Savitribai Phule Pune University
(Formerly University of Pune)

Two Year Post Graduate Degree Programme in Mathematics
(Faculty of Science & Technology)

New Syllabi

for

M.Sc.- Mathematics Part-I & Part-II

(For Colleges Affiliated to Savitribai Phule Pune University, Pune)

(As per National Education Policy- 2020)

To be implemented from the Academic Year 2023-2024.

Preamble

The board of studies in Mathematics of Savitribai Phule Pune University, Pune made a rigorous attempt to revise the curriculum of postgraduate Programme M.Sc. to align it with National Education Policy-2020 and UGC Quality Mandate for Higher Education Institutions-2021. The process of revamping the curriculum started with the series of Meetings, workshops, webinars and discussions with sub-committees conducted by the University to orient the teachers about the key features of the Education Policy, enabling them to revise the curriculum in sync with the Policy. Appropriate orientation of the faculty about the vision and provisions of NEP-2020 made it easier for them to appreciate and incorporate the vital aspects of the Policy in the revised curriculum focused on 'creating holistic, thoughtful, creative and well-rounded individuals equipped with the skill sets of 21st century for the 'development of an enlightened, socially conscious, knowledgeable, and skilled citizen of the nation'.

With NEP-2020 in background, the revised curricula will articulate the spirit of the policy by emphasizing upon- integrated approach to learning; innovative pedagogies and assessment strategies; multidisciplinary and Interdisciplinary education; creative and critical thinking; student-centric participatory learning; imaginative abilities and flexible curricular structures to enable creative combination of disciplines for the study. The Credit framework for designing Post Graduate Programme prepared by the University as per the guidelines of State Government is followed as it is and the curriculum is further modified as per the needs specified in NEP. The curriculum is developed to trigger the inquisitiveness, discussion, analytical ability and quest for discovery among learners. Mathematics is a powerful tool for understanding and communicate globally that organizes our lives and prevents chaos, which helps us understand the world and provides an effective way of building mental discipline. Along with mathematical skills, it is also expected that students will learn life skills like argumentation, communication and general social values which are necessary to life rich, productive and meaningful life. Additionally, the knowledge of mathematical modelling and computational training which the students acquire during the Programmemakes them highly sought after. In keeping with the demands of industry and academia, the syllabus is updated regularly, with inputs taken from various stakeholders including students, alumni and parents at different stages of the modification/addition of the syllabus. The new curriculum provides a synoptic overview of possible career paths mapped by a postgraduate degree in Mathematics Teaching, Research, Engineering, Computer Programming, Statistician, Competitive examination, and many more.

******

Important Highlights

(1) **Title of the Programme**: M.A./M. Sc. (Mathematics)

(2) **Duration**: 02 years (Four semesters) Full-time Post - Graduate Degree Programme

(3) **Total number of credits**: 88 credits

(4) **Programme Structure of M.A./M.Sc. (Mathematics)**: For M.A./M.Sc. (Mathematics) Degree, a student has to earn the minimum 88 credits from at least FOUR semesters. The structure of the Programme is as follows:

(a) In each of the four semesters I, II, III, and IV, the Department will offer at least 22 credits.

(b) In each semester, there will be three mandatory courses each of 4 credits, and one elective course. Also, in each of the semesters I,II, and III, there will be a mandatory course of 2 credits.

(c) Each course of 4 credits, other than OJT and RP.

(d) A student has to attend 1-hour classroom teaching per week for one credit of theory and 2 hours lab work/problem-solving session/ related activities per week for one credit of practical.

(e) Practical sessions (lab work/problem-solving session/related activity) will be conducted in batches. A batch for such sessions will be of size maximum of 12 students.

(f) The Department may conduct necessary lectures/workshops as a part of OJT.

(I) Each course of 4 credits (T + P) will carry 100 marks and the evaluation of the course will be carried out by considering T and P Separately. There will be Continuous Assessment (CA) and End Term Examination for each course.

(II) The CA will be based on minimum two internal tests (IT). In addition, a teacher may consider one or more of the following.

(i) Home Assignment(s)

(ii) Seminar/Presentation (Individual / Group)

(iii) Laboratory assignment

(iv) Group Discussions / Oral

(v) Research Paper Review

(vi) Technology Demonstration

(III) For passing a course, a student has to score a minimum of 40% marks in each of the CA and ETE separately and a minimum of 40% marks in the combined grading of CA and ETE. If a student fails to score a minimum of 40% marks in CA or ETE in a course, then the result of such a course will be FAIL.

(IV) For both OJT and RP, the CA will be based on grades awarded by guide/mentor while the ETE will be based on presentation/oral/discussion/ any other criterion decided by

Sem I- Research Methodology (RM) - 4 credits
Sem II- On Job Training (OJT) - 4 credits
Sem III - Research Project (RP1) - 4 credits
Sem IV- Research Project (RP2)- 6 credits.

(6) **Exit Option:** After successful earning of 44 credits offered by the Department for the first two semesters (First year-I, II Sem), a student will have the option of exit from the Programme. In this case, the student will be conferred with PG Diploma in Mathematics.

(7) **ATKT Rules**: A student who wishes to take admission to the second year (register for third or fourth semester) of M. A. /M. Sc. (Mathematics) Programme must have earned at least 22 credits from the total credits of two semesters of the first year of M. A./M.Sc. (Mathematics).

(8) **Research Project (RP-1 & RP-2):**

Procedures and guidelines for the conduct of the Research project:

(a) A student is supposed to register for the course RP-1 and RP-2 separately in a group of 2 to 4 students.
(b) A student will carry out the academic activity for the course throughout the semester.
(c) The course is to be completed under the supervision and guidance of a teacher. Each teacher of the Department of Mathematics, Savitribai Phule Pune University is expected to guide at least one group of students.
(d) The respective teacher is expected to engage a group of students for at least 4 hours/week for RP-1 and at least 6 hours/week for RP-2.
(e) Every group will submit a dissertation at the end of the semester duly signed by all group members and the respective teacher.

(9) **On Job Training (OJT)** In this course, the students are expected to do the On Job Training (OJT) in appropriate Industries/Government sectors/Institute etc. to get hands on experience. The department may conduct necessary lectures/workshops/seminars as a part of OJT. The course will be conducted as per the guidelines of the College/ University and Government of Maharashtra.

(10) **Eligibility:**(B.Sc. / B. A.) Mathematics/(B. Sc./ B. E./ B. Tech.) with Mathematics subject at least at second year).

****

Programme Outcomes (POs)

Name of the Programme: M.Sc. Mathematics

PO-No.	**ProgrammeOutcomes** *The Student will be*	Component
PO-1	Capable of delivering basic disciplinary knowledge gained during the Programme.	Basic Knowledge
PO-2	Capable of describing advanced knowledge gained during the Programme	In-depth Knowledge
PO-3	Able to gain knowledge with the holistic and multidisciplinary approach across the fields.	Holistic and multidisciplinary Education
PO-4	Capable of analyzing the results critically and applying acquired knowledge to solve the problems	Critical thinking and Problem-Solving abilities
PO-5	Capable to identify, formulate, investigate and analyze the scientific problems and innovatively design and create product solutions to professional and real life problems.	Creativity and innovation
PO-6	Able to develop a research aptitude and apply knowledge to find the solution of burning research problems in the concerned and associated fields at global level.	Research aptitude and global Competency
PO-7	Able to Learn interdisciplinary and multidisciplinary skill sets and advanced techniques to apply them for better livelihood of mankind.	Skills enhancement
PO-8	able to learn and work in a groups and capable of leading a team even.	Leadership and Teamwork abilities
PO-9	Able to acquire lifelong learning skills which will lead important to better opportunities and improve quality of life.	Environmental and human health awareness
PO-10	Inculcate the professional and ethical attitude and ability to relate with social problems.	Ethical thinking and Social awareness
PO-11	Capable to establish independent start-up/innovation Centre etc.	Lifelong learning skills and Entrepreneurship

******

Programme Specific Outcomes (PSOs)

PSO. No.	**ProgrammeSpecific Outcomes** *The student-*
PSO-1	will have a strong foundation in both pure and applied mathematics.
PSO-2	will have the knowledge of the fundamental axioms in mathematics and capability of developing ideas based on them and inculcate mathematical reasoning.
PSO-3	will be able to apply mathematical skills for solving problems and can prepare himself forvarious competitive exams.
PSO-4	will acquire the knowledge of a wide range of mathematical techniques and application of mathematical methods/tools in science, social science, engineering and technology
PSO-5	will have basic knowledge of Programming and computational techniques as required for employment.
PSO-6	will be able to develop analytical skills, critical thinking, creativity, communication, and presentation skills through assignments, seminars, Training, and Research project.

Course Structure of the Programme: M.Sc./ M.A. Mathematics Part- I
Approved by B.O.S.
2023-24

Year	Level	Sem.	Course Type	CourseCode	CourseTitle	Remark	Credit	No. of Hours
1	6.0	I	Core	MTS-501MJ	LinearAlgebra	Theory	**2**	30
			Core	MTS-502 MJP	Practical Based on Linear Algebra	Practical	**2**	60
			Core	MTS-503 MJ	GroupTheory	Theory	**4**	60
			Core	MTS-504 MJ	Ordinary differential Equations	Theory	**4**	60
			Core	MTS-505 MJ	Programming withPython	Practical	**2**	60
			Elective (Choose Any one)	MTS-511(A)MJ	Advanced Numerical Analysis	Theory	**2**	30
				MTS-512(A) MJP	Practical Based on Advanced Numerical Analysis	Practical	**2**	60
				MTS-511(B)MJ	Number Theory	Theory	**2**	30
				MTS-512(B) MJP	Practical Based on Number Theory	Practical	**2**	60
				MTS-511(C)MJ	Combinatorics	Theory	**2**	30
				MTS-512(C)MJP	Practical Based on Combinatorics	Practical	**2**	60
				MTS-511(D)MJ	Lattice Theory	Theory	**2**	30
				MTS-512(D) MJP	Practical Based on Lattice Theory	Practical	**2**	60
			RM	MTS-508 RM	Research Methodology	Theory	**2**	30
				MTS-509 RMP	Practical based on Research Methodology	Practical	**2**	60

		II	Core	MTS-551MJ	Topology	Theory	**2**	30
			Core	MTS-552 MJP	Practical Based on Topology	Practical	**2**	60
			Core	MTS-553 MJ	Ring Theory	Theory	**4**	60
			Core	MTS-554 MJ	AdvancedCalculus	Theory	**4**	60
			Core	MTS-555 MJP	Data Science	Practical	**2**	60
			Elective (Choose any one)	MTS-561(A)MJ	GraphTheory	Theory	**2**	30
				MTS-562(A)MJP	Practical Based on Graph Theory	Practical	**2**	60
				MTS-561 (B)MJ	DynamicalSystems	Theory	**2**	30
				MTS-562(B)MJP	Practical Based on Dynamical Systems	Practical	**2**	60
				MTS-561(C)MJ	CodingTheory	Theory	**2**	30
				MTS-562 (C)MJP	Practical Based onCodingTheory	Practical	**2**	60
				MTS-561(D)MJ	Operations Research	Theory	**2**	30
				MTS-562(D)MJP	Practical Based On Operations Research	Practical	**2**	60
			OJT/FP	MTS-581 MJP	On Job Training or Field Project	Practical	**4**	120

Course Structure of the Programme: M.Sc./M.A. Mathematics Part- II

2024-25

Year	Level	Semester	Course Type	Course Code	CourseTitle	Remark	Credit	No. of Hours
2	6.5	III	Core	MTS-601MJ	Complex Analysis	Theory	**2**	30
			Core	MTS-602 MJP	Practical Based on Complex Analysis	Practical	**2**	60
			Core	MTS-603 MJ	Field Theory	Theory	**4**	60
			Core	MTS-604 MJ	Differential Geometry	Theory	**4**	60
			Core	MTS-605 MJ	Machine learning using Python	Practical	**2**	60
			Elective (Choose any one)	MTS-611(A)MJ	Mathematical Statistics	Theory	**2**	30
				MTS-612(A)MJP	Practical Based on Mathematical Statistics	Practical	**2**	60
				MTS-611(B)MJ	Algebraic Topology	Theory	**2**	30
				MTS-612(B)MJP	Practical Based on Algebraic Topology	Practical	**2**	60
				MTS-611(C)MJ	Integral Transforms and Special Functions	Theory	**2**	30
				MTS-612(C)MJP	Practical Based on Integral Transforms and Special Functions	Practical	**2**	60
				MTS-611(D)MJ	Mechanics	Theory	**2**	30
				MTS-612(D)MJP	Practical Based on Mechanics	Practical	**2**	60
			RP	MTS-631 MJ	Research Project	Practical	**4**	120
			Core	MTS-651MJ	Functional Analysis	Theory	**2**	30
				MTS-652 MJP	Practical based on Functional Analysis	Practical	**2**	60
			Core	MTS-653 MJ	Partial Differential Equations with Boundary Value Problems	Theory	**4**	60

		IV	Core	MTS-654 MJ	Measure Theory and Integration	Theory	**4**	60
			Elective (Choose any one)	MTS-661(A)MJ	Commutative Algebra	Theory	**2**	30
				MTS-662(A)MJP	Practical based on Commutative Algebra	Practical	**2**	60
				MTS-661(B)MJ	Financial Mathematics	Theory	**2**	30
				MTS-662(B)MJP	Practical based on Financial Mathematics	Practical	**2**	60
				MTS-661(C)MJ	Algebraic Curves	Theory	**2**	30
				MTS-662(C)MJP	Practical based on Algebraic Curves	Practical	**2**	60
				MTS-661(D)MJ	Optimization Techniques	Theory	**2**	30
				MTS-662(D)MJP	Practical based on Optimization Techniques	Practical	**2**	60
			RP	MTS-681 RP	ResearchProject	Practical	**6**	180

****

Details of Syllabus:

M.Sc./M.A. Mathematics Part- I

Semester-I

MTS-501MJ, MTS 502 MJP: LINEAR ALGEBRA

Course Objectives:

