory, Boundary value problems and Green?s functions.

II. **Partial Differential Equations (PDEs):** Classification (elliptic, parabolic, hyperbolic), Solution techniques for Laplace, Heat, and Wave equations, Green's functions, Introduction to nonlinear PDEs, Shock waves and conservation laws, Method of characteristics for nonlinear PDEs, Weak solutions and Sobolev spaces (overview), Fundamental solutions and Green's functions.

III. **Integral equations(IEs):** Introducion to integral equations, classification and connections with differential equations. Fredholm and Volterra integral equations of the first and second kind, resolvent kernels, Neumann series, and Hilbert-Schmidt theory. Methods for solving integral equations with degenerate and symmetric kernels, applications to boundary value problems and mathematical physics. Nonlinear integral equations, iterative methods of solution, and their applications in modeling complex systems.

IV. **Dynamical Systems(DSs):** Autonomous and non-autonomous systems, Phase portraits and invariant manifolds, Bifurcation theory (saddle-node, Hopf bifurcations), Chaos and attractors.

V. **Fractional Differential Equations (FDEs):** Fractional derivatives and integrals (Riemann Liouville, Caputo), Existence and uniqueness of solutions, Mittag-Leffler functions and Laplace transform methods, Stability and asymptotic behavior.

Reference Books:

1. Lawrence Perko, Differential Equations and Dynamical Systems, Springer, 2013.

2. Lawrence C. Evans, Partial Differential Equations, AMS, 2010.

3. Igor Podlubny, Fractional Differential Equations, Academic Press, 1998.

4. Earl E Coddington and Norman Levinson,Theory of Ordinary Differential Equations, McGraw-Hill, 1984.

5. Ram P. Kanwal, Linear Integral Equations Theory and Technique, Springer New York, 2012.

******

5.6 SPAL-6: Topics in Discrete Mathematics

Any **TWO** of the following four topics can be covered as part of this course.

I. **Algebraic Combinatorics:** Chains in distributive lattices, the incidence algebra of a locally finite poset, the Mobius inversion formula, techniques for computing Mobius functions, applications to counting, lattices and their Mobius algebras.

II. **Polya Theory:** Group actions, Burnside's Lemma, Cycle Index, Polya's Enumeration Theorem, Applications to counting problems

III. **Computational Complexity:** Computational Models, Problems and Algorithms, Complexity classes, Reductions, Notions of NP-completeness, NP-hardness etc

IV. **Geometric Combinatorics:** Hyperplane Arrangements, combinatorial properties of collections of hyperplanes in space, Zaslavsky's theorem for enumeration, lattice polytopes and the counting of integer points within them, the theory of Ehrhart polynomials and their reciprocity.

Reference Books:

1. D. B. West, Introduction to Graph Theory, 2nd Edition (Pearson)

2. Richard Stanley, Enumerative Combinatorics Vol 1 (Cambridge University Press, Second Edition).

3. G. Gratzer, General Lattice Theory.

******

5.7 SPAL-7: Topic in Algebraic Graph Theory

Any **TWO** of the following four topics can be covered as part of this course.

I. **Spectral Graph Theory:** Adjacency matrix of a graph, some examples, adjacency matrix and walks, spectral characterization of bipartite graphs, bounds for the eigenvalues, other matrices related to graphs, graph Laplacians, and its properties, Laplace eigenvalues, Matrix tree theorem, Perron-Frobenius theorem, applications of Perron-Frobenius theory

II. **Algebraic Graphs:** Automorphisms of graphs: Graph automorphisms, Automorphisms of typical graphs, permutation groups, Abstract groups, Cayley graphs, some graphs arising from algebraic structures such Cayley Graphs, zero-divisor graphs associated to rings, Examples and basic properties of such graphs

III. **Expander Graphs:** Ihara zeta function of graph, Ihara-Bass determinant formula, Graph theory prime number theorem, Isoperimetric number, Expander graphs, Ramanujan Graphs

IV. **Random Walks on Graphs:** Random walks on graphs and Markov chains, stationary distribution, convergence speed to the stationary distribution, applications to ranking algorithms

Reference Books:

1. L. W. Beinke and R. J. Wilson, Topics in Algebraic Graph Theory, Cambridge University Press, 2005.

2. Audrey Terras, Zeta Functions on Graphs, Cambridge University Press, 2010.

3. Ayman Badawi, David F. Anderson, T. Asir and T. Tamizh Chelvam, Graphs from Rings, Springer Nature, 2021.

******

5.8 SPAL-8: Topics in Computational Mathematics and Data Science

Any **TWO** of the following four topics can be covered as part of this course.

