Savitribai Phule Pune University

(Formerly University of Pune)

Two Year Degree Program in Mathematics

(Faculty of Science & Technology)

Revised Syllabi for

M.Sc./M.A.(Mathematics) Part-I

(For Colleges Affiliated to SavitribaiPhulePune University)

Choice Based Credit System Syllabus

To be implemented from Academic Year 2019-2020

Title of the Course: M.Sc./M.A. (Mathematics)

Preamble :

Savitribai Phule Pune University has decided to change the syllabi of various faculties from June,2019. Taking into consideration the rapid changes in science and technology and new approaches in different areas of mathematics and related subjects, Board of studies in Mathematics after a thorough discussion with the teachers of Mathematics from different colleges affiliated to University of Pune has prepared the syllabus of M.Sc./M. A. Semester - I and Semester- II (w.e.f. 2019-20) Mathematics course under the Choice Based Credit System (CBCS). The model curriculum as developed by U. G. C. is used as a guideline for the present syllabus.

Aims and Objectives of the new curriculum :

i) To maintain updated curriculum.

ii) To take care of fast development in the knowledge of mathematics.

iii) To enhance the quality and standards of Mathematics Education.

iv) To provide a broad common frame work, for exchange, mobility and free dialogue across the Indian Mathematical and associated community.

v) To create and aptitude for Mathematics in those students who show a promise for higher studies and creative work in Mathematics.

vi) To create confidence in others, for equipping themselves with that part of Mathematics which is needed for various branches of Sciences or Humanities in which they have aptitude for higher studies and original work.

Structure of the course:

There are five compulsory courses in semester I and five compulsory courses in semester II.

Sr. No.	Courses		
	Semester-I	**Semester-II**	**Credit**
1	MTUT111: Linear Algebra	MTUT121: Complex Analysis	4
2	MTUT112: Real Analysis	MTUT122: General Topology	4
3	MTUT113: Group Theory	MTUT123: Ring Theory	4
4	MTUT114: Advanced Calculus	MTUT124. Advanced Numerical Analysis	4
5	MTUT115:Ordinary Differential Equations	MTUT125: Partial Differential Equation	4

Equivalence of previous syllabus with new syllabus:

Semester I and Semester II

Old Courses	Equivalent New Couses
MT 501: Real Analysis	MTUT112: Real Analysis
MT 502: Advanced Calculus	MTUT114: Advanced Calculus
MT 503: Group Theory	MTUT113: Group Theory
MT 504: Numerical Analysis	MTUT124: Advanced Numerical Analysis
MT 505: Ordinary Differential Equations	MTUT115: Ordinary Differential Equations
MT 601: Complex Analysis	MTUT121: Complex Analysis
MT 602:General Topology	MTUT122: General Topology
MT 603: Rings and Modules	MTUT123: Rings and Modules
MT 604: Linear Algebra	MTUT111: Linear Algebra
MT 605:Partial Differential Equations	MTUT125: Partial Differential Equations

Details of Syllabus:

Semester I

MTUT111: LINEAR ALGEBRA

Unit I. Vector Spaces. **[06 Hours]**

1.1 Vector Spaces,
1.2 Subspaces and linear dependence,
1.3 The concepts of basis and dimension.

Unit II.Linear Transformation and Matrices. **[06 Hours]**

2.1 Linear Transformations
2.2 Addition and multiplication of matrices.
2.3 Linear Transformations and matrices.

Unit III.Vector Spaces with an Inner product. **[10 Hours]**

3.1 The concept of symmetry.
3.2 Inner Product.

Unit IV.The Theory of a single Linear Transformation. **[10 Hours]**

4.1 Basic Concepts
4.2 Invariant Subspaces
4.3 The Triangular form theorem
4.4 The rational and Jorden canonical forms.

Unit V. Dual Vector Spaces and Multilinear Algebra. **[14 Hours]**

5.1 Quotient spaces and dual vector spaces
5.2 Bilinear forms and duality
5.3 Direct sums and tensor products

5.3 A proof of the elementary divisor theorem.

Unit VI. Orthogonal and Unitary Transformations [14 Hours]

6.1 The structure of orthogonal transformations
6.2 The principal axis theorem
6.3 Unitary transformation and the spectral theorem.