1. To make students understands the concept of Vectors and Vector Spaces
2. To impart the Applications of Eigen values and Eigen Vectors in various fields like Cryptography, Economics, Computer Graphics etc.
3. To make students understand the applicability of Matrices in Linear Algebra
4. To teach the Utility / Practical use of Linear Transformation
5. To make student aware about Linear Algebra and its Applications
6. To interpret the various forms of data using Linear Algebra

Course Outcomes:

1. Can imagine the results of basic operations on vectors in $\mathbb{R}^2, \mathbb{R}^3$ geometrically and differentiate between Finite and Infinite Dimensional Vector Spaces
2. Can differentiate between Eigen Values and Eigen Vectors along with its Applications along with real life examples of the difference between Linear and Non – Linear Transformation
3. Can recognize the invariant and Non – invariant subspaces under the given linear operator
4. Can apply the Cayley – Hamilton Theorem to calculate the values of matrices that are raised to a large exponent
5. Can understand the construction of Inner Product Spaces and Bilinear spaces
6. Can tell the applications of Linear Algebra to real life

Course Contents:

Unit 1. Vector Spaces **[12 Hours]**

1.1 Vector Spaces
1.2 Subspaces
1.3 Bases and Dimension
1.4 Coordinates
1.5 Applications in Cryptography
1.6 Applications in Economics

Unit 2.Linear Transformation **[15 Hours]**

2.1 Linear Transformations
2.2 The Algebra of Linear Transformations
2.3 Isomorphism
2.4 Representation of Transformations by Matrices
2.5 Linear Functionals
2.6 The Double Dual

Unit 3.Elementary Canonical Forms **[15 Hours]**

3.1 Introduction
3.2 Characteristic Values
3.3 Annihilating Polynomials
3.4 Invariant Subspaces and Diagonalization

Unit 4.The Rational and Jordan Forms **[15 Hours]**

4.1 Cyclic Subspaces and Annihilators
4.2 Cyclic Decomposition and the Rational Form
4.3 The Jordan Form

Unit 5.Inner Product Spaces **[15 Hours]**

5.1 Inner Products
5.2 Inner Product Spaces
5.3 Linear Functionals and Adjoints
5.4 Unitary Operators
5.5 Normal Operators

Unit 6 Bilinear Forms **[18 Hours]**

6.1 Bilinear Forms
6.2 Symmetric Bilinear Forms
6.3 Skew – Symmetric Bilinear Forms
6.4 Application to Input-Output Economic Models
6.5 Application to Markov Chain

Recommended Book:

1. **K. Hoffman and R. Kunze**, Linear Algebra, 2nd Edition [Prentice Hall of India Pvt. Ltd.]
   Unit 1: 2.1 – 2.4, Unit 2: 3.1 – 3.6, Unit 3: 6.1 – 6.4, Unit 4: 7.1 – 7.3, Unit 5: 8.1 – 8.5,
   Unit 6: 10.1 – 10.3
2. **Applications:**
   1.5 [2.5 Applied Finite Mathematics by R. Sekhon and R. Bloom, Libre Texts]
   1.6 [2.6 Applied Finite Mathematics by R. Sekhon and R. Bloom, Libre Texts]
   6.4 [2.8 LINEAR ALGEBRA with Applications, by W. Keith Nicholson]
   6.5 [2.9 LINEAR ALGEBRA with Applications, by W. Keith Nicholson]

Reference Books:

1. **P B Bhattacharya, S K Jain and S R Nagpaul** :A First Course in Linear Algebra [New Age International Publishers].
2. **Vivek Sahai and Vikas Bist** Linear Algebra [Narosa Publishing House].
3. **S. Kumaresan**Linear Algebra: A Geometrical Approach [PHI Learning Pvt., Ltd.]
4. **A. R. Rao and P. Bhimasankaram** Linear Algebra [Tata McGraw – Hill Publishing Company Ltd., New Delhi]
5. **Kuldeep Singh,** Linear Algebra Step by Step [Oxford University Press]

MTS-503 MJ: GROUP THEORY

Course Objectives: *Students will*

1. learn group and subgroup.
2. learn cyclic group and properties of permutations.
3. learn external direct product of a finite number of groups.
4. learn group homomorphism and Cayley's theorem.
5. learn isomorphism and their properties.
6. learn Sylow theorems and their applications.

Course Outcomes:

1. Recognize the mathematical objects that are groups, and classify them as abelian, cyclic and permutation groups, etc.
2. Identify the various algebraic structures with their corresponding binary operations.
3. Learn about structure preserving maps between groups and their consequences.
4. Analyze consequences of Lagrange's theorem.
5. Apply Sylow theorems for groups of finite orders.
6. Explain the significance of the notion of cosets, normal subgroups, and factor groups.

Course Contents:

Unit 1. Introduction to Groups **[16 Hours]**

1.1 Basic Axioms and Examples.
1.2 Dihedral Groups
1.3 Symmetric Groups
1.4 Matrix Groups
1.5 The Quaternion Group.
1.6 Homomorphism and Isomorphisms.
1.7 Group Actions.

Unit 2. Subgroups **[12 Hours]**

2.1 Definition and Examples.
2.2 Centralizers and Normalizers, Stabilizers and Kernels.
2.3 Cyclic Groups and Cyclic Subgroups.
2.4 Subgroups Generated by Subsets of a Group.
2.5 The Lattice of Subgroups of a Group.

Unit 3 Quotient Groups and Homomorphisms **[10 Hours]**

3.1 Definition and Examples.
3.2 More on Cosets and Lagrange's Theorem.
3.3 The Isomorphism Theorems.
3.4 Composition Series.
3.5 Transpositions and Alternating Group.

Unit 4 Sylow Theorems and Group Actions [08 Hours]

4.1 Group Actions and Permutation Representations.

4.2 Group Acting on Themselves by Left Multiplication – Cayley's Theorem.

4.3 Group Acting on Themselves by Conjugation – The Class Equation.

4.4 Automorphisms.

4.5 The Sylow Theorems.

4.6 The Simplicity of A_n.

Unit 5 Direct and Semidirect Products and Abelian Groups [14 Hours]

5.1 Direct Products.

5.2 The Fundamental Theorem of Finitely Generated Abelian Groups.

5.3 Table of Groups of Small order.

5.4 Recognizing Direct Products

5.5 Semidirect Product.

Recommended Book:

David S. Dummit, Richard M. Foote, Abstract Algebra, 3rd Edition, John Wiley and Sons (Indian Edition).Chapter-1, 2, 3 (Expect Holder Programme), 4, 5.

Reference Books:

1. **Joseph Gallian**, Contemporary Abstract Algebra, 9th Edition, Cengage Learning India Pvt. Ltd. ISBN-109360502527
2. **P.B. Bhattacharya, S.K.Jain and S.R. Nagpaul,** Basic Abstract Algebra (Cambridge University Press, Second Edition), 1995 (Indian Edition).
3. **I. S. Luthar, I. B. S. Passi**, Algebra (Vol 1), Groups; Narosa Publication House.
4. **I. N. Herstein**, Topics in Algebra, Wiley Eastern Ltd.
5. **J. B. Fraleigh**, A First Course in Abstract Algebra, 7th Edition, Pearson EditionLtd.

MTS-504 MJ: ORDINARY DIFFERENTIAL EQUATIONS

Course Objectives:

1. This course aims at providing knowledge of ordinary differential equations.
2. A student should be able to apply skills and knowledge to translate information presented verbally into differential equations form. Analyze and use appropriate methods to obtain the solution.
3. A student should get adequate exposure to global and local concerns that explore many aspects of differential equations.
4. A student should be able to solve the mathematical models occurring in various fields which contain differential equations.

Course Outcomes:

1. Students are able to find solutions of linear equations of first order.
2. Students can find solutions for homogeneous and non-homogeneous equations of second order.

3. Explain Euler's equation, Legendre's equation, and Bessel's equation.
4. Understand the Existence and Uniqueness of solutions.
5. Students learn a system of differential equations.

Course Contents:

Unit 1 : Linear equations with constant coefficients: **[14 Hours]**

1.1 Linear equations of the first order
1.2 The equation y'+ ay=0
1.3 The equation y'+ ay=b(x)
1.4 The general linear equations of first order
1.5 Second order homogeneous equations
1.6 Initial value problems for second order equations
1.7 Linear dependence, and independence
1.8 Formula for the Wronskian
1.9 Non-homogeneous equations of order two
1.10 Homogeneous equations of order n
1.11 Non-homogeneous equations of order n
1.12 Algebra of constant coefficients equations

Unit 2:Linear equations with variable coefficients: **[12 Hours]**

2.1 Initial value problems for the homogeneous equation
2.2 Solutions of the homogeneous equation
2.3 Wronskian and linear independence
2.4 Reduction of the order of the homogeneous equation
2.5 non-homogeneous equations
2.6 Homogeneous equations with analytic coefficients
2.7 The Legendre equation

Unit 3 Linear Equations with regular singular points: **[10 Hours]**

3.1 The Euler equation
3.2 Second order equation with regular singular points
3.3 The exceptional cases
3.4 The Bessel equation
3.5 Regular singular point at infinity

Unit 4 Existence and uniqueness of solutions to first-order equations: **[10 Hours]**

4.1 Equations with variables separated
4.2 Exact equations
4.3 Method of successive approximations
4.4 Lipschitz condition
4.5 Approximation to, and Uniqueness of solutions

Unit 5 System of First-Order Equations: **[14 Hours]**

5.1 First-order systems
5.2 Linear first-order systems
5.3 Constant systems – eigenvalues and eigenvectors
5.4 Nonhomogeneous systems
5.5 System as a vector equation
5.5 Existence and uniqueness of solutions of a system
5.6 Existence and uniqueness of linear systems

Recommended Book:

1. **E. A. Coddington**, An Introduction to Ordinary Differential Equations(Prentice-Hall).
   Unit 1 : Chapter 1: sections 4 to 7, Chapter 2: sections 1 to 7, 10, 12.
   Unit 2 : Chapter 3 : sections 1 to 8. Unit 3 : Chapter 4: sections 1 to 3 , 6, 7, 9.
   Unit 4 : Chapter 5 : sections 1 to 5 , 8. Unit 5: Chapter 6 : section 1, 5, 6, 7.
2. **Shair Ahmad Antonio Ambrosetti**, A Text Book on Ordinary Differential Equations, Second Edition, Springer Unit 5: Chapter 7: 7.1 (revision) 7.2 to 7.6

Reference Books:

G.F. Simmons and S.G. Krantz, Differential Equations(Tata McGraw-Hill).

****

MTS-505 MJP: PROGRAMMING WITH PYTHON

:Course Objectives:

1. Prime objective is to give students a basic introduction to Programming and problem solving with computer language Python. And to introduce students not merely to the coding of computer Programmes, but to computational thinking, the methodology of computer Programming, and the principles of good Programme design including modularity and encapsulation.
2. To understand problem solving, problem solving aspects, Programming and to know about
3. various Programme design tools.
4. To learn problem solving with computers.
5. To learn basics, features and future of Python Programming.
6. To acquaint with data types, input output statements, decision making, looping and
7. functions in Python.

Course Outcomes:

1. Inculcate and apply various skills in problem solving.
2. Choose most appropriate Programming constructs and features to solve the problems in diversified domains.
3. Exhibit the Programming skills for the problems those require the writing of well-documented Programmes including use of the logical constructs of language, Python.
4. Demonstrate significant experience with the Python Programme development environment.
5. To learn features of Object Oriented Programming using Python.
6. To acquaint with the use and benefits of files handling in Python.

Course Contents:

Unit 1 Revision **[12Hours]**

Variables and Simple Data Types

1.1 Variables
1.2 Strings
1.3 Numbers Introducing Lists
1.4 What is a List?
1.5 Changing, Adding and Removing Elements
1.6 Organizing a List
1.7 Looping Through an Entire List
1.8 Avoiding Indentation Errors
1.9 Making Numerical Lists
1.10 Working with Part of a List
1.11 Tuples If Statements
1.11 Conditional Tests
1.12 If Statements
1.13 Using if Statements with Lists Dictionaries
1.14 Working with Dictionaries
1.15 Looping Through a Dictionary
1.16 Nesting

Unit 2: User Input and While Loops **[12 Hours]**

2.1 How the input () Function works
2.2 Introducing while Loops
2.3 Using a while Loop with Lists and Dictionaries

Unit 3 :Functions **[12 Hours]**

3.1 Defining a Function
3.2 Passing Arguments
3.3 Return Values
3.4 Passing a List
3.5 Passing an Arbitrary Number of Arguments
3.6 Storing Your Functions in Modules

Unit 4 :Classes **[12 Hours]**

4.1Creating and using a Class
4.2 Working with Classes and Instances
4.3 Inheritance
4.4 Importing Classes

Unit 5 :Files and Exceptions **[12 Hours]**

5.1Reading from a File
5.2 Writing to a File
5.3 Exceptions
5.4 Storing Data

Recommended Book:

Eric Matthes,Python Crash Course 2nd Edition.

Chap 2-10.

Reference Books:

1. **Magnus Lie Hetland**Beginning Python, From Novice to Professional Third Edition
2. **Michael T. Goodrich, Roberto Tamassia, Michael H. Goldwasser**: Data Structures & Algorithms in Python

****

MTS-511 (A)MJ , 512(A)MJP : ADVANCED NUMERICAL ANALYSIS

Course Objectives:

1. To introduce the numerical techniques for solving the rootfinding problems, systems of linear equations.
2. To comprehend the numerical techniques for approximating the eigenvalues and eigenvectors of $n \times n$ matrix.
3. To introduce the polynomial interpolation , numerical techniques of differentiation and integration.
4. To acquaint the students with the knowledge of various numerical techniques and methods of solving ordinary differential equations.