I. **Principal Component Analysis (PCA):** Definition and purpose of PCA in dimensionality reduction and feature extraction. Examples in various fields like image processing, pattern recognition, and data compression, dimensionality reduction, noise reduction, feature extraction, data as matrices and vectors, covariance matrix, Eigenvalues and Eigenvectors and understanding their role in PCA, PCA using the Singular Value Decomposition (SVD), principal components based on explained variance, Projecting data onto the principal components, PCA in Python, using libraries like NumPy and scikit-learn to implement PCA.

II. **Gradient Descent Method:** The concept of finding the best solution (minimum or maximum) for a given problem, defining cost function to quantify the error or discrepancy between predicted and actual values in a model, distinguishing between local and global optima and the challenges associated with finding the global minimum, gradient descent as a method for finding the minimum of a function by iteratively moving in the direction of the steepest descent, the gradient as a vector of partial derivatives, indicating the direction of the steepest ascent, the formula for updating model parameters using the gradient and a learning rate, the role and importance of the learning rate, Learning how to determine when to stop the iterative process.

III. **Regression Analysis:** Definition of regression analysis, dependent and independent variables, correlation vs. causation, Simple linear regression, multiple linear regression, polynomial regression, the method for finding the best-fitting line, Testing the significance of the regression coefficients, Using R, Python, or other software to perform regression analysis, Creating scatter plots, residual plots, and other visualizations to explore data and assess model fit, Interpreting the results of regression analysis and communicating them effectively.

IV. **Markov Chains:** Discrete-Time Markov Chains, recurrence and transience, random walks on graphs and Markov chains, stationary distribution, power method to compute eigenvalues, applications to ranking algorithms, google page rank algorithm.

Reference Books:

1. Gareth James, D. Witten, T. Hastie and R. Tibshirani, An Introduction to Statistical Learning (Second Edition) Springer.

2. Christopher M. Bishop, Pattern Recognition and Machine Learning, Springer.

******

5.9 SPAL-9: Topics in Integral Transforms

Any **TWO** of the following four topics can be covered as part of this course.

I. **Mathematical Foundations:** Review of Lebesgue Integration and Lp spaces, Test functions and Distributions, Operations with Distributions, Schwartz Space, Tempered Distributions, Convolution of Distributions, General Theory of Linear Integral Transforms: Kernels, Inversion, and Uniqueness

II. **The Fourier Transform:** Definition and Properties of Fourier Transform, Fourier Inversion Theorem and Plancherel Theorem, Convergence and Summability (FejÃ©r, Abel) and Applications of Fourier Transforms to: Solutions of Ordinary Differential Equations, Solutions of Integral Equations, Solutions of Partial Differential Equations, Fourier Cosine and Sine Transforms with Example, Properties of Fourier Cosine and Sine Transforms, Applications of Fourier Cosine and Sine Transforms to Partial Differential Equations

III. **Specialized Transforms for Specific Geometries:** Hankel Transform: Definition, Examples, Connection to Bessel functions and Applications of Hankel Transforms to Partial Differential Equations. Legendre Transforms: Definition of the

Legendre Transform and Examples, Basic Operational Properties of Legendre Transforms, Applications of Legendre Transforms to Boundary Value Problems Hermite Transforms: Definition of the Hermite Transform and Examples, Basic Operational Properties, Apllications of Hermite Transforms The Radon Transform: Definition and Properties of the Radon Transform, The Radon Transform of Derivatives, Derivatives of the Radon Transform, Convolution Theorem for the Radon Transform, Inverse of the Radon Transform and the Parseval Relation, Applications of the Radon Transform