Recommended Book:
Linear Algebra.An Introductory Approach.By Charles W. Curtis.
Chapter 2: Section- 3, 4, 5. ; Chapter 3: Section-11, 12, 13. ; Chapter 4: Section- 14, 15
Chapter 7: Section- 22, 23, 24, 25.; Chapter 8: Section- 26, 27, 28, 29;
Chapter 9: Section- 30, 31, 32 .

MTUT112: REAL ANALYSIS

Unit-I.Lebesgue Measure: [22 Hours]
1.1 Lebesgue Outer Measure
1.2 σ- algebra of Lebesgue Measurable Sets
1.3 Outerand Inner Approximation of Lebesgue Measurable Sets
1.4 Countable Additivity
1.5 Continuity
1.6 Borel-Cantelli Lemma
1.7 Non-measurable Set, Cantor Set, Cantor-Lebesgue Function.

Unit-II.Lebesgue Measurable Functions: [18 Hours]
2.1 Definition and algebra of Lebesgue Measurable Functions
2.2 Sequential Point wise Limits and Approximations by Simple Functions
2.3 Littlewood'sThreePrinciples
2.4 Egoroff's Theorem
2.5 Lusin's Theorem.

Unit-III. Differentiation and Integration: [20 Hours]
3.1 Continuity of Monotone Functions
3.2 Lebesgue's Differentiation Theorem
3.3 Functions of Bounded Variation
3.4 Jordan's Theorem, Absolutely Continuous Functions
3.5 Integration of Derivatives
3.6 Differentiation of Indefinite Integral
3.7 Fundamental Theorem of Calculus.

Recommended Book:
Real Analysis-Fourth Edition, Authors: H. L. Royden, P. M. Fitzpatrick.
Sections: Chapter 2 - sections 2.1 to 2.7, Chapter 3 - sections 3.1 to 3.3,
Chapter 6 - sections 6.1 to 6.5.

Reference Books:

1. Real Analysis: Authors: Elias M. Stein, Rami Shakarchi.
2. Basic Real Analysis: Author: Anthony W. Knapp.
3. Beginning Functional Analysis: Authors: Karen Saxe (Springer International Edition)

MTUT113: GROUP THEORY

Unit 1: Groups **[25 Lectures]**

1.1. The Definition of a Group
1.2. Subgroups
1.3. Isomorphisms
1.4. Homomorphisms
1.5. Equivalence Relations and Partitions
1.6. Cosets
1.7. Restriction of a Homomorphism to a Subgroup
1.8. Products of Groups
1.9. Modular Arithmetic
1.10. Quotient Groups

Unit 2: Symmetry **[35 Lectures]**

2.1. Symmetry of Plane Figures
2.2. The Group of Motions of the Plane
2.3. Finite Groups of Motions
2.4. Discrete Groups of Motions
2.5. Abstract Symmetry: Group Operations
2.6..The Operation on Cosets
2.7. The Counting Formula
2.8. Permutation Representations
2.9. Finite Subgroups of the Rotation Group
2.10. The Operations of a Group on Itself
2.11. The Class Equation of the Icosahedral Group
2.12. Operations on Subsets
2.13.TheSylow Theorems
2.14. The Groups of Order 12
2.15. Computation in the Symmetric Group

Text Book:

M. Artin, Algebra, (Prentice Hall), Second edition.
Chapter 2: Sec 1 to 10.
Chapter 5: Sec 1 to 9.
Chapter 6: Sec 1 to 6.