Course Outcomes:

1. Solve the rootfinding problems with an arbitrary nonlinear function.
2. Use the direct and iterative techniques for the solution of systems of linear algebraic equations.
3. Determine the value of the interpolating polynomial at a single value of the independent variable.
4. Apply the numerical techniques of differentiation and integration for engineering problems.
5. Understand various numerical techniques for approximating the solution of initial value problems of ordinary differential equations.

Course Contents:

Unit 1. Root Finding Methods: [12 Hours]

1.1 Convergence; Floating Point Number Systems; Floating Point Arithmetic.

1.2 Fixed Point Iteration Schemes; Newton's Method; Secant Method; Accelerating Convergence.

Unit 2. System of Equations: [20 Hours]

2.1 Gaussian Elimination; Pivoting Strategies, Vector and matrix norms.

2.2 Error Estimates and Condition Number; LU decomposition; Direct Factorization.

2.3 Iterative Techniques for Linear Systems: Basic Concepts and Methods.

Unit 3. Eigenvalues and Eigenvectors [12 Hours]

3.1 The Power Method

3.2 The Inverse Power Method.

3.3 Reduction to Symmetric Tridiagonal Form.

3.4 Eigenvalues of Symmetric Tridiagonal Matrices.

Unit 4. Interpolation (and curve fitting **[12 Hours]**

4.1 Lagrange Form Of The Interpolating Polynomial

4.2 Neville's Algorithm

4.3 The Newton Form Of The Interpolating Polynomial

4.4 Optimal Points For Interpolation

Unit 5. Differentiation and Integration **[16 Hours]**

5.1Numerical Differentiation, Part II

5.2 Numerical Integration – The Basics and Newton-Cotes Quadrature; Composite Newton-Cotes Quadrature

Unit 6. Initial Value Problems of Ordinary Differential Equations **[18 Hours]**

6.1 Euler's Method; Higher-Order One-Step Methods: Taylor Methods.

6.2 Runge-Kutta Methods.

6.3 Multistep Methods.

6.4 Convergence and Stability Analysis.

Recommended Book:

Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Prentice Hall, Eleventh impression, ISBN 978-81-317-0942-9.

Sections: 1.2 – 1.4, 2.3 – 2.6, 3.1 – 3.6, 3.8, 4.1, 4.2, 4.4, 4.5, 5.1–5.4, 6.2, 6.4, 6.5, 7.2 – 7.6

Reference Books:

1. **K.E. Atkinson**, An Introduction to Numerical Analysis, Second Edition, John Wiley & Sons.
2. **J. L. Buchanan, P. R. Turner**, Numerical Methods and Analysis,McGraw-Hill, New York, N.Y., 1992 cop
3. **M.K. Jain, S.R.K. Iyengar, R.K. Jain,** Numerical Methods for Scientific &Engineering Computation, New Age International, 5th edition.
4. **Grewal, B.S., and Grewal, J.S.,** "Numerical Methods in Engineering and Science", Khanna Publishers, 10th Edition, New Delhi, 2015.
5. **J.N. Sharma**, Numerical Methods for Engine
6. **J.N. Sharma**, Numerical Methods for Engineers and Scientists, Alpha Science, second edition.

****

MTS-511(B)MJ , MTS-512(B)MJP : NUMBER THEORY

Course Objectives:

- ❖ Recall, define basic concepts of set of integers and divisibility.
- ❖ Discuss congruence relation and its applications and use different techniques of numerical calculations.
- ❖ Learn different number theoretic functions, and use it for solving problems.
- ❖ Understanding Gauss Reciprocity and its use in solving problems.
- ❖ Solving linear Diophantine equations and linear congruences.
- ❖ Studying Algebraic Number Fields, Algebraic Integers, Quadratic Fields.

Course Outcomes:

1. The basics of divisibility and modular arithmetic.
2. And be able to solve congruence problems for degree one and some special cases of degree two, using the Chinese Remainder Theorem.
3. The concept of Legendre and Jacobi symbol, quadratic residues and law of quadratic reciprocity.
4. The different number theoretic functions.
5. The method of solving linear Diophantine equations.
6. The concept of Algebraic number fields, Algebraic integers, Quadratic Fields.

Course Contents:

Unit 1. Divisibility **[9 Hours]**

1.1 Introduction
1.2 Divisibility
1.3 Primes

Unit 2. Congruences **[9 Hours]**

2.1 Congruences
2.2 Solution of Congruences
2.3 The Chinese Remainder Theorem

Unit 3. Some functions of Number Theory **[15 Hours]**

3.1 Greatest integer function
3.2 Arithmetic functions
3.3 The Mobius Inversion formula

Unit 4. Quadratic Reciprocity **[18 Hours]**

4.1 Quadratic residues
4.2 Quadratic reciprocity
4.3 The Jacobi Symbol

Unit 5. Diophantine Equations **[09 Hours]**

5.1 Diophantine equations $ax + by = c$
5.2 Pythagorean triplets.

Unit 6. Algebraic Numbers **[30 Hours]**

6.1 Algebraic Numbers
6.2 Algebraic Number Fields
6.3 Algebraic Integers
6.4 Quadratic Fields

Recommended Books:

Niven, H. Zuckerman and H.L. Montgomery, An Introduction to Theory of Numbers, 5th Edition, John Wiley and Sons. (§1.1- §1.3, §2.1- §2.3, §3.1- §3.3, §4.1 -§4.3, §5.1 and §5.3, §9.1- §9.5)

Reference Books:

1. **David M. Burton**, Elementary Number Theory (Second Ed.), Universal Book Stall, New Delhi, 1991.
2. **K.IrelandandM.Rosen**,AClassicalIntroductiontoModernNumberTheory(SecondEdition,Springer)

****

MTS-511(C) MJ & 512(C)MJP : COMBINATORICS

Course Objectives:

1. A student should be able to understand counting methods for arrangements and selections.
2. A student should acquire sufficient competencies to solve problems on Binomial identities, distribution problems etc.
3. Able to explain various counting principles and Binomial Identities.
4. A student should acquirethe knowledge of the Inclusion-Exclusion Principle to solve combinatorial problems.
5. A student should able to use recurrence relation.

Course Outcomes:

1. To learn general counting methods for arrangements and selection.
2. Test and validate Binomial identities, distribution problems. Explain various counting principles and Binomial Identities to solve different problems.
3. Students will able to use generating functions and recurrence relations to solve problems.
4. Apply Inclusion-Exclusion Principle to solve combinatorial problems.
5. Understand inclusion-exclusion with Venn diagrams and inclusion – exclusion formula.
6. To learn restricted positions and Rook polynomials.

Course Contents:

Unit 1. General Counting Methods for Arrangements and Selections [20 Hours]

1.1Two Basic Counting Principles.
1.2 Simple Arrangements and Selections.
1.3 Arrangements and Selections with Repetitions.
1.4 Distributions.
1.5 Binomial Identities

Unit 2. Generating Functions [20 Hours]

2.1 Generating Functions Models.
2.2 Calculating Coefficients of Generating Functions.
2.3 Partitions.
2.4 Exponential Generating Functions.
2.5 A Summation Method

Unit 3. Recurrence Relations **[25 Hours]**

3.1 Recurrence Relations Models.
3.2 Divide and Conquer relations
3.3 Solutions of Linear Recurrence Relations.
3.4 Solution of Inhomogeneous Recurrence Relations.
3.5 Solution with Generating Functions

Unit 4. Inclusion-Exclusion **[25 Hours]**

4.1 Counting with Venn Diagrams.
4.2 Inclusion-Exclusion Formula.
4.3 Derangements and Simple Examples
4.4 Restricted Positions and Rook Polynomials.
4.5 Pigeonhole Principle and Examples.

Recommended Book:

Alan Tucker,Applied Combinatorics (Sixth Edition), John Wiley & Sons, New York (2012)
Sections: - 5.1-5.5, 6.1-6.5, 7.1-7.5, 8.1-8.3, Appendix-4.

Reference books:

1. **V. Krishnamurthy**, Combinatorial, Theory and Applications, East West Press, New Delhi (1989) Scientific, (1996)
2. **Chen Chaun-Chong, Koh Khee-Meng** : Principles and Techniques in Combinatorics (World Scientific)
3. **Kenneth Rosen**: Discrete Mathematics & It's Applications, Tata Mc-Graw Hill
4. **K. D. Joshi**: Foundations of Discrete Mathematics, Wiley

****

MTS-511(D)MJ , 512(D)MJP : LATTICE THEORY

Course Objectives:

1. To introduce partially ordered sets and their properties.
2. To understand Lattices as algebraic structures
3. Homomorphism between lattices and Boolean algebra.
4. To understand Stone lattices.

Course Outcomes:

1. Recall the algebra and ordered properties to understand POSETs and lattices.
2. Understand different types of lattices.
3. Apply Dedekind`s and Birkhoff's criterias.
4. Analyze Stone lattices.
5. Explain the use of homomorphisms.
6. Create examples and counter-examples of different types of lattices.

Course Contents:

Unit 1. Partially Ordered Sets and Lattices **[25 Hours]**

1.1 Two definitions of lattices, Hasse diagrams,
1.2 Homomorphism, isotone maps, ideals,
1.3 Congruence relations, congruence lattices,
1.4 The isomorphism theorem, product of lattices, complete lattice, ideal lattice,
1.5 Distributive–modular inequalities and identifies,
1.6 Complements, pseudo-complements
1.7 Boolean lattice of pseudo-complements, join and meet-irreducible elements

Unit 2. Characterization theorems and representation theorems **[25 Hours]**

2.1 Dedekind`s modularity criterion Birkhoff`s distributivity criterion,
2.2 Hereditary subsets, rings of sets
2.3 Stone theorems, Nachbin theorem, statements of Hashimoto's theorem.

Unit 3. Distributive Lattices **[20 Hours]**

3.1 Distributive lattices with pseudo-complementation,
3.2 Stone lattices, Characterization of stone lattices, Stone Algebra,
3.3 Characterization of Stone Algebra

Unit 4. Modular Lattices: **[20 Hours]**

4.1 Modular lattices, isomorphism theorem,
4.2 Upper and lower covering conditions,
4.3 Kuros-Ore theorem, independent sets.

Recommended Books:G. Gratzer, General Lattice Theory, Birkhauser, 2nd Edition 1998.
Section 1.1-1.4, 1.6, 2.1, 2.6, 4.1

Reference Books:

1. **Birkhoff, G.**, Lattice Theory, Amer. Math. Soc., Providence, R. I. Vol. 25, Third Edition, 1967.
2. **Blyth, T. S.,** Lattices and Ordered Algebraic Structures, Springer-Verlag, London, 2005.
3. **Davey, B. A. and Priestley, H. A.,** In Introduction to Lattices and Order, Cambridge University Press, Cambridge, 2002.
4. **Stern, M.,**Semimodular Lattices, Theory and Applications, Cambridge University Press, 1999.

******

MTS-508 RM & 509 RMP : RESEARCH METHODOLOGY

Course Description

The Research Methodology course is designed to equip students in Mathematics with the essential skills and knowledge required to conduct rigorous and effective research in their field. This course provides an overview of various research methods, techniques, and tools commonly used in mathematical research, with an emphasis on developing critical thinking, problem-solving abilities, and research ethics. Students will also gain hands-on experience in formulating research questions, designing experiments, analyzing data, and presenting and writing research findings.

Course Outcomes:*the student will*

- develop a comprehensive understanding of different research methodologies and their applications in mathematics.
- cultivate critical thinking and analytical skills necessary for identifying research problems and formulating research questions.
- provide practical experience in designing experiments, collecting and analyzing data, and interpreting research results.
- foster effective communication skills for presenting research findings orally and in written form.
- promote ethical research practices and awareness of responsible conduct in mathematical research.

Course Contents:

Unit 1. Foundations of Research: [15 Hours]

Meaning, Objectives, Motivation, Utility, Concept of theory, Research Problem Identification, Developing a Research Plan – Exploration, Description, Diagnosis, Experimentation, Determining Experimental and Sample Designs. Writing of Proofs, quantifiers etc.

Unit 2. Research Design: [15 Hours]

Defining research objectives and questions, Analysis of Literature Review – Primary and Secondary Sources, Web sources for critical Literature Review such as MathSciNet, ZMATH, Scopus, Web of Science, reviewing literature and identifying research gaps.

Unit 3. Research Methods: [15 Hours]

Scientific methods, Logical Methods: Deductive, Inductive, logical methods. Quantitative research methods, Qualitative research methods, Data Collection Techniques, Surveys and questionnaires, Interviews and focus groups, Observations and case studies, Experimental methods, Data Analysis and Interpretation, Statistical analysis techniques in mathematics, Qualitative data analysis methods, Visualization and interpretation of results.

Unit 4. Research Writing and Presentation: [15 Hours]

Scientific/ technical Writing Structure and Components, Importance of Effective Communication. Preparing Research papers for journals, Seminars and Conferences – Design of paper using TEMPLATE, Calculations of Impact factor of a journal, citation Index, ISBN & ISSN. Preparation of Project Proposal – Time frame and work plan – Budget and Justification – Preparation and Publication of Research paper, Thesis writing. Project Reports for various funding, Writing Statement of Purpose for PhD/Post Doc etc, Writing a review of paper, Presenting research findings orally and visually, Research Collaboration and Communication, Collaborative research practices, Effective communication in mathematical research, Participating in conferences and seminars,

Unit 5. Research Ethics and Responsible Conduct: [15 Hours]

Ethics and Ethical Issues – Ethical Committees – Commercialization – copy right – royalty – Intellectual Property rights and patent law – Track Related aspects of intellectual property Rights – Reproduction of published material – Plagiarism and software to detect plagiarism– Citation and Acknowledgement – Reproducibility and accountability.

Unit 6. Mathematical Software and Paraphrasing Software: [15 Hours]

Basic Latex, Beamer, Overleaf, Grammarly, QuillBot, ChatGPT, and SAGE. Particularly, introduction to SAGE: Overview of the SAGE software, installation, and user interface. Basic Algebraic Manipulations: Symbolic algebra, equations, simplifications, and algebraic manipulations. Calculus Computations: Differentiation, integration. Linear Algebra with SAGE: Matrix operations, solving linear systems, eigenvalue calculations. Discrete Mathematics with SAGE: Combinatorics, graph theory, number theory, and cryptography.

