SYBA
SEMESTER -III

Subject: Mathematical Statistics (General-II)

ST-23243: DISCRETE PROBABILITY DISTRIBUTIONS AND TIME SERIES

Unit 1. Negative Binomial Distribution: **(10)**

1.1Probability mass function (p.m.f.)

$$P(X = x) = \binom{x+k-1}{x} p^k q^x \quad ; \quad x = 0,1,2,\ldots \; ; \; 0 < p < 1 \; ; \; q = 1 - p \; ; \; k > 0$$

$$= \quad 0 \qquad ; \quad otherwise.$$

Notation: $X \sim NB(k, p)$.

1.2 Nature of probability curve, negative binomial distribution as a waiting time distribution.

1.3 Moment generating function(MGF). Cumulant generating function(CGF), mean, variance, skewness, kurtosis (recurrence relation between moments is not expected), additive property of NB(k, p).

1.4 Relation between geometric distribution and negative binomial distribution. Poisson approximation to negative binomial distribution. Real life situations.

Unit 2. Multinomial Distribution: **(14)**

2.1Probability mass function (p.m.f.)

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_k = x_k) = \frac{n!\; {p_1}^{x_1} {p_2}^{x_2} \ldots . {p_k}^{x_k}}{x_1!\; x_2! \ldots . x_k!}; \; x_i = 0, 1, 2, \ldots, n - \sum_{1}^{i-1} x_r,$$

$$i = 1,2, \ldots . , k$$

$$x_1 + x_2 + \cdots + x_k = n;$$

$$0 < p_i < 1; \; i = 1, 2, \ldots , k;$$

$$p_1 + p_2 + \cdots + p_k = 1;$$

$$= \quad 0 \qquad ; otherwise.$$

$$Notation: (X_1, X_2, \ldots, X_k) \sim MD(n, p_1, p_2, \ldots, p_k), \; \underline{X} \sim MD(n, \underline{p}),$$

$$where \; \underline{X} = (X_1, X_2, \ldots, X_k), \qquad \underline{p} = (p_1, p_2, \ldots, p_k).$$

2.2 Joint MGF of $(X_1, X_2, \ldots, X_k)$, use of MGF to obtain means, variances, covariances, total correlation coefficients.

2.3 Variance – covariance matrix, rank of variance – covariance matrix and its interpretation.

2.4 Univariate marginal distribution, distribution of $Xi + Xj$, conditional distribution of Xi given $Xj = r$, conditional distribution of Xi given $Xi + Xj = r$, real life situations and applications.

Unit. 3 Truncated Distributions: (8)

3.1 Concept of truncated distribution, truncation to the right, left and on both sides.

3.2 Binomial distribution left truncated at $X = 0$ (value zero is discarded), its p.m.f., mean and variance.

3.3 Poisson distribution left truncated at $X = 0$ (value zero is discarded), its p.m.f., mean and variance. Real life situations and applications.

Unit.4 Time Series: (16)

4.1 Meaning and utility of time series, components of time series: trend, seasonal variations, cyclical variations, irregular (error) fluctuations or noise.

4.2 Exploratory data analysis: Time series plot to (i) check any trend & seasonality in the time series (ii) capture trend.

4.3 Methods of trend estimation and smoothing: (i) moving average, (ii) curve fitting by least square principle (Linear and second degree), (iii) exponential smoothing.

4.4 Choosing parameters for smoothing and forecasting.

4.5 Forecasting based on exponential smoothing.

4.6 Measurement of seasonal variations: i) simple average method, ii) ratio to moving average method, iii) ratio to trend where linear trend is calculated by method of least squares, (To be taken in practical).

4.7 Fitting of autoregressive model $AR\ (p)$, where $p = 1,2$.

4.8 Case studies of real life Time Series: Price index series, share price index series, economic time series: temperature and rainfall time series, wind speed time series, pollution levels.