Reference Book:

J. S. Milne, Group Theory: Lecture Notes

MTUT114: ADVANCED CALCULUS

Unit 1. Differential Calculus **[18 Lectures]**1.1: Differentiability in Several Variables
1.2: The Chain Rule
1.3: The Mean Value Theorem
1.4: Functional Relations and Implicit Functions
1.5: Higher Order Partial Derivatives
1.6: Taylor's Theorem
1.7: Critical Points
1.8: Extreme Value Problems
1.9: Vector Valued Functions and Their Derivatives

Unit 2. The Implicit Function Theorem and Its Applications **[12 Lectures]**
2.1 : The Implicit Function Theorem
2.2 : Curves in the Plane
2.3 : Surfaces and Curves in Space
2.4 : Transformations and Coordinate Systems

Unit 3. Integral Calculus **[12 Lectures]**
3.1: Integration on the line
3.2 : Integration in Higher Dimensions
3.3 : Multiple Integrals and Iterated Integrals
3.4 : Change of Variables for Multiple Integrals

Unit 4. Line and Surface Integrals; Vector Analysis **[18 Lectures]**
4.1: Arc Length and Line Integrals
4.2: Green's Theorem
4.3 : Surface Area and Surface Integrals
4.4 : Vector Derivatives
4.5 : The Divergence Theorem
4.6 :Stoke's Theorem

Recommended Book:
Gerald B. Folland, Advanced Calculus, Pearson(2002).
Articles : 2.2 - 2.10, 3.1 - 3.4, 4.1 - 4.4, 5.1 - 5.5, 5.7.

Reference Books:
1.Pattrick M. Fitzpatrick, Advanced Calculus, AMS, undergraduate Texts in Mathematics, Indian Edition .
2. Michael D. Spivak, Calculus on Manifolds:
A Modern Approach to Classical Theorems of Advanced Calculus, Harper Collins.
3. T.M.Apostol, Calculus Vol. II, John Wiley and Sons.
4. James Stewart, Calculus, Books/ Cole.
5.T. M. Apostol: Mathematical Analysis , Narosa publishing house .
6.W. Rudin: Principles of Mathematical Analysis , Mc-Graw Hill.

MTUT115: ORDINARY DIFFERENTIAL EQUATIONS

Unit I: Linear equations of the first order **[04 hours]**

1.1 Linear equations of the first order
1.2 The equation y'+ay=0
1.3 The equation y'+ay=b(x)
1.4 The general linear equations of first order

Unit II: Linear equations with constant coefficients **[12 hours]**

2.1 Second order homogeneous equations
2.2 Initial value problems for second order equations
2.3 Linear dependence and independence
2.4 Formula for the Wronskian
2.5Non homogeneous equations of order two
2.6 Homogeneous equations of order n
2.7Non homogeneous equations of order n
2.8 Algebra of constant coefficients equations

Unit III:Linear equations with variable coefficients **[12 hours]**

3.1 Initial value problems for the homogeneous equation
3.2 Solutions of the homogeneous equation
3.3Wronskian and linear independence
3.4 Reduction of order of the homogeneous equation
3.5 Non homogeneous equations with analytic coefficients
3.6 Homogeneous equations
3.7Legendre equation

Unit IV:Linear Equations with regular singular points **[12 hours]**

4.1 Euler equation
4.2 Second order equation with regular singular points
4.3 Exceptional cases
4.4 Bessel's equation
4.5 Regular singular point at infinity

Unit V:Existence and uniqueness of solutions to first order equations **[12 hours]**

5.1 Equations with variables separated
5.2 Exact equations
5.3 Method of successive approximations
5.4 Lipschitz condition
5.5 Approximation to, and uniqueness of, solutions

Unit VI :Existence and uniqueness of solutions to systems and n-th order equations **[10 hours]**

6.1Complex n-dimensional space
6.2 Systems as vector equations
6.3 Existence and uniqueness of solutions to systems
6.4Existence and uniqueness for linear systems
6.5 Equations of order n

Recommended Book: An Introduction to Ordinary Differential Equations,E. A. Coddington, Prentice- Hall.
Chapter- 1.4 -1.7; 2.1-2.12; 3.1-3.8; 4.1- 4 .4, 4.6- 4.8; 5.1 -5.8; 6.4-6.8.
Reference Books :
G. F. Simmons and S. G. Krantz, DfferentialEquations (Tata McGraw-Hill).