Course Assessment:

The course assessment will be done at the college/institute level that includes but is not limited to a combination of the following methods:

- Research proposals and progress reports
- Research presentations
- Critical analysis of published mathematical research papers
- Participation in class discussions and activities
- Final research project or paper

Note: The syllabus provided above is a general outline and can be adapted and expanded based on the specific requirements of the institution offering this subject in Mathematics Programme and the expertise of the instructor.

References:

- **Kothari, C.R.**(2008), Research Methodology: Methods and Techniques. Second Edition. New Age International Publishers, New Delhi.
- **Dilip Datta,** LaTeX in 24 Hours, A Practical Guide for Scientific Writing, Springer
- **Eva O. L. Lantsoght,** The A-Z of the PhD Trajectory -A Practical Guide for a Successful Journey, Springer Cham, 2018.

****

Semester- II

MTS-551 MJ & 552 MJP: TOPOLOGY

Course Objectives:

1. Explore the foundations of mathematics (logic and set theory) at a level and depth appropriate for someone aspiring to study higher-level mathematics and to become a professional mathematician.
2. Understand open sets in general context.
3. Present an introduction to the field of topology, with emphasis on those aspects of the subject that are basic to higher mathematics.
4. Understand Topological properties of a space.

Course Outcomes:

1. Understand terms, definitions and theorems related to topological spaces
2. Demonstrate knowledge and understanding of concepts such as open and closed sets, interior, closure and boundary, connectedness, compactness, countability and separation axioms.
3. Create new topological spaces from existing topological spaces.
4. Use continuous functions and homeomorphisms to understand structure of topological spaces
5. Understand difference and interrelationship between Metric Spaces and Topological Spaces.
6. Apply theoretical concepts in topology to understand real world applications.

Course Contents:

Unit 1. Topological Spaces **[27 Hours]**

1.1 Infinite Sets and Axiom of Choice
1.2 Well Ordered Sets
1.3 Topological Spaces
1.4 Basis for a Topology
1.5 Order Topology
1.6 Product Topology on $X \times Y$
1.7 Subspace Topology

Unit 2. Continuous Functions **[18Hours]**

2.1 Closed Sets and Limit Points
2.2 Continuous Functions
2.3 The Product Topology
2.4 Metric Topology
2.5: Metric Topology(continued)

Unit 3.Connected and Compact Spaces **[25Hours]**

3.1 Connected spaces
3.2 Connected Subspaces of Real Line
3.3 Components and Local Connectedness

3.3 Compact spaces
3.4 Compact Subspaces of the Real Line
3.5 Limit point compactness
3.6 : Local Compactness

Unit 4.Countably and Separation Axioms **[20 Hours]**

4.1 Countability Axioms
4.2 Separation axioms
4.3 Normal Spaces
4.4 Urysohn Lemma
4.5 Tietze Extension Theorem
4.6 The UrysohnMetrization Theorem
4.7 Tychonoff's Theorem

Recommended Book :J. R. Munkres, *Topology: A First Course*, (Pearson Prentice Hall, third edition), 2007.

Ch1: Sec. 9, 10. Ch 2: Sec.12 to 21. Ch3: Sec. 23 to 29. Ch 4: Sec. 30 to 35. (statement with applications) . Ch 5: Sec. 37.

Reference Books:

1. **K. D. Joshi**, Introduction to General Topology - (New Age International)
2. **J. L. Kelley.**General Topology - (GTM, Springer,1975)
3. **L. A. Steen and J. A. Seebach**Counterexamples in Topology -. (Springer)
4. **C. Adams and R. Franzosa**Introduction to Topology, Pure, and Applied (Pearson , 2009).

****

MTS-553 MJ : Ring Theory **[04 Credits]**

Course Objectives:

1. Understand the basic concepts of Ring theory.
2. Define, Euclidean Domains, Principal Ideal Domains.
3. Define, Unique Factorization Domains:.
4. Understand basic properties of Unique Factorization Domains.
5. Apply Polynomial Ring over fields.

Course Outcomes:

1. Students understand the fundamental concept of Rings, Fields, subrings, integral domains and the corresponding Homomorphism’s.

2. Students learn in detail about polynomial rings. Matrix ring and group ring.
3. Students are able to determine or classify rings into UFD, PID and ED
4. Understanding the concepts of ring of polynomials and irreducibility tests for polynomials over ring of integers

Course Contents:

Unit 1: Introduction to Rings: [17 Hours]

1.1 Revision of Basic definition and properties of Rings
1.2 Examples of Rings
1.3 Ring Homomorphism's, Quotient Rings and Ideals
1.4 Properties of ideals
1.5 Rings of Fractions

Unit 2: Euclidean Domains, Principal Ideal Domains [15 Hours]

2.1 Euclidean Domains
2.2 Principal Ideal Domains

Unit 3: Unique Factorization Domains: [10 Hours]

3.1 Unique Factorization Domains
3.2 Fundamental Theorem of Arithmetic
3.3 Factorization in the Gaussian Integers

Unit 4: Polynomial Rings: [18 Hours]

4.1Basic properties of Polynomial Rings
4.2 Polynomial Ring over fields I
4.3Irreducibility Criteria
4.4 Polynomial rings that are Unique Factorization Domain
4.5 Polynomial Ring over fields II

Recommended Book: 1. **David S. Dummit, Richard M. Foote,** Abstract Algebra (3rdEdition) John Wiley and Sons

Unit 1: Chapter 7: 7.1 to 7.5 Unit 2: Chapter 8: 8.1 to 8.2
Unit 3: Chapter 8: 8.3 Unit 4: Chapter 9: 9.1 to 9.5

Reference Books:

1.**Michael Artin,** Algebra (2nd Edition) Person Education Limited.
2. **C. Musili**,Rings and Modules, 2nd Revised Edition Narosa Publishing House.
3. **Jain and Bhattacharya,**Basic Abstract Algebra, 2nd Edition, Cambridge University Press.
4. **N. Jacobson**, Basic Algebra, Volume 1, Dover publication

****

MTS-554 MJ: ADVANCED CALCULUS

Course Objectives:

1. Understand the basic concepts of differential calculus of scalar and vector fields.
2. Apply the chain rule to derivatives of scalar and vector fields.
3. Calculate line integrals and understand their applications.
4. Define and evaluate double integrals.
5. Apply Green's theorem and the change of variables formula to double integrals.
6. Understand the basic concepts of surface integrals

Course Outcomes:

1. Be able to define and differentiate scalar and vector fields.
2. Be able to calculate directional derivatives, partial derivatives, and higher-order partial derivatives.
3. Be able to apply the chain rule to derivatives of scalar and vector fields.
4. Be able to calculate line integrals and understand their applications.
5. Be able to define and evaluate double integrals.
6. Be able to apply Green's theorem and the change of variables formula to double integrals.
7. Be able to define and calculate surface integrals.
8. Be able to apply the theorem of Stokes and the divergence theorem.

Course Contents:

Unit 1. Differential Calculus of Scalar and Vector Fields [20 Hours]

1.1 FunctionsfromR^{n}toR^m, Scalarandvectorfields,Limitsandcontinuity.

1.2 The derivative of a scalar field with respect to a vector, Directional derivativesand partial derivatives, Partial derivatives of higher order, Inverse functiontheoremand ImplicitFunction theorem.

1.3 Directional derivatives and continuity, The total derivatives, The gradient of ascalarfield, A sufficient condition for differentiability.

1.4 A chain rule for derivatives of scalar fields, Applications to geometry, LevelSets, Tangent planes, Derivatives of vector fields, Differentiability impliesContinuity, The chain rule for derivatives of vector fields, Matrix form of thechainrule.

Unit 2. Line Integrals [10 Hours]

2.1 Paths and line integrals, other notations for line integrals, Basic properties of lineintegrals.

2.2 The concept of work as a line integral, Line integrals with respect to arc length,further applications of line integrals.

2.3 Open connected sets. Independence of the path, The first and secondfundamental theorem of calculus for line integrals, Necessary and sufficientconditions for a vector field to be a gradient, Necessary conditions for a vectorfieldtobe a gradient.

Unit 3. Multiple Integrals **[15 Hours]**

3.1 Partitions of rectangles. Step functions, The double integral of a step function,The definition of the double integral of a function defined and bounded on aRectangle, Upper and lower double integrals, Evaluation of double integral byrepeated one- dimensional integration, Geometric interpretation of the double integrals as a volume.

3.2 Inerrability of continuous functions; Inerrability of bounded functions with Discontinuities, Double integrals extended over more general regions, Applications to area and volume.

3.3 Green's theorem in the plane, Some applications of Green's Theorem, A necessary and Sufficient condition for a two-dimensional Vector Field to be a gradient.

3.4 Change of variables in a double integral, special cases of the transformationformula with proof, General case of the transformation formula with proof, Extensions to higher dimensions, Change of variables in an n-fold integral.

Unit 4. Surface Integrals **[15 Hours]**

4.1 Parametric representation of a surface, the fundamental vector product, thefundamental vector product as a normal to the surface, Area of a parametricsurface.

4.2 Surface integrals, Change of parametric representation, Other notations forsurfaceintegrals

4.3 The theorem of Stokes, Curl and divergence of a vector field, Properties of curland divergence, the divergence theorem (Gauss' theorem) and applications ofdivergencetheorem.

Recommended Book:

1. **Tom M. Apostol,***Calculus Volume II (Second Edition)*,Indian Reprint 2016 (JohnWiley& Sons,Inc) ISBN: 978-81-265-1520-2.
   Unit1:Chapter8:8.1,8.4,8.6-8.22; Unit2:Chapter10:10.2to10.11,10.14to10.16
   Unit3:Chapter11:11.2 to11.15,11.19 to11.22,11.26 to11.33
   Unit4:12.1to12.15,12.19,12.20.
2. For"InverseFunctionTheorem"and"ImplicitFunctionTheorem",**TomM.Apostol,** *Mathematical Analysis 2nd Edition* Narosa Publication 20th Reprint2002. ISBN978-81-85015-66-8.

Reference Books:

1. **Munkres, J. R.**Analysis on manifolds. Addison-Wesley Publishing Company (1991).
2. **Spivak M.**Calculus on manifolds, Addison-Wes.Publishing Company (1965)
3. **GeraldB.Folland**,AdvancedCalculus,PearsonEdition2012.
4. **ADevinatz**, Advanced Calculus(Holt, Reinhart &Winston) 1968.
5. **James J. Callahan**, Advanced Calculus-A Geometric View, Springer, 2010
6. **David V. Widder**, Advanced Calculus (Second Edition), Dover, 1989
7. **Sudhir R. Ghorpade and Balmohan V. Limaye**, A course in Multivariable Calculus and Analysis , Springer, 2009
8. **Jarold Marsden, Anthony Tromba and Alan Weinstein**, Basic Multivariable Calculus, W H Freeman and Co Ltd, 2001.

****

MTS-555 MJP : DATA SCIENCE

Course Objectives:

1. Introduction to Data Science is a comprehensive course that introduces students to the fundamental concepts, techniques, and tools used in the field of data science.
2. The course explores the role of data science in the era of big data and provides a strong foundation in statistical analysis and predictive modelling.
3. Students will gain hands-on experience through lab sessions using Python Programming and learn how to effectively preprocess and analyze data, build predictive models, and evaluate their performance.
4. By the end of the course, students will have a solid understanding of the key principles of data science and be able to apply them to real-world scenarios.

Course Outcomes:

1. Understand the need, benefits, and applications of data science in the context of big data.
2. Recognize the importance of mathematics and statistics as foundational disciplines for data science.
3. Develop skills in data preprocessing, including handling missing values, data wrangling, and data visualization.
4. Learn various supervised and unsupervised machine learning techniques for predictive modelling.
5. Gain proficiency in evaluating and selecting appropriate evaluation metrics for machine learning models.
6. Apply the concepts and techniques learned in the course to practical scenarios through lab sessions.

Course Contents:

Pre-requisite: Programming in Python

Unit 1: Data Science in a big data world: **[12 Hours]**

1.1 Need, benefits and uses of data science and big data
1.2 Overview of the data science process
1.3 The big data ecosystem and data science
1.4 Challenges in big data world
1.5 Importance of Mathematics and Statistics in data science

Unit 2: Statistical Foundation for Data Science. **[18 Hours]**

2.1Analysis of Variance
2.2Data and data representation Techniques
2.3 Measure of Central Tendency and Variability
2.4 Exploratory Data Analysis

2.4 Introduction to probability and probability distributions
2.5 Methods of Estimation
2.6 Testing of Hypothesis.

Unit 3: Data Pre-processing. **[15 Hours]**

3.1 Data and data quality
3.2 Missing Value Analysis and Data wrangling
3.3 Label encoding and feature selection
3.4 Data Visualization techniques
3.5 Data integration and reshaping
3.6 Graph mining methods
3.7 Tex mining techniques

Unit 4: Predictive Modeling. **[15 Hours]**

4.1 Supervised Learning.

4.1.1 Regression Analysis: Linear, Non-linear and correlation
4.1.2. Time Series Analysis: ARIMA, SARIMA, VERMAX
4.1.3. Classification Techniques: Logistic regression, Decision trees, Random forest, Support Vector Machine

4.2 Unsupervised Learning.

4.2.1 Clustering: K-means, Hierarchical clustering, density-based clustering
4.2.2 Dimensionality reduction using PCA and t-SNE
4.2.3 Association rules mining

4.3 Evaluation metrics for Machine Learning models.

Note: Lab sessions on different statistical and machine learning techniques covered in Course. **[60 Hours]**

Recommended Books:

1. **Foster Provost and Tom Fawcett,** *Data Science for Business*. O'REILLY publications, 2013.
Unit 1: Chapter 1 Unit 3: Chapters 6, 7, 8, 10 Unit 4: Chapters 3, 4, 5
2. **Peter Bruce, Andrew Bruce & Peter Gedeck,** *Practical Statistics for Data Scientists*, 2nd Edition. Unit 2: Chapters 1, 2, 3, 4 Unit 4: Chapters 5, 6, 7.