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SYBA
SEMESTER -IV

Subject: Mathematical Statistics (General-II)

ST-23244: TESTS OF SIGNIFICANCE AND STATISTICAL METHODS

Unit.1 Tests of Significance: (6)

1.1 Random sample from a distribution as $i.i.d.$ r.vs. $X_1, X_2, X_3, \ldots\ldots, X_n$.

1.2 Statistic and Parameter, sampling distribution, standard error with illustrations.

1.3 Statistical Inference: point estimation, Estimator and estimate. Unbiased estimator (definition and illustration only).

1.4 Statistical hypothesis, null and alternative hypothesis, Simple and composite hypothesis, one sided and two sided alternative hypotheses.

1.5 Critical region, $type-I$ and $type-II$ error, level of significance, $p-value$. Two sided confidence interval.

1.6 Testing of hypothesis / Decision using critical region approach, $p-value$ approach and confidence interval approach.

Unit. 2 Tests for population means: (6)

2.1Test for single population mean $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$, $H_1: \mu > \mu_0$, $H_1: \mu < \mu_0$. (variance known)

2.2 Test for no difference of two population means $H_0: \mu_1 = \mu_2$ against $H_1: \mu_1 \neq \mu_2$,, $H_1: \mu_1 > \mu_2$,, $H_1: \mu_1 < \mu_2$,. (variances known)

2.3 Construction of two sided confidence interval for μ and $\mu_1 - \mu_2$

Unit. 3 Tests for population proportions: (6)

3.1 Test for single population proportion $H_0: P = P_0$ against $H_1: P \neq P_0$, $H_1: P > P_0$, $H_1: P < P_0$.

3.2 Test for no difference of two population means $H_0: P_1 = P_2$ against $H_1: P_1 \neq P_2$, $H_1: P_1 > P_2$, $H_1: P_1 < P_2$

3.3 Construction of two sided confidence interval for P and $P_1 - P_2$.

Unit. 4 Multiple Linear Regression Model: (16)

4.1Definition of multiple correlation coefficient $R_{Y.X_1X_2}$ Derivation of the expression for multiple correlation coefficient. Properties of multiple correlation coefficient.

i) $0 \leq R_{Y.X_1X_2} \leq 1$, ii) $R_{Y.X_1X_2} \geq \min\{r_{yx_1}, r_{yx_2}\}$.

4.2 Coefficient of multiple determination $R^2_{Y.X_1X_2}$ as proportion of variation explained by the linear regression with interpretation. Interpretation of i) $R^2_{Y.X_1X_2} = 1$ and ii) $R^2_{Y.X_1X_2} = 0$.

4.3 Partial correlation coefficient: Definition of partial correlation coefficient $r_{yx_1.x_2}$ and $r_{yx_2.x_1}$ Property of partial correlation coefficient ($-1 \leq r_{yx_1.x_2}, r_{yx_2.x_1} \leq 1$).

4.4 Notion of multiple linear regression. Yule's notation (trivariate case)(statement only). Fitting of regression plane of Y on X_1 and X_2, $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$ by the method of least squares; obtaining normal equations, solution of normal equations. Definition and interpretation of partial regression coefficients β_1 and β_2. (relations are not expected)

4.5 Residual: Definition, order, derivation of variance, properties (Only statement).

Unit. 5 Demography: **(08)**

5.1Vital events, vital statistics, methods of obtaining vital statistics, rates of vital events, sex ratios, dependency ratio.

5.2 Death/Mortality rates: Crude death rate, specific (age, sex etc.) death rate, standardized death rate (direct and indirect), infant mortality rate.

5.3 Fertility/Birth rate: Crude birth rate, general fertility rate, specific (age, sex etc.) fertility rates, total fertility rate.

5.4 Growth/Reproduction rates: Gross reproduction rate, net reproduction rate.

5.5 Interpretations of different rates, uses and applications.

5.6 Trends in vital rates as revealed in the latest census.

Unit. 6 Queuing Model: **(06)**

6.1 Introduction to $M/M/1: FIFO$ queuing model as an application of exponential distribution, Poisson distribution and geometric distribution

6.2 Inter arrival rate (λ), service rate (μ), traffic intensity ($\rho = \frac{\lambda}{\mu} < 1$), queue discipline, probability distribution of number of customers in queue, average queue length, average waiting time in queue and in system(only expression not derivation).

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