SEMESTER-II

MTUT121: COMPLEX ANALYSIS

Unit I: Basic Properties of Complex Numbers: **[08 hours]**

1.1 Arithmetic of Complex Numbers
1.2 Geometry of Complex Numbers
1.3 Path Connectivity
1.4 The Fundamental Theorem of Algebra

Unit II Complex Differentiability and Conformality: **[10 hours]**

2.1 Definition and Basic Properties
2.2 Polynomials and Rational Functions
2.3 Analytical Functions: Power Series
2.4 Cauchy- Riemann Equations
2.5 Review of Calculus of Two Real Variables
2.6 Fractional Linear Transformation

Unit III: Contour Integration: **[12 hours]**

3.1 Definition and Basic Properties
3.2 Existence of Primitives
3.3 Cauchy-Goursat Theorem
3.4 Cauchy's Theorem via Green's Theorem
3.5 Cauchy's Integral Formulae
3.6 Analyticity of Complex Differentiable Functions
3.7 A Global Implication: Liouville
3.8 Mean Value and Maximum Modulus
3.9 Harmonic Functions

Unit IV: Zeros and Poles: **[12 hours]**

4.1 Zeros of Holomorphic Functions
4.2 Open Mapping Theorem
4.3 Singularities
4.4 Laurent Series
4.5 Residues
4.6 Winding Number
4.7 The Argument Principle

Unit V: Application to Evaluation of Definite Real Integrals: [10 hours]

5.1 Trigonometric Integrals
5.2 Improper Integrals
5.3 Jordan's Inequality
5.4 Bypassing a Pole

Unit VI: Local And Global Properties: [08 hours]

6.1 Schwarz's Lemma
6.2 Local Mapping
6.3 Homotopy and Simple Connectivity
6.4 Homology Form of Cauchy's Theorem

Recommended Book:

1**. Anant R. Shastri, Basic Complex Analysis of One Variable**, Macmillan Publishers India,2010 . Ch. 1: 1.1,1.3,1.6,1.8 , Ch. 2: 2.1,2.2,2.3 Ch. 3: 3.1, 3.2,3.7, Ch. 4: 4.1 to 4.8 , Ch. 5: 5.1 to 5.7, Ch. 6: 6.1 to 6.4

Reference Books:

1. J. W. Brown and R.V. Churchill, Complex Variables and Applications, Indian Edition. (Eighth Edition)
2. John. B. Conway, Functions of One Complex Variable, Springer International Student Edition. (Second Edition)
3. S. Ponnusamy, Foundation of Complex Analysis, Narosa Publications. (Second Edition)
4. L.V. Alfors, Complex Analysis, McGraw Hill, 1979.

MTUT122: GENERAL TOPOLOGY

Unit 1.Prerequisites [10 hours]

1.1 : Cartesian Products
1.2 : Finite Sets
1.3 : Countable and Uncountable Sets
1.4 : Infinite Sets and Axiom of Choice
1.5 : Well Ordered Sets

Unit 2. Topological Spaces and Continuous Functions [20 hours]

2.1 : Topological Spaces
2.2 : Basis for a Topology, Order Topology, Subspace Topology
2.3 : Product Topology
2.4 : Closed Sets and Limit Points
2.5: Continuous Functions
2.6 : Metric Topology
2.7 : Quotient Topology

Unit 3.Connected and Compact Spaces **[15 hours]**

3.1 : Connected spaces
3.2 : Connected Subspaces of Real Line
3.3 : Components and Local Connectedness
3.4 : Compact spaces
3.5 : Compact Subspaces of the Real Line
3.6 : Limit point compactness
3.7 : Local Compactness

Unit 4. Countablity and Separation Axioms **[15 hours]**

4.1 :Countability Axioms
4.2 : Separation axioms and Normal Spaces
4.3 :Urysohn Lemma
4.4 :Tietze Extension Theorem
4.5 : The UrysohnMetrization Theorem
4.6 :Tychonoff's Theorem.