Reference Books:

1. **Peter Bruce, Andrew Bruce & Peter Gedeck**, Practical Statistics for Data Scientists, 2nd Edition
2. **Jiawei Han, Micheline Kamber & Jian Pei**, Data Mining, Concepts and Techniques, 3rd Edition
3. **Ethem Alpaydin,** Introduction to Machine Learning, Edition 2, The MIT Press.
4. **S. C. Gupta,**Fundamentals of Statistics, Himalaya Publishing House

******

MTS-561(A) MJ & 562 (A) : MJP: GRAPH THEORY

Course Objectives:

1. Students will achieve command of the fundamental definitions and concepts of graph theory.
2. Students will understand and apply the core theorems and algorithms, generating examples as needed, and asking the next natural question.
3. Students will achieve proficiency in writing proofs, including those using basic Graph theory proof techniques such as bijections, minimal counterexamples, and Loaded induction.
4. Students will work on clearly expressing mathematical arguments, in discussions and in their writing.
5. Students will become familiar with the major viewpoints and goals of graph theory: classification, externality, optimization and sharpness, algorithms, and duality.
6. Students will be able to apply their knowledge of graph theory to problems in other areas, possibly demonstrated by a class project.

Course Outcomes:

1. Understanding of fundamental concepts: Students should have a strong understanding of the fundamental concepts of graph theory, including vertices, edges, paths, cycles, connectivity, and isomorphism. They should be able to analyze and manipulate graphs using these concepts.
2. Graph Algorithms: Students should be proficient in implementing and analyzing graph algorithms. This includes algorithms for graph traversal (e.g., breadth-first search), shortest paths (e.g., Dijkstra's algorithm), and minimum spanning trees (e.g., Prim's algorithm, Kruskal's algorithm).
3. Graph Theory Applications: Students should be able to apply graph theory concepts and algorithms to solve real-world problems in various domains. This includes applications in computer science, network analysis, social networks, logistics, optimization, and scheduling.
4. Graph Properties and Structures: Students should be familiar with important properties and structures of graphs, such as bipartiteness, Eulerian and Hamiltonian paths and cycles, tree structures, and graph coloring. They should understand the relationships between these properties and be able to apply them to problem-solving.
5. Graph Optimization Problems: Students should be able to formulate graph optimization problems, such as the traveling salesman problem, maximum matching, and graph partitioning. They should understand the complexity of these problems and be able to design and analyze efficient algorithms to solve them..
6. Advanced Topics in Graph Theory: Students should be exposed to advanced topics in graph theory, including graph embedding, spectral graph theory, random graphs, graph minors, and algorithmic graph theory. They should be able to explore and understand research papers in these areas.

7. Research Skills: Students should develop research skills, including literature review, identifying research gaps, formulating research questions, and designing experiments or theoretical investigations. They should be able to contribute to the field of graph theory through independent research or project work.
8. Overall, the course aims to equip postgraduate students with a comprehensive understanding of graph theory, its applications, and its theoretical foundations. It prepares them for further research in graph theory or related areas, as well as for careers in fields such as computer science, data analysis, network analysis, operations research, and optimization.

Course Contents:

Unit 1:An Introduction to Graphs: **[20 Hours]**

The Definition of a Graph, Graphs as Models, More Definitions, Vertex Degrees, Subgraphs, Paths and Cycles, Matrix Representation of Graphs, Fusion.

Unit 2:Trees and Connectivity: **[15 Hours]**

Definitions and Simple Properties, Bridges, Spanning Trees, Connector Problems, Shortest Path Problems, Cut Vertices and Connectivity.

Unit 3:Euler Tours and Hamiltonian Cycles: **[15 Hours]**

Euler Tours, The Chinese Postman Problem, Hamiltonian Graphs, The Travelling Salesman Problem.

Unit 4:Matchings: **[15 Hours]**

Matching and augmenting paths, Hall's Marriage Theorem, The Personnel Assignment Problem.

Unit 5:Colouring: **[15 Hours]**

Vertex colouring, Vertex colouring algorithms, Critical graphs, Cliques, Edge colouring.

Unit 6:Directed Graphs: Definitions, In degree and Out degree. **[10 Hours]**

Recommended Book : John Clark and D. A. Holton, *A First Look at Graph Theory*, World Scientific (1995).

Unit 1: 1.1-1.8Unit2: 2.1-2.6 ; Unit 3: 3.1-3.4

Unit4: 4.1-4.4; Unit. 5: 6.1-6.5 ; Unit. 6: 7.1-7.2

References: 1. **Harary,** Graph Theory, Narosa Publishers (1989).

2. **Nora Hartsfield and Gerhard Ringel**, Pearls Theory, Academic Press (1990).
3. **Narsingh Deo**, Graph Theory with applications to computer science and engineering, Prentice Hall (2009).
4. **Douglas B. West**, Introduction to Graph Theory, Prentice- Hall (1999).

****

MTS-561(B) MJ & 562 (B): DYNAMICAL SYSTEMS

Course Objectives:

1. Students able to find solution system of linear differential equations.
2. Students able to draw phase portrait of dynamical systems in two dependent variables.
3. Students able to find equilibrium points and identify nature of phase portrait in neighbourhood of the equilibrium point.
4. Students able to decompose the phase portrait into stable, unstable, centre manifold.
5. Students able to find normal forms of dynamical systems.
6. Students able to find bifurcation of one dimensional maps.

Course Outcomes:

1. Explain the behavior of the systems of differential equations.
2. Implement techniques to describe the behaviour of dynamical systems.
3. Differentiate the nature of dynamical systems.
4. Judge the technique to solve and behaviour of dynamical systems.
5. Develop criteria to solve and discuss the nature of dynamical systems.

Course Contents:

Unit 1: Linear System of Differential Equations: [15 Hours]

1.1 Introduction to system of differential equations.
1.2 Fundamental set of solutions.
1.3 Solutions and phase portraits of linear homogeneous system with constant coefficients
1.4 Non homogeneous systems with time-dependent forcing.

Unit 2:Flow of Nonlinear system of Differential Equations: [10 Hours]

2.1 Solutions of nonlinear systems of differential equations.
2.3 Existance and uniqueness of solution.
2.4 Gronwall inequality.

Unit 3: Fixed Points and Stability of Differential System: [15 Hours]

3.1 Limit sets.
3.2 Stability of fixed Points.
3.3 Stability and fixed points of one dimensional equation.
3.4 Nullclines for planar systems.
3.5 Linearization and stability.
3.6 Competitive Population model.

Unit 4: Phase Portraits Using Scalar Functions: [20 Hours]

4.1 Predator-Prey Systems
4.2 Systems with undamped forces
4.3 Lyapunov functions
4.4 Bounding functions
4.5 Gradient systems.

Unit 5: Periodic Orbits and Bifurcations: [18 Hours]

5.1 Introduction to periodic orbits, limit cycles, Poincare map.
5.2 Poincaré-Bendixson theorem.
5.3 Self-Excited oscillator.
5.4 Andronov-Hopf bifurcation.
5.5 Homoclinic bifurcation.
5.6 Poincaré map.

Unit 6:Introduction to Discrete Dynamical Systems: [12 Hours]

6.1 One dimensional maps
6.2 Periodic points of one dimensional maps
6.3 Iteration Using the Graph
6.4 Stability of Periodic Points.

Recommended Book:R. Clark Robinson, *An Introduction to Dynamical Systems-Continuous and Discrete,* (Second Edition), American Mathematical Society (AMS), 2012.
Section: 2.1-2.5, 3.1-3.3, 4.1-4.8, 5.1-5.7, 6.1-6.9, 8.1, and 9.1-9.3.

Reference Books:

1. **Lawrence Perko,**Differential Equations and Dynamical Systems (Third Edition), Springer, 2001.
2. **James D. Meiss**, Differential Dynamical Systems, Society for Industrial and Applied Mathematics (SIAM), 2007.
3. **Steven H. Strogatz**, Nonlinear Dynamics and Chaos- With Applications to Physics, Biology, Chemistry, and Engineering, CRC Press, 2015.
4. **Morris W. Hirsch, Stephen Smale, Robert L. Devaney**, Differential Equations, Dynamical Systems, and an Introduction to Chaos, (Third Edition), Elsevier, 2016.
5. **Stephen Lynch**, Dynamical Systems with Applications using Python, Birkhauser, 2018.
6. **Fritz Colonius, Wolfgang Kliemann**, Dynamical Systems and Linear Algebra, American Mathematical Society (AMS), 2014.

****

MTS-561(C) MJ & 562 (C): CODING THEORY

Course Objectives:

1. To understand the role and significance of information and coding theory in theory and practice.
2. To understand how theory of finite fields and linear algebra comes to applied in designing of codes.
3. To understand different techniques of error detection and correction.
4. To understand applications of coding theory.

Course Outcomes: *The student will have to*

1. Understand the need for encoding and decoding strategies and the concept of error correction.
2. Understand how finite fields and linear algebra can be used to construct codes.
3. Understand the significance various bounds in coding theory .
4. Learn some applications of coding theory.

Course Contents:

Unit I: Error detection, correction and decoding: **[18 Hours]**

Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

Unit II: Finite Fields: **[7 Hours]**

Basics of finite fields, construction of finite field as quotient of polynomial ring, minimal polynomials.

Unit III: Linear codes: **[30 Hours]**

Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cosets, Nearest neighbor decoding for linear codes.

Unit IV: Bounds in Coding Theory: **[20 Hours]**

Main Coding theory problem, lower bound, sphere covering bound, Hamming bound and perfect codes, Binary hamming codes**,** Singleton bound and MDS codes.

Unit V: Cyclic codes: **[15 Hours]**

Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes.

Recommended Book:San Ling and ChaoingXing,*Coding Theory- A First Course.* Cambridge University Press :Unit-1:2.1, 2.2, 2.3, 2.4, 2.5 Unit-2:3.1,3.2, 3.3,3.4 Unit-3 :4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 Unit-4: 5.1, 5.2, 5.3, 5.4, Unit-5: 7.1, 7.2, 7.3, 7.4

Reference Books: 1.**Raymond Hill**, A First Course in Coding Theory (Oxford)

2. **J. H. van Lint**, An Introduction to Coding Theory (Springer, Third Edition)

****

MTS-561(D) MJ & MTS 562(D): OPERATIONS RESEARCH

Course Objectives:

1. To identify the concept of Linear Programming Problem.
2. To apply methods to solve Integer Programming problems and analysis of the solutions.
3. To understand the effect of variations in input data through sensitivity analysis.
4. To analyze the primal-dual relationship of a linear Programming problem and compute the dual.
5. To understand concept of multi-objective optimization problem by Goal Programming Approach.
6. To understand the concept of Non-linear Programming and methods of solving the Nonlinear Programming problems.

Course Outcomes:

1. The successful completion of this course, students will able to formulate real life problems into linear Programming problem.
2. Understand the importance of sensitivity analysis in managerial decision making.
3. Analyze the effect of variations in input data of linear Programming problem through sensitivity analysis.
4. Understand the importance of duality in linear Programming problem.
5. Understand the concept of multi-objective decision problems.
6. Use the quadratic Programming models for real life problems.

Course Contents:

Unit I: Linear Programming Problem [18 Hours]

1.1 Standard form of Linear Programming Problem.

1.2 Simplex method

1.3 Big-M method

1.4 Types of linear Programming solutions

1.5 Duality in linear Programming problem

a) Primal-Dual Relationship

b) Economic interpretation of dual variables and constraints

c) Managerial significance of duality d) Solution of primal linear Programming problem using dual linearProgramming problem

Unit II: Sensitivity Analysis in Linear Programming[18 Hours]

2.1 Changes in objective function coefficients

2.2 Changes in availability and resources

2.3 Changes in the input-output coefficients

Unit III: Integer Linear Programming[18 Hours]

3.1 Types of Integer Programming Problems

3.2 Gomory's all integer cutting plane method

3.3 Gomory's mixed- integer cutting plane method 3.4 Branch and Bound Method.

Unit 4: Goal Programming [18Hours]

4.1 Difference between linear Programming and goal Programming

4.2 Concept of Goal Programming

4.3 Goal Programming Model Formulation

4.4 Graphical solution method for Goal Programming

4.5 Modified Simplex method of Goal Programming

Unit 5: Non-Linear Programming Problem [18 Hours]

5.1 The general non-linear Programming problem

5.2 Graphical solution method

5.3 Quadratic Programming

a) Kuhn-Tucker Conditions b) Wolfe's Modified Simplex Method c) Beale's Method

Recommended Book: J. K. Sharma, *Operations Research*, (Third Edition, Macmillan India Ltd.),2008.

Chapter 4: 4.1, 4.2, 4.3, 4.4.2, 4.5, 4.6 Chapter 5: 5.1, 5.2, 5.3, 5.4, 5.5

Chapter 6: 6.1, 6.2.1, 6.2.2, 6.2.3 Chapter 7: 7.1, 7.2, 7.3, 7.4, 7.5, 7.6

Chapter 8: 8.1, 8.2, 8.3, 8.4, 8.5, 8.6 Chapter 24: 24.1,24.2, 24.3, 24.4, 24.5

Reference Books:

1. **Hamdy A. Taha**, Operations Research, (Eighth Edition, Prentice Hall of India), 2008.
2. **P. K. Gupta and D. S. Hira,** Operations Research, (Fifth Edition, S. Chand), 2014.

****

MTS-581:On Job Training (OJT)/ FIELD PROJECT **[04 Credits = 120 Hrs]**

In this course,the students are expected to do the On Job Training (OJT) in appropriate Industries/Government sectors/Institute etc. to get hands on experience. The department may conduct necessary lectures/workshops/seminars as a prerequisite for OJT. The course will be conducted as per the guidelines of the Institute /the University and Government of Maharashtra.