IV. **Wavelets and Wavelet Transform:** Limitations of Fourier methods: time?frequency localization problem, Short-Time Fourier Transform (STFT), Gabor transform and the Heisenberg uncertainty principle, Continuous Wavelet Transforms, Discrete Wavelet Transforms, Examples of Orthonormal Wavelets, Fast Wavelet Transform (Mallat?s algorithm)., Applications of Wavelet Transform in Signal Processing, Image Processing and Numerical Solution of PDEs

Reference Books:

1. Davies, B. (2002). Integral Transforms and Their Applications (3rd ed.). Springer.
2. L. Debnath & D.D. Bhatta, Integral Transforms and Their Applications, CRC Press.
3. S. Kesavan (1989). Topics In Functional Analysis And Applications, Wiley. (For theory of Distributions)
4. Folland, G. B. (2009). Fourier analysis and Its Applications. American Mathematical Society.
5. Mallat, S. (2009). A Wavelet Tour of Signal Processing: The Sparse Way (3rd ed.). Academic Press.
6. HÃ¶rmander, L. (2003). The Analysis of Linear Partial Differential Operators I (2nd ed.). Springer.
7. C. Sidney Burrus, Ramesh Gopinath & Haitao Guo ? Introduction to Wavelets and Wavelet Transforms: A Primer (1998).
8. Hernandez & Weiss, A First Course on Wavelets (1996)

******

5.10 SPAL-10: Topics in Mathematics for Machine Learning

Any **TWO** of the following four topics can be covered as part of this course.

I. **Linear Algebra & Analytic Geometry for ML:** Vector spaces, norms, and inner products, Eigen decomposition and Singular Value Decomposition (SVD) and their applications (PCA, matrix factorization), Matrix Calculus: derivatives of functions with vector/matrix input/output. Positive (semi-)definite matrices and their role in optimization.

II. **Applications to Image Processing:** The Dirac distribution and convolution, Linear integral transforms , Images as linear systems , Introduction to linear integral transforms, 1D Fourier transform, 2D Fourier transform , Sampling and the Shannon constraint , Discrete cosine transform , Wavelet transform, Eigen-analysis , Other orthogonal image transforms, Images as stochastic processes, Image formation physics, Images as radiometric measurements , Image capture and geometric optics , Lens aberrations and radial distortion , Image capture from a radiometric point of view , Surface reflectance.

III. **Data structures for Image Analysis:** Levels of image data representation, Traditional image data structures , Matrices , Chains , Topological data , Relational structures , Hierarchical data structures , Pyramids , Quad trees , Other pyramidal structures

IV. **Image pre-processing:** Pixel brightness transformations, Position-dependent brightness correction, Gray-scale transformation, Geometric transformations , Pixel coordinate transformations , Brightness interpolation Contents, Local pre-processing, Image smoothing , Edge detectors , Zero-crossings of the second derivative , Scale in image processing , Canny edge detection , Parametric edge models , Edges in multispectral images , Local pre-processing in the frequency domain , Line detection by local pre-processing operators , Detection of corners (interest points) , Detection of maximally stable extremal regions Image restoration , Degradations that are easy to restore , Inverse filtration , Wiener filtration

Reference Books:

1. Deisenroth, M. P., Faisal, A. A.,& Ong C. S. (2020), Mathematics for Machine Learning. Cambridge University Press.

2. Bishop, C. M. (2006), Pattern Recognition and Machine Learning. Springer.

3. Goodfellow, I., Bengio, Y., & Courville, A. (2016), Deep Learning. MIT Press.

4. Bracewell, R. N. (2000), The Fourier Transform and Its Applications (3rd Ed.). McGraw-Hill.

5. Pratt, W. K. (2007). Digital Image Processing (4th Ed.). Wiley-Interscience.

6. Snyder, D. L., & Qi, H. (2004), Machine Vision: Theory, Algorithms, Practicalities. Morgan Kaufmann

7. Kropatsch, W. G., & Bischof, H. (Eds.), (2001). Digital Image Analysis: Selected Techniques and Applications. Springer.

8. Gonzalez, R. C., & Woods, R. E. (2018), Digital Image Processing (4th Ed.). Pearson.

******

Savitribai Phule Pune University

(Formerly University of Pune)