Recommended Book:

J. R. Munkres, Topology: A First Course, (Prentice Hall, Second Edition), 2000.
Chapter1 : Sec. 5 to 7, Sec. 9 to 10., Chapter 2: Sec.12 to 22., Chapter 3 : Sec. 23 to 29.
Chapter 4 : Sec. 30 to 35 , Chapter 5 : Sec. 37.

Reference Books:

1. K Janich. Topology.Springer, 1984.
2. M A Armstrong. Basic Topology.Springer, 1983.
3. O Viro, O Ivanov, V Kharlamov, and N Netsvetaev. Elementary Topology: Problem Textbook, AMS Publication, 2008.
4. K. D. Joshi, Introduction to General Topology, John Wiley & Sons.

MTUT123: RINGS AND MODULES

Unit I : Rings **[16 hours]**

1.1 Basic Terminologies
1.2 Rings of Continuous functions
1.3 Matrix Rings, Polynomial Rings, Power Series Rings, Laurent Rings, Boolean Rings, Some Special Rings,
1.4 Direct Products
1.5 Several Variables
1.6 Opposite Rings
1.7 Characteristic of a Ring.

Unit II : Ideals **[12 hours]**

2.1 Definitions
2.2 Maximal Ideals

2.3 Generators
2.4 Basic Properties of Ideals
2.5 Algebra of Ideals
2.6 Quotient Rings
2.7 Ideals in Quotient Rings
2.8 Local Rings.

Unit III :Homomorphisms of Rings **[10 hours]**

3.1 Definitions and Basic Properties
3.2 Fundamental theorems
3.3 Endomorphism Rings
3.3 Field of Fractions, Prime fields.

Unit IV : Factorisation Domains **[12 hours]**

4.1Division in Domains
4.2Euclidean Domains
4.3 Principal Ideal Domains
4.4 Factorisation Domains
4.5 Unique Factorisation Domains
4.6 Eisenstein's Crieterion.

Unit V : Modules **[10 hours]**

5.1Definitions and Examples
5.2 Direct Sums
5.3 Free Modules
5.4 Quotient Modules
5.5 Homomorphism
5.6 Simple Modules
5.7 Modules over PID.

Recommended Book:

C. Musili, Rings and Modules, 2nd Revised Edition, Narosa Publishing House.
(Chapters 1, 2, 3, 4, 5)

Reference Books :

1. Dummit and Foote, Abstract Algebra, second edition (Wiley India).
2. Luther and Passi, Algebra II, Narosa Publishing House.
3. Jain and Bhattacharya, Basic Abstract Algebra, 2nd Edition, Cambridge University Press.
4. Joseph Gallian, Contemporary Algebra, 7th Edition, Narosa Publishing House.

MTUT124: ADVANCED NUMERICAL ANALYSIS

Unit 1: **Core Linear Algebra** **[12 Lectures]**

1.1 Basic concepts and problems in Matrix and Linear Algebra

1.2 Emphasis onto some special matrices including Permutation, Hessenberg, Companion, Nonderogatory, Diagonally dominant

1.3 Positive definite type of matrices, Difference between vector and matrix norm.

Unit 2: **Floating Point Numbers and Error in Computations** **[16 Lectures]**

2.1 Calculating errors at the time of various numerical calculation

2.2 Calculating the error Bounds for floating - Point Matrix computations

2.3 Basic algorithms for computing Norm of a vector, Inner product of two vectors, solution of an Upper Triangular system and other systems.

2.4: Finding the condition on stability of the algorithm and accuracy of the solution.