****

M.Sc.- Mathematics, Part-II

(For Colleges Affiliated to Savitribai Phule Pune University, Pune)

(As per National Education Policy- 2020)

To be implemented from the Academic Year 2024-2025.

Semester-III

MTS 601MJ , MTS-602 MJP: COMPLEX ANALYSIS

Course Objectives:

1. To understand concept of Analytic function and confidently apply the Cauchy-Riemann Equations.
2. To study techniques of complex variables and functions together with their derivative, Contour integration and transformation.
3. To study complex power series and classification of singularities.
4. To study calculus of residue and its applications in the evaluation of integrals.
5. To apply concepts in complex analysis to solve problems in diverse areas such as Engineering, Physics, and Applied Mathematics.
6. To understand the modulus of a complex valued functions and results regarding that,

Course Outcomes:

By the end of the course, students will be able to:

CO1: Represent complex numbers algebraically and geometrically.
CO2: Analyze limit, continuity and differentiation of functions of complex variables.
CO3: Understand Cauchy-Riemann equations, analytic functions and various properties of analytic functions.
CO4: Understand Cauchy theorem and Cauchy integral formula and apply these to evaluate complex contour integrals.
CO5: Classify singularities and poles and evaluate complex integration using the residue theorem.
CO6: Understand conformal mapping, Mobius transformation.
CO7: Prepare themselves for Research or Competitive Examinations: SET, NET/CSIR, GATE etc.

Course Content:

UNIT 1: The Complex number system **[04 Hrs]**

1.1 The field of Complex numbers
1.2 The Complex plane
1.3 Polar representation and roots of complex numbers
1.4 Lines and half planes in the Complex plane

UNIT 2: Elementary properties and examples of Analytic functions [**16Hrs]**

2.1 Power Series
2.2 Analytic Functions
2.3 Analytic functions as mapping, Conformal Mapping, Mobius transformation

UNIT 3: Complex Integration [**16Hrs]**

3.1 Power series representation of analytic functions, Cauchy's Estimate
3.2 Zeros of analytic function, Liouville's Thm, Fundamental Theorem of Algebra, Maximum Modulus Theorem
3.2 The index of a closed curve
3.3 Cauchy's Theorem and Integral formula, Morera's Theorem
3.4 The homotopic version of Cauchy's Theorem and simple connectivity
3.5 Counting zeroes; the Open Mapping Theorem
3.6 Goursat's Theorem

UNIT 4: Singularities [**16Hrs]**

4.1 Classification of singularities, Casorati-Weierstrass Theorem, Residues, Residue Theorem
4.2 The Argument Principle, Meromorphic Function, Rouche's Theorem

UNIT 5: The Maximum Modulus Theorem **[08Hrs]**

5.1 The Maximum Principle
5.2 Schwarz's Lemma
5.3 The gamma function
5.4 The Riemann zeta function

Recommended Book:

John B. Conway: Functions of one complex variable (Narosa Publishing house)

UNIT1: Sections 1.2 to 1.5; **UNIT2:** Sections 3.1 to 3.3; **UNIT3:** Sections 4.2 to 4.8
UNIT4: Sections 5.1 to 5.3 ; **UNIT 5:** Sections 6.1 to 6.2 and 7.7 to 7.8.

Reference Books:

1. S. Ponnusamy: Foundation of Complex Analysis, Narosa Publications, Second Edition.
2. E. Stein and Shakarchi, Complex Analysis, Overseas Press (India) Ltd.
3. B. Choudhary : The Elements of Complex Analysis, Wiley Eastern Ltd.
4. R. V. Churchill, J. W. Brown, Complex Variables and Applications, McGraw Hill.
5. R.P.Agrawal, K.Perera,S.Pinelas: An Introduction to Complex Analysis, Springer.

****

MTS 603 MJ: FIELD THEORY

Course Objectives:

1. To become Capable in working with field extensions, including understanding the notion of algebraic and transcendental elements.
2. To grasp the idea of splitting fields and their role in algebraic closures.
3. To understand the applications of field theory concepts to the classical problem of constructability in geometry.
4. To overview the concept of Galois theory and its applications in solving polynomial equations.
5. To aware the students towards proficiency in working with polynomial rings over fields.
6. To acquire the essential skills for conducting independent research in the field of algebra.

Course Outcome:

After the completion of this course, students will be able

CO1: to understand basic concepts in Field Theory
CO2: to grasp the idea of polynomial rings over the field
CO3: to collect, present, analyse and interpret the theory of splitting field and its role in finding algebraic closure.
CO4: to overview the concept of Galois theory and its applications
CO5: to solve classical problem of constructability in geometry
CO6: to develop essential skills for conducting independent research in the area of algebra.

Course Content:

UNIT 1: **[30Hrs]**

1.1 Basic Theory of Field Extensions
1.2 Algebraic Extensions
1.3 Classical Straightedge and compass constructions
1.4 Splitting Fields and Algebraic Closures
1.5 Separable and Inseparable Extensions
1.6 Cyclotomic Polynomials and Extensions

UNIT 2 : **[30Hrs]**

2.1 Basic Definitions
2.2 The Fundamental Theorem of Galois Theory
2.3 Finite Fields
2.4 Cyclotomic Extensions and Abelian Extensions over Q

Recommended Book:

1. David S. Dummit, Richard M. Foote, Abstract Algebra, 2nd Edition [Wiley India]
   Chapter: 13 **[13.1 To 13.6],** Chapter: 14 **[14.1 To 14.3, 14.5]**

Reference Books:

1. Joseph Gallian, Contemporary Abstract Algebra, 9th Edition, Cengage Learning India Pvt. Ltd.
2. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra (Cambridge University Press, 2nd Edition), 1995 (Indian Edition).
3. I. S. Luthar, I. B. S. Passi, Algebra (Vol 1), Groups; Narosa Publication House.
4. I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd.
5. J. B. Fraleigh, A First Course in Abstract Algebra, 7th Edition, Pearson Edition Ltd.

****

MTS 604 MJ: DIFFERENTIAL GEOMETRY

Course Objectives:

1. To become Capable in working with fundamental concepts of Differential Geometry.
2. To grasp the idea of curvature of curve and surfaces.
3. To understand the applications of differential geometry in various fields.
4. To overview the concept of arc length and line integrals.
5. To aware the students towards proficiency in working with orientation of surfaces.
6. To acquire the essential skills for conducting independent research in the field of Differential Geometry.

Course Outcome:

CO 1: To define and understand fundamental concepts such as curves, surfaces, tangent vectors, normal vectors, and curvature

CO 2: To learn how to express curves and surfaces using different coordinate systems

CO 3: To define and study geodesics as curves that represent the shortest paths between points on surfaces

CO 4: To study vector fields and their role in describing the motion of objects on surfaces

CO 5: To find arc lengths and line integrals

CO 6: To explore applications of Differential Geometry in various fields such as physics, computer graphics, and relativity

Course Content:

UNIT 1: **[15Hrs]**

1.1. Graphs and Level Sets,
1.2. Vector Fields
1.3. The Tangent Space
1.4. Surfaces ,

UNIT 2 : **[20Hrs]**

2.1 Vector Fields on Surfaces
2.2 Orientation
2.3 The Gauss Map
2.4 Geodesics ,

UNIT 3: **[20Hrs]**

3.1 Parallel Transport
3.2 The Weingarten Map
3.3 Arc Length and Line Integrals

UNIT 4: 4.1 Curvature of Surfaces **[05Hrs]**

Recommended Book:

1. J.A. Thorpe, Elementary Topics in Differential Geometry, First Indian Reprint, Springer Publication. Chapters: 1 to 12.

Reference Books:

1. Erwin Kryszig, Differential Geometry, Dover Publications Inc.
2. Christian Bar, Elementary Differential Geometry, Cambridge University Press.
3. Andrew Pressley, Elementary Differential Geometry, Springer.
4. T.J. Willmore, An Introduction to Differential Geometry, Dover Publications Inc.

****

MTS 605 MJ: MACHINE LEARNING USING PYTHON

Course Objectives:

1. To provide a thorough understanding of the fundamentals of machine learning, covering both theoretical concepts and practical applications.
2. To become familiar with various libraries related to machine learning.
3. To become familiar with data pre-processing techniques.
4. To become familiar with supervised machine learning algorithms.
5. To develop hands-on experience in implementing machine learning algorithms.

Course Outcomes:

Upon successful completion of this course the student will be able to

CO1: well-prepared to apply machine learning techniques to various domains.

CO2: contribute to real-world projects.

CO3: continue their learning journey in the rapidly evolving field of machine learning.

Course Contents:

UNIT 1: An Overview of Machine Learning [08 Hrs]

1.1 What & why behind machine learning

1.2 Types of Machine Learning – Supervised, Unsupervised, Semi supervised and Reinforcement

1.3 Main challenges of Machine Learning

1.4 Testing and Validating

1.5 Hyper parameter Tuning and Model Selection

UNIT 2: Python Libraries for Machine Learning [12 Hrs]

2.1 NumPy: The NumPy ndarray, Creating ndarrays, Data types for ndarrays, indexing and slicing, Transposing array and Swapping axes, Universal functions, Random number generation.

2.2 Pandas: Getting started with pandas, Series, DataFrame, Essential Functionality: Reindexing, Dropping entries, Sorting, Summarizing and Computing Descriptive Statistics, Handling Missing Data: filtering out missing data, filling in missing data.

2.3 Matplotlib**:** Figures and subplots, Plotting Functions in Pandas, Colors, Markers, Line styles, Legend, Title, Plotting functions in Pandas: Line plot, Bar plots, Histogram and Density plots, Scatter plots.

UNIT 3: Training Models –I [10 Hrs]

3.1 Linear Regression: The Normal Equation

3.2 Gradient Descent: Batch Gradient Descent, Stochastic Gradient Descent.

3.3 Polynomial Regression

3.4 Logistic Regression: Estimating Probabilities, Training and Cost Function, Classification of iris dataset.

UNIT 4: Training Models-II [10 Hrs]

4.1 Some Sample Datasets:

4.2 K Nearest Neighbors: k-Neighbors classification, classification of forge dataset, k-Neighbors regression, prediction of wave dataset.

4.3 Decision Trees: Building decision tree, Classification of Cancer dataset.

4.4 Random Forest: Classification of two_moons dataset and cancer dataset.

UNIT 5: Prediction of Housing Prices **[10Hrs]**

5.1 Get the data
5.2 Discover & visualize the data to gain insights
5.3 Preparing the data for machine learning: Cleaning, Handling categorical values, Feature scaling
5.4 Select and Train a model: Linear Regression model, Decision Tree Regressor model and Random Forest Regressor model.
5.5 Fine-tuning the model: Grid Search, Randomized Search

UNIT 6: Classification of MNIST dataset **[10 Hrs]**

6.1 MNIST dataset
6.2 Training a Binary Classifier
6.3 Performance Measures: Measuring accuracy using Cross Validation, Confusion Matrix, Precision and Recall
6.4 Multiclass Classification: OneVsOne classifier, OneVsRest classifier, Random Forest classifier.
6.5 Multilabel Classification: KNeighbors Classifier

Recommended Books:

1. Aurelien Heron. Hands-on Machine Learning with Scikit-Learn, Keras and Tensorflow. O.Reilly Publication, 2nd edition, 2019.
   Unit 1: Chapter 1; **Unit 3**: Chapter 4; **Unit 5**: Chapter 2, **Unit 6**: Chapter 3.
2. Wes McKinney. Python for Data Analysis. O' Reilly publication, 1st edition, 2012.
   Unit 2: Chapter 4, Chapter 5 and Chapter 8.
3. Andreas C. Muller & Sarah Guido. Introduction to Machine Learning With Python, O'Reilly publication, 1st edition, 2017. **Unit 4**: Chapter 2.

Reference Books:

1. Dipanjan Sarkar, Raghav Bali and Tushar Sharma. Practical Machine Learning with Python. APress publication, 1st edition, 2018.
2. Marc Peter Deisenroth, A. Aldo Faisal and Cheng Soon Ong. Mathematics for Machine Learning. Cambridge University Press, 1st edition, 2020.
3. Sandeep Nagar, Introduction to Python for Engineers and Scientists. APress publication, 1st edition, 2018.

****

MTS-611(A) MJ, MTS-612(A) MJP: MATHEMATICAL STATISTICS

Course Objectives:

1. To introduce fundamental concepts of probability, random variables and their distributions.
2. To study various discrete and continuous probability distributions:
3. To develop skills in hypothesis testing and parameter estimation:
4. To introduce correlation and regression models

Course Outcomes:

Upon completion of the course, students should be able to:

CO1. Demonstrate proficiency in fundamental concepts of probability, random variables and probability distributions.

CO2. Compute and interpret statistics such as mean, variance, and covariance for random variables.

CO3. Conduct tests related to population parameters such as mean and variance.

CO4. Critically evaluate assumptions and limitations of statistical methods and models.

Course Content:

UNIT 1: Introduction to Probability & Random Variables [7Hrs]

1.1 Sample space, events, probability of an event, addition and multiplication theorem of probability, conditional probability.

1.2 Concept of a random variable, discrete probability distribution, continuous probability distribution, joint probability distribution.

1.3 Independent random variables, Mean of a random variables, Variance and Covariance, Mean and Covariance of linear combinations of random variable

UNIT 2: Some Discrete & Continuous Probability Distributions [8Hrs]

2.1 Bernoulli, Binomial, Poisson distribution, Negative binomial distribution, Geometric distribution,

2.2 Continuous Uniform distribution, Normal distribution, Central Limit Theorem, Gamma distribution, Exponential distribution, Chi-squared distribution, Student's t distribution, F distribution.

UNIT 3: Testing of Hypothesis [8Hrs]

3.1 Parameter, Parameter Space, Statistic, Null & Alternative hypotheses, Type I & Type II errors, level of significance, P-value, Test Statistic

3.2 Statistical Tests:

i) Chi-Square test of Goodness of fit **(to be covered in practical only).**

ii) Chi-Square Test of Independence of Attributes **(to be covered in practical only).**

iii) Test for the specified value of mean of Normal population (known and unknown variance)

iv) Test for the equality of population means of two Normal populations. (known and unknown equal variances)

v) Paired t-test

vi) Test for the specified value of variance of Normal population;

vii) Test for the equality of population variances of two Normal populations

Unit IV: Correlation & Regression Models **[7Hrs]**

4.1 Karl Pearson’s Coefficient of Correlation, Numerical Problems.

4.2 Simple Linear Regression Model, Least square principle, estimation of parameters by least square method of estimation.

4.3 Multiple Linear Regression Model, estimation of parameters by least square method of estimation.

4.4 Testing significance of parameters and overall regression model, Verifying the assumptions of regression model (to be covered in practical only).

4.5 Introduction to Logistic Regression Model (to be covered in practical only).

Recommended Books:

1. Gupta, S.C. and Kapoor, V. K. (2020). Fundamentals of Mathematical Statistics, Tenth Edition, Sultan Chand and Sons Publishers, New Delhi.**Unit 1, Unit 3, Unit 4:** 4.1.
2. Ross, S. M.(2014), Introduction to Probability and Statistics for Engineers and Scientists. 5th edition, Academic Press. **Unit 2.**
3. Rice, J. A. (2006). Mathematical Statistics and Data Analysis. Belmont, CA: Duxbury Press. **Unit 3**: 3.2
4. Montgomery, D.C., Peck, E.A. and Vining, G.G. (2015) Introduction to Linear Regression Analysis. John Wiley & Sons, Hoboken. **Unit 4:** except 4.1.

Reference Books:

1. Ross, S. M. (2013). A First Course in Probability. Pearson. **[Examples in Unit 2.]**
2. Walpole, R.E., Myers, R.H., Myers, S.L. and Ye, K. (2007) Probability & Statistics for Engineers & Scientists. 9th Edition, Pearson Education, Inc. **[Examples in Unit 2.]**

****

MTS-611(B) MJ, MTS-612(B) MJP: ALGEBRAIC TOPOLOGY

Course Objectives:

1. Be able to use tools from abstract algebra to study topological spaces.
2. Be able to find algebraic invariants that classify topological spaces up to homeomorphism, though usually most classify up to homotopy equivalence.
3. Understand the concepts of fundamental groups, covering spaces.
4. Discussing some classical groups and their fundamental groups.

Course Outcomes:

CO1: After successful completion of the course, the student should be able to:

CO2: Compute algebraic invariants associated to topological spaces and maps between them.

CO3: Prove topological results by using algebraic methods.

CO4: Apply methods from algebraic topology to problems in a broader mathematical context.

Course Content:

UNIT-I: The Fundamental Group **[12Hrs]**

1.1 Homotopy of Paths.
1.2 The Fundamental Group.
1.3 Covering Spaces.
1.4 The Fundamental Group of the Circle.
1.5 Retractions and Fixed Points.
1.6 The Fundamental Theorem of Algebra.
1.7 The Borsuk-Ulam Theorem.
1.8 Deformation Retracts and Homotopy Type.
1.9 The Fundamental Group of S_n.

UNIT-II: The Seifert-van Kampen Theorem **[10 Hrs]**

2.1 Direct Sums of Abelian Groups (only revision).
2.2 Free Products of Groups (only revision).
2.3 Free Groups
2.4 The Seifert-van KampenTheorem.
2.5 The Fundamental Group of a Wedge of Circles.
2.6 The Fundamental Groups of the Torus and the Dunce Cap.

UNIT-III: Classification of Covering Spaces **[08 Hrs]**

3.1 Equivalence of Covering Spaces.
3.2 The Universal Covering Spaces.
3.3 Covering Transformations.
3.4 Existence of Covering Spaces.
3.5 Applications.

Recommended Book:

James R. Munkres, Topology, Second Edition, Pearson Prentice Hall.
Chapter 9: Sections: 51, 52, 53, 54, 55, 56, 57, 58, 59.
Chapter 11: Sections: 67, 68 (Only revision), 69, 70, 71, 73.
Chapter 13: Sections: 79, 80, 81, 82.

Reference Books:

1. Allen Hatcher, Algebraic Topology, Cambridge University Press, 2002.
2. M.A. Armstrong, Basic Topology, Springer International Edition, 2004.
3. J. J. Rotman, An Introduction to Algebraic Topology, Springer, 1988.
4. E. H. Spanier, Algebraic Topology, Springer, 1994.
5. B K Lahiri, A first course in Algebraic Topology, 2nd Edition.

****

MTS- 611 (C) MJ, MTS- 612(C) MJP: INTEGRAL TRANSFORMS AND SPECIAL FUNCTIONS

Course Objectives:

The objectives of this course are

1. To understand Integral transforms like Laplace transform, Fourier transform
2. To understand inverse integral transforms and convolution theorems.
3. To be able to solve ODE and IVP's .
4. To able to solve Partial differential equations and boundary value problems.
5. To understand Gamma functions, Beta functions and Bessel functions, Hankel function and modified Bessel functions

Course Outcomes:

After completion of this course students will be able to,

1. use Laplace transform in ordinary differential equations and initial value problems / boundary value problems.
2. use Laplace transform in partial differential equations and initial value problems / boundary value problems.
3. Use Fourier transform, Finite Fourier transform, Discrete Fourier transform and the fast Fourier transform to solve BVP's.
4. Use beta and Gamma functions.
5. Use Bessel function, Hankel function, Modified Bessel function and Kelvin function.

Course Content:

UNIT I: Integral Transforms: **[15Hrs]**

Laplace Transform, Existence of the Transform, The Gamma Function and Laplace Transforms, Transforms of Derivatives, Derivatives of Transforms, The Inverse Laplace Transform, Solutions of ODE and IVPs, Partial Fractions, The Unit Step Function, Shifting Properties, The Dirac Delta Function, Convolution, Laplace Transform Methods for PDEs,

Finite Fourier Transforms, Fourier Transforms, The Discrete Fourier Transform, The Fast Fourier Transform

UNIT 2: **Special Functions:** **[15Hrs**

Gamma and Beta functions, Properties of Beta and Gamma functions, Definition of Gamma function for negative values of the argument, Examples.

Bessel's function of first kind and second kind, Generating Function, Integral representation, Recurrence relations, Hankel functions, Equations reducible the Bessel's equations, Modified Bessel's functions, Recurrence relation and Integral representations for modified Bessel's functions, Kelvin function, Examples

Recommended Book:

1. J. Ray Hanna, John H. Rowland, Fourier Series, Transforms, and Boundary Value Problems, Dover Publications (2008), **Chapter 7: 7.1-7.21**
2. W. W. Bell , Special functions for Scientists and Engineers, D. Van Nostrand Company LTD London, **Chapter 2; 2.1 to 2.4; Chapter 4; 4.1 to 4.10**

Reference Books:

1. Larry C. Andrews, Field guide to Special Functions for Engineers; SPIE Field guides Volume FG18, John E Greivenkamp, Seires Editor.
2. Dr. M. D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand and Company LTD.
3. Lokenath Debnath and Dambaru Bhatta, Integral Transforms and their applications, third edition, CRC Press.
4. Joel L. Schiff, The Laplace Transform: Theory and Applications, Springer.

****

MTS 611(D) MJ, MTS-612(D) MJP: MECHANICS

Course objectives:

The objectives of this course are

1. Acquire an understanding of Euler's variational principles and apply them to solve practical problems.
2. Develop proficiency in utilizing D'Alembert's Principle, Lagrange's equations, Hamilton's Principle, Hamilton's equations, and Hamilton-Jacobi equation to formulate and solve differential equations for diverse real-world systems.
3. Familiarize the concepts such as Poisson's brackets, Lagrange's bracket, canonical transformations, and related topics.

Course outcomes:

CO1: Discover extremals for a given integral through variational methods.

CO2: Analyze and determine the Lagrangian, Hamiltonian, and Routhian for a given dynamic system.

CO3: Derive equations of motion for a system utilizing various methods, including D'Alembert's equation, Lagrange's equations of motion, Hamilton's principle, Hamilton's equations of motion, and the Hamilton-Jacobi method.

CO4: Discriminate whether a given transformation is canonical or non-canonical.

Course Content:

UNIT 1: Variational Problems With Fixed and Boundaries: [12 Hrs]

The Concept of variation and its properties, Euler's equation, Variational problems for functional of the form, Functionals dependent on Higher order derivatives, Functional dependent on functions on several independent variables, Variational problems in parametric form, Functional of the form $I[y(x)] = \int_{x_1}^{x_2} F(x, y, y')dx$, Variational problem with a movable boundary for a functional dependent on two functions.

UNIT 2: Survey of the Elementary Principle: [08Hrs]

Mechanics of a particle, Mechanics of a system of particle, Constraints, D'Alembert's principle and Lagrange's equation, Velocity-dependent potential and the dissipation function, Simple applications of the Lagrangian formulation.

UNIT 3: Variational Principles and Lagrange's Equation: [10 Hrs]

Hamilton's principle, Derivation of Lagrange's equations from Hamilton's principle, Extension of Hamilton's principle to nonholonomic system, Advantages of a variational principle formulation, Conservation theorems and symmetry properties.

UNIT 4: The Hamilton Equation of Motion: [10Hrs]

Legendre transformations and the Hamilton equations of motion, Cyclic coordinates and conservation theorems, Routh's procedure and oscillations about steady motion, Derivation of Hamilton's equation from a variational principle, The principle of least action.

UNIT 5: Canonical Transformations: [08Hrs]

The equations of canonical transformation, Examples of canonical transformations, Poisson brackets and other canonical invariants, Equations of motion, infinitesimal canonical transformations and conservations theorems in the poisons bracket formulation.

UNIT 6: Hamilton-Jacobi Theory: [12Hrs]

Hamilton-Jacobi equation for Hamilton's principle function, The Harmonic oscillator problem as an example of the Hamilton-Jacobi method, The Hamilton- Jacobi equation for characteristic function, Separation of variables in the Hamilton-Jacobi equation

Recommended Books:

1. Herbert Goldstein, *Classical Mechanics*, Narosa Publishing House, (1993) (Reprint).
   Chap 1: 1 to 6; Chap 2: 1 to 6; Chap 8: 1 to 3, 5, 6; Chap 9: 1, 2, 4, 5 ; Chap 10: 1 to 4
2. A. S. Gupta, *Calculus of Variation with Application*, Prentice-Hall of India Private Limited, New Delhi, (2005).
   Chap 1: 1.1 to 1.6, Chap 2: 2.1, 2.2

Reference Books:

1. G. Aruldhas, *Classical Mechanics*, Phi learning Pvt Ltd (First Edition), (2009).
2. Madhumangal Pal, *A Course on Classical Mechanics, Narosa Book Dist. Pvt. Ltd.2008.,*
3. Gupta, Kumar, Sharma, *Classical Mechanics*, Pragati Prakashan, (2010).

****

MTS 631 MJP: Research Project **04 Credits**

Semester-IV

MTS-651 MJ, MTS 652 MJP: FUNCTIONAL ANALYSIS

Course objectives:

The objectives of this course are

1. To develop understanding of basic concepts in functional analysis
2. To introduce central theorems in functional analysis
3. To study Hilbert space theory
4. To study basic operator theory
5. To introduce spectral properties of linear bounded operators

Course Outcomes:

On completion of the course, the students will be able to:

CO1: Understand the basic concepts and fundamental principles of functional analysis.
CO2: Independently prove and thoroughly explain central theorems.
CO3: Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concepts from functional analysis.
CO4: Gain mastery in basic Hilbert space theory and basic operator theory.
CO5: Study in detail the spectral properties of bounded linear operators.

Course Contents:

UNIT 1: Normed linear spaces and Examples, Normed linear spaces as metric spaces, Banach spaces, Hilbert Spaces, Bounded linear maps. **[20Hrs]**

UNIT 2: Riesz representation theorem for Hilbert spaces,Finite dimensional spaces, Quotient spaces. **[15Hrs]**

UNIT 3: Five pillars of functional analysis: Hahn-Banach theorem, Open Mapping theorem, Bounded Inverse theorem, Closed Graph theorem, Uniform Boundedness principle. **[25Hrs]**

UNIT 4: General Results on Compact Operators, Compact self-adjoint operators on Hilbert spaces, Dual spaces, Adjoint operators, Hilbert space adjoint. **[20Hrs]**

UNIT 5: Concept of spectrum of an operator, Spectrum of some standard operators, Finite Dimensional spectral theorem. **[10Hrs]**

Note:. In practical sessions, it is expected to discuss problems based on theory taught.

Recomended Book:

S. Kumaresan and D. Sukumar, *Functional Analysis A first course*, Narosa Publication.

Unit-1: Chapter 1: 1.1-1.5.; **Unit-2**: Chapter 1: 1.6 (except 1.6.1, 1.6.2), 1.7, 1.9.

Unit-3: Chapter 2 All sections except 2.5.1, 2.5.2, 2.6.1.3, 2.6.1.4 & 2.6.1.5.

Unit-4: Chapter 3: 3.1 & 3.4 , Chapter: 4.1-4.3. **Unit-5**: Chapter 5: 5.3, Chapter 6: 6.1.

Reference Books:

1. Ervin Kreyszig, *Introductory Functional Analysis* John Wiley and Sons, 1978.
2. B.V. Limaye, *Functional Analysis* (Second Edition), New Age International (P) limited publishers, 2013.
3. S. Kesavan, *Functional analysis*, Hindustan Book Agency, 2009.
4. P.K. Jain, Khalil Ahmad, Om P. Ahuja, *Functional Analysis*, New Age International (P) limited publishers, 2004.
5. B. Bollobas, *Linear Analysis an introductory course*, 2nd edition, Cambridge Mathematical Texts, Cambridge University Press, 1999.
6. K. Yosida, *Functional Analysis*, Springer-Verlag, 1995.

MTS 653 MJ : PARTIAL DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS

Course objectives:

This course is a study of concept of different types linear and nonlinear partial differential equations. Moreover this course also aims to study the elliptic, Parabolic and hyperbolic boundary value problems.

Course Outcomes:

On completion of the course, student will be able to

CO1: Understand aims and objectives of PDE and Boundary Value Problems

CO2: Familiarize students with mathematical tools and techniques in PDE and BVP.

CO3: Classify and solve the partial differential equations

CO4: Understand canonical forms for different types of differential equations.

CO5: Understand existence and uniqueness theorems

CO6: Solve elliptic, parabolic and hyperbolic boundary value problems by using variable method.

Course Content:

UNIT 1: Partial Differential Equations of First Order [10 Hrs]

1.1 Formation of Partial Differential Equation
1.2 Solution of Partial Differential Equation of First order
1.3 Compatible System of First Order Equations
1.4 Charpit's Method
1.4.1 Special Types of First Order Equations

UNIT 2 : Fundamental Concepts of Partial Differential Equations [16 Hrs]

2.1 Classification of Second Order PDE
2.2 Canonical Forms
2.2.1 Canonical Form for Hyperbolic Equations
2.2.2 Canonical Form for Parabolic Equations
2.2.1 Canonical Form for Elliptic Equations
2.3 Linear Partial Differential Equations with Constant Coefficients
2.3.1 General Method for Finding CF of Reducible Non-homogeneous Linear PDE
2.3.2 General Method for Finding CF of Irreducible Non-homogeneous Linear PDE
2.3.3 Methods for Finding the Particular Integral.

UNIT 3: Elliptic and Parabolic Boundary Value Problems [22 Hrs]

3.1 Occurrence of the Laplace, Poisson and Diffusion Equations
3.1.1 Derivation of Laplace Equation
3.1.2 Derivation of Poisson Equation
3.2 Boundary Value Problems
3.3 Greens First and Second Identity
3.4 Separation of Variables method for Elliptic and Parabolic BV
3.5 Dirichlet and the Neumann Problem for a Rectangle
3.6Interior and Exterior Dirichlet Problem for a Circle
3.7 Interior Neumann Problem for a Circle
3.8 Boundary Conditions for Parabolic Equations
3.9 Elementary Solution of the Diffusion Equation
3.10 Dirac Delta Function

UNIT 4: Hyperbolic Boundary Value Problems **[12 Hrs]**

4.1 Occurrence of the Wave Equation

4.2 Derivation of the One-Dimensional Wave Equation

4.3Solution of the One-Dimensional Wave Equation

4.4 The Initial Value Problem D' Alembert's Solution

4.5 Vibrating String – Variable Separable Solution

Recommended Book:

K. Sankara Rao: Introduction to Partial Differential Equations, Third Edition PHI Learning Private Limited, Delhi (2013).

Unit 1: Chapter 0; 0.4, 0.5, 0.10, 0.11. **Unit 2**: Chapter 1; 1.1 to 1.3, 1.6, 1.7.

Unit 3 Chapter 2: 2.1 to 2.3, 2.6 to 2.10. Chapter 3: 3.1 to 3.5

Unit 4 Chapter 4: 4.1 to 4.5.

Reference Books:

1. Mark A. Pinsky Partial Differential Equations and Boundary Value Problems with Applications, Third Edition, Pure and Applied Undergraduate Texts, -15, American Mathematical Society providence, Rhode Island.
2. Nakhle' H. Asmar, Partial Differential Equations with Fourier Series and Boundary Valur Problems, Second Edition, Pearson Prentice Hall.

****

MTS 654 MJ : MEASURE THEORY AND INTEGRATION

Course Objectives: The aim of this course is

1. To study Measurable sets in and measurable functions.
2. To study Lebesgue dominated convergence theorem and Lebesgue monotone convergence theorem to evaluate integrals.
3. To study connection between differentiation and integration of Lebesgue integrable functions.
4. To study the completion of L_p space.

Course Outcomes: On completion of the course, student will be able

CO1. To understand measurable sets and measurable functions.

CO2. To apply Lebesgue dominated convergence theorem and Lebesgue monotone convergence theorem to evaluate integrals.

CO3. To analyse and explore the relationship between differentiation and integration of Lebesgue integrable functions.

CO4. To construct inter-connections between types of convergence of sequence of measurable functions.

UNIT 1 : **Preliminaries**: **[04 Hrs]**

1.1 Countability.

1.2 Properties of Open Sets, G_δ and F_σ sets.

1.3 Cantor Set.

UNIT 2 : **Measure on Real Line**: **[12 Hrs]**

2.1 Lebesgue Outer Measure.

2.2 Measurable Sets and Regularity.

2.3 Measurable Functions.
2.4 Non-measurable sets and Borel Sets.

UNIT 3 : Integration of Functions on Real Variable: **[12 Hrs]**

3.1 Integration of Non-Negative Functions.
3.2 General Integral.
3.3 Integration of Series.

UNIT 4 : Differentiation: **[12 Hrs]**

4.1 Functions of Bounded Variation.
4.2 Lebesgue Differentiation Theorem.
4.3 Differentiation and Integration.

UNIT 5: Inequalities and Lp spaces: **[10 Hrs]**

5.1 The L^p Spaces,
5.2 Jensen's Inequalities, Holder's and Minkowski inequalities.
5.3 Completion of L^p.

UNIT 6 : Convergence: **[10 Hrs]**

6.1 Convergence in Measure.
6.2 Almost Uniform Convergence.

Recommended Book:

1. G. de Barra, Measure Theory and Integration, New Age International Ltd, Publishers (2000).. **Unit 1.** 1.1,1.5,1.6,1.7 ; **Unit 2**. 2.1 to 2.5; **Unit 3**. 3.1 to 3.3; **Unit 4.** 4.3, 4.4,4.5; **Unit 5.** 6.1, 6.3 to 6.5 ; **Unit 6.** 7.1 to 7.2.

Reference Books:

1. H. L. Roydon, Real Analysis (Third Ed.), Prentice Hall (1995).
2. Real Analysis, E. Stein and R. Shakharchi, New Age International Publishers, Princeton.

****

MTS-661(A) MJ , MTS-662(A)MJP: COMMUTATIVE ALGEBRA

On completion of the course the student should have the following learning outcomes defined in terms of knowledge, skills and general competence:

Course objectives:

The student

1. Knows basic definitions concerning elements in rings, classes of rings, and ideals in commutative rings.
2. Know constructions like tensor product and localization, and the basic theory for this.
3. Know basic theory for noetherian rings and Hilbert basis theorem.
4. Know basic theory for integral dependence, and the Noether normalization lemma.
5. Have insight in the correspondence between ideals in polynomial rings, and the corresponding geometric objects: affine varieties.
6. Know basic theory for support and associated prime ideals of modules, and know primary decomposition of ideals in noetherian rings.
7. Know the theory of Gröbner bases and Buchbergers algorithm.
8. Know the theory of Hilbert series and Hilbert polynomials.
9. Know dimension theory of local rings.

Course objectives:

CO1: Can use algebraic tools which are important for many problems and much theory development in algebra, algebraic geometry, number theory, and topology.

CO2: Have solid experience and training in reasoning with abstract and general algebraic structures.

CO3: Has insight in the most important algebraic theory which is used in other parts of mathematics.

CO4: Has insight in the mathematics that is used in computer algebra.

CO5: See the usefulness of abstract theory development so that different parts mathematics, like number theory and algebraic geometry, can be described in the same framework

Course Content:

UNIT 1: Rings and Ideals **[06 Hrs]**

A brief review of rings, ideals and homomorphisms, Operations on ideals, Extension and contraction of ideals, Nil radical and Jacobson radical.

UNIT 2: Modules **[08 Hrs]**

Modules, sub modules, homomorphism, direct sum and products of modules, exact sequences Tensor product of modules and algebras and basic properties

UNIT 3: Modules of Fractions and Primary Decomposition **[08 Hrs]**

Rings and modules of fractions, Primary decomposition,

UNIT 4: Integral Dependence and Valuation Rings **[08 Hrs]**

Integral dependence, Going up and going down theorems, Valuation rings; Noetherian rings, Artin rings

Recommended Book:

M. F. Atiyah & I. G. Macdonald, Introduction to Commutative Rings, Addison Wesley **[Chapter 1 to 8]**

Reference Books:

1. Zarinski and P. Samuel, Commutative Algebra with a view towards Algebraic Geometry, Springer
2. Irving Kapalansky– Commutative Rings
3. N. S. Gopalakrishnan – Commutative Algebra, Oxonian Press

****

MTS-661(B)MJ, MTS-662(B)MJP: FINANCIAL MATHEMATICS

Course Objectives:

1. Identify basic terminologies in Mathematical Finance.
2. State the concepts of Risk Free and Risky Assets.
3. Differentiate between forward and Futures.
4. Analyse the concept of Risk and apply it to build portfolio from various securities.
5. Describe the principle of No Arbitrage and Fundamental Theorem of Asset Pricing

Course Outcomes:

After successful completion of the course, the student should be able to:

CO1: Understand risk-free and risky assets.

CO2: Model dynamics of the stock prices.

CO3: Make strategy to invest in different derivatives.

Course Content:

UNIT-1: **Risk free Assets** **[05 Hrs]**

1.1 Time Value of Money- Simple Interest,
1.2 Periodic Compounding,
1.3 Stream of Payments,
1.4 Continuous Compounding,
1.5 How to compare Compounding Methods,
1.6 Money Market- Zero Coupon Bonds,
1.7 Coupon Bonds,
1.8 Money Market Account

UNIT-2: Risky Assets **[05 Hrs]**

2.1 Dynamics of Stock Prices- Returns,
2.2 Expected Returns,
2.3 Binomial Tree Model- Risk Neutral Probability,
2.4 Martingale Property
2.5 Other Models- Trinomial Tree Model
2.6 Continuous Time Limit

UNIT-3: Discrete Time Market Models **[06 Hrs]**

3.1 Stock and Money Market Models- Investment Strategies,
3.2 Principle of No Arbitrage,
3.3 Application to Binomial Tree Model,
3.4 Fundamental Theorem of Asset Pricing
3.5 Extended Models

UNIT-4: Portfolio Management **[08 Hrs]**

4.1 Concept of Risk
4.2 Two Securities- Risk and Expected Return on Portfolio
4.3 Several Securities- Risk and Expected Return on Portfolio,
4.4 Efficient Frontier
4.5 Capital Asset Pricing Model,
4.6 Beta Factor,
4.7 Security Market Line

UNIT-5: Forward and Futures **[06 Hrs]**

5.1 Forward Contracts- Forward Price,
5.2 Value of a Forward Contract
5.3 Futures-Pricing, Hedging with Futures

Recommended Book:

1. Mathematics for Finance: An introduction to Financial Engineering, Marek Capinski, Tomasz Zastawniak, Springer Publications. **Unit1.** 2.1 to 2.2, **Unit 2**. 3.1 to 3.3, **Unit 3.** 4.1 to 4.2, **Unit 4.** 5.1 to 5.4, **Unit 5.** 6.1 to 6.2.

Reference Books:

1. The Calculus of Finance, Amber Habib, Universities Press
2. Investment Science, David Luenberger, Oxford University Press
3. John Hull, Option Futures and other derivatives, Prentice Hall.

****

MTS-661(C) MJ, MTS-662(C)MJP: ALGEBRAIC CURVES

Course Objectives:

1. To learn basics of Algebraic Geometry
2. To be able to apply these ideas to the one-dimensional situation of Algebraic Curves
3. To understand Bezout's theorem

Course Outcomes:

CO1: Understand the correspondence between algebraic sets/varieties and ideals associated to them.

CO2: Understand Hilbert's Nullstellensatz.

CO3: Understand how local rings along with other algebraic machinery is useful in defining the intersection numbers.

CO4: Understand the statement and proof of Bezout's theorem.

Course Content:

UNIT-1 Affine Algebraic Sets **[15 Hrs]**

Affine Space and Algebraic Sets, vanishing ideals of a set points, Hilbert Basis Theorem, irreducible components of algebraic sets, Hilbert's Nullstellensatz, integral elements and field extensions

UNIT-2 Affine Varieties **[15 Hrs]**

Coordinate rings and polynomial maps, rational functions, local rings, discrete valuation rings, forms, operations with ideals, ideals with finite number of zeros

UNIT-3 local Properties of Plane Curves **[12 Hrs]**

Multiple points and tangent lines, multiplicities and local rings, intersection numbers

UNIT-4 Projective Varieties and Projective Place Curves **[18 Hrs]**

Projective spaces, projective algebraic sets, affine and projective varieties, projective plane curve, Bezout's theorem, illustrations of Bezout's theorem

Recommended Book:

Algebraic Curves (An Introduction to Algebraic Geometry), William Fulton. Publisher: Available Freely at the following link
https://dept.math.lsa.umich.edu/~wfulton/CurveBook.pdf

Unit 1. 1.1 to 1.10; **Unit 2**. 2.1 to 2.9, **Unit 3**. 3.1 to 3.3; **Unit 4**. 5.1 to 5.3

Reference Books:

1. Phillip Griffiths, Introduction to Algebraic Curves, (AMS Mathematical Monographs)
2. M. F. Atiyah, I.G. McDonald, Introduction to Commutative Algebra

****

MTS 661(D) MJ, MTS 662 (D)MJP : OPTIMIZATION TECHNIQUES

Course Objectives

1. To understand the principles of the zero-sum game, two-person games and to apply the various methods to select and execute various optimal strategies.
2. To understand the techniques of project management.
3. To understand the meaning of inventory control as well as various forms and functional role of inventory.
4. To derive necessary and sufficient conditions for obtaining multivariable optimization problems with equality/inequality constraints.
5. To understand the steps of decision-making process.

Course Outcomes

The successful completion of this course students will able to

CO1: Know about the optimal strategies which can be used in conflict and competitive environment.
CO2: Identify the critical activities and critical path which may affect project completion time.
CO3: Take decision under various decision-making environments.
CO4: Calculate the economic order quantity for minimizing the total inventory cost.
CO5: Use differential calculus-based methods to obtain an optimal solution of problems.

UNIT 