Savitribai Phule Pune University
Department of Mathematics
Ph. D./M. Phil. (Mathematics)
Entrance Examination Syllabus

Research Methodology : Mathematics

Unit I : **Logic and Set Theory** : Sets, operations on sets, functions, pigenhole principle, well-ordering principle, principle of mathematical induction, number system, fundamental theorem of arithmetic, GCD and LCM of numbers, polynomials, modular arithmetic, reasoning, deductive arguments, Venn diagrams, Russell's paradox.

Unit II : **Analytical Geometry** : Equations of straight line, circles, conics, plane, space curves, tangent and normals to curves.

Unit III : **Differential Calculus** : Sequence, series, convergence tests, limit, limit points, lim sup, lim inf, real valued functions of one variable, continuity, differentiability.

Unit IV : **Integral Calculus** : Indefinite integration, definite integral and area under curve, differentiation under integral sign, improper integrals.

Unit V : **Basic Algebra** : Quadratic equations, their roots and properties, permutation and combinations and related formuale, binomial theorem.

Unit VI : **Matrices and Determinant** : Types of matrices, algebra of matrices, properties of matrix multiplication, inverse of a matrix, concept of rank, system of linear equations, Cramers's rule. Determinants, minors and cofactors and their properties, solutions of a linear systems of equations, condition of consistency.

Unit VII : **Vector Algebra** : Representation of vectors, coplanar and non-coplanar vectors, resultant of two vectors, angle between two vectors, scalar product, dot product, gradient, curl and divergence.

Unit VIII : **Complex Numbers** : Algebra of complex numbers, square root of a complex number, geometrical representation of a complex number, polar form, De Movier's theorem, roots of unity, Cauchy-Riemann equations.

Unit IX : **Statistics and Probability** : Random experiment, sample space, types of events, concept of probability, independent events, conditional probability, Baye's theorem, mean, mode, variance.

Unit X : **Basic Differential Equations** : Basic concepts, formation, modeling of elementary differential equations, classification of differential equations, order and degree, linear first order differential equations, exact equations, applications of first order differential equations.

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Subject Specific : Mathematics

Unit I : **Algebra** : Groups, Group action, Sylow Theorems, Rings, PID, UFD, Fields, Field Extension, Finite Fields.

Unit II : **Linear Algebra** : Finite dimensional Vector Spaces, Algebra of Linear transformations, Diagonaziblity, Jordan canonical form, Primary decomposition theorem, Cayley Hamilton theorem, Hermitian and Unitary transformations, Spectral theorem.

Unit III : **Real Analysis** : Real valued functions of one variable, bounded functions, limits of functions, monotonic functions, continuous functions, uniformly continuous functions, sequences of functions, series of functions, differentiability of function, Rolle's theorem, Mean value theorem, Riemann integral and properties, Fundamental theorem of Calculus, Taylor series, Fourier series, topology of $\mathbb{R}^n$ (convergence, continuity, compactness, connectedness, completeness), Bolzano-Weierstrass theorem, Cantor's intesection theorem, Heine-Borel theorem, Weierstrasss approximation theorem, Baire category theorem.

Unit IV : **Complex Analysis** : Topology of complex plane, Power series and radius of convergence, analytic functions, chain rule, branch points, Cauchy-Riemann equations, Mobius transformations, complex integration, Riemann-Stieltjes integral, Fundamental theorem of algebra, maximum modulus theorem, winding number, Cauchy theorem and integral formula and its applications, Morera's theorem, Open mapping theorem, Goursat's theorem, classification of singularities, Laurent series, residue theorem.

Unit V : **Differential Equations** : Linear Differential Equations with constant and variable coefficients, Existence and uniqueness of solution, Picard's iteration theorem, Boundary value problems, Applications of differential equations, System of linear differential equations.

Unit VI : **Advanced Calculus** : Functions of several variables, Limit, Continuity, Differentiability, Chain rule, maxima and minima, Implicit function theorem, Inverse function theorem, integration, Stoke's theorem.

Unit VII : **Measure and Integration** : Measure on the real line, Lebesgue outer measure, Measurable sets, Measurable functions, Borel and Lebesgue measurability, Integration of measurable functions, functions of bounded variation, Lebesgue's differentiation theorem, L^p spaces, convex functions, Jensen's inequality, Hölder's and Minkowski's inequality, completness of L^p spaces.

Unit VIII : **Functional Analysis** : Normed linear spaces, continutity of linear maps, Hahn-Banach theorem, Banach spaces, Uniform boundedness principle, Closed graph and Open mapping theorems, Bounded inverse theorem, Spectrum of bounded operator, Duals and Transposes, Duals of $L^p([a,b])$ and $C([a,b])$, Inner product spaces, Orthonormal sets, Approximation and optimization, Projection, Riesz-representation theorem, Bounded operators and adjoints, self adjoint, Normal, Unitary operators.

Unit IX : **Topology** : Topological of metric spaces, Continuity, Convergence, Homeomorphism, Compactness, Connectedness, Axioms, Subspaces, Product Spaces, Quotient spaces, Tychonoffs theorem, Urysohns metrization theorem.

Unit X : **Discrete Mathematics** :Partially ordered sets, Lattices, Complete lattices, Distrbutive lattices, Complements, Boolean algebra, Boolean expressions, Application to switching circuits, Elements of Graph Theory, Eulerian and Hamiltonian graphs, Planar graphs, Directed graphs, Trees, Permutations and Combinations, Pigeonhole principle, Principle of inclusion and exclusion, Derangements.