Unit 3: Gaussian Elimination and LU factorization with Applications [16 Lectures]

3.1 LU factorization without and with Pivoting and stability of method

3.2 Householder Transformations and applications to QR factorization and Hessenberg Reduction.

3.3 Solving linear system through numerical method with existence, uniqueness and invariance of solution.

3.4 Applications to Electrical Circuit Problems, ODE, PDE.

Unit 4: Least Squares Solutions to Linear System and Error Analysis [16 Lectures]

4.1 Geometric interpretation of the Least Squares problem

4.2 Polynomial fitting method with applications leading to an over determined system and itsexistence, uniqueness.

4.3 Basic laws of Floating - Point Arithmetic

4.4 Error analysis for Forward Elimination and Backward substitution.

Recommended Book:

Title: Numerical Linear Algebra and Applications (2nd Edition) by Biswa N. Datta by PHI
Chapter0: (0.1- 0.3) ; Chapter1: 1.1- 1.8 ; Chapter 2: 2.1- 2.7; Chapter3: 3.1- 3.4
Chapter5: 5.1-5.4 ;5.6- 5.7; Chapter6: 6.1- 6.3; 6.9- 6.10 ; Chapter7: 7.1- 7.5
Chapter11: 11.1- 11.3.

Reference Books:

1.Numerical Linear Algebra by L. N. Trefethen
(SIAM: Society for Industrial and Applied Mathematics)
2. Applied Numerical Linear Algebra by James Demmel
(SIAM: Society for Industrial and Applied Mathematics)
3. Numerical Linear Algebra by V. Sundarapandian
(Prentice Hall India Learning Pvt. Ltd.)
4. Numerical Linear Algebra by G. Allaire, Sidi Mahmoud Kaber, K. Trabelsi.
(Springer Publications)

MTUT125: PARTIAL DIFFERENTIAL EQUATIONS

Unit I. Introduction ToPartial Differential Equations of First Order [12 hours]

1.1 Genesis of first order P.D.E.
1.2 Compatible systems
1.3Charpit's method
1.4 Jacobi's method
1.5 Nonlinear first order P.D.E.

Unit II.Fundamental Concepts [16 hours]

2.1 First order partial differential equations
2.2 Classification of Second Order PDE
2.3 Canonical Forms:
2.3.1.Canonical Form for Hyperbolic Equation
2.3.2.Canonical Form for Parabolic Equation
2.3.3.Canonical Form for Elliptic Equation
2.4 Linear Partial Differential Equations with Constant Coefficients:
2.4.1.General Method for Finding CF of Reducible Non-homogeneous Linear PDE
2.4.2.General Method to Find CF of Irreducible Non-homogeneous Linear PDE

Unit III. Elliptic And Parabolic Differential Equations [20 hours]

3.1 Occurrence of the Laplace and Poisson Equations
3.1.1.Derivation of Laplace Equation
3.1.2.Derivation of Poisson Equation
3.2 Boundary Value Problems (BVPs)
3.3 Green's first and second identities
3.4 .Dirichlet Problem for a Rectangle
3.5 Occurrence of the Diffusion Equation
3.6 Boundary Conditions
3.7 Elementary Solutions of the Diffusion Equation
3.8 Dirac Delta Function
3.9Separation of Variables Method

Unit IV. Hyperbolic Differential Equations [12hours]

4.1Occurrence of the Wave Equation
4.2Derivation of One-dimensional Wave Equation
4.3 Solution of One-dimensional Wave Equation by Canonical Reduction
4.4 The Initial Value Problem; D'Alembert's Solution

Recommended Books:

1. An Elementary Course in Partial Differential Equations, T Amarnath,Narosa Publication. Chapter 1: 1.2, 1.6, 1.7, 1.8,1.11.
2. Intoduction to Partial Differential Equations, K.SankaraRao (Third Edition) PHI

Learning Private Limited:
Chapter 1:1.1,1.2,1.3 (1.3.1-1.3.3),1.4 (1.4.1,1.4.2) ; Chapter 2: 2.1(2.1.1,2.1.2); 2.2,2.3, 2.6, Chapter 3: 3.1- 3.5; 4.1-4.4.

Reference Books

1. Elements of Partial Differential Equations, Ian Sneddon, Dover Publication
2. An Introduction to Partial Differential Equations, YehudPinchor&Jaco Rubinstein, Cambridge University Press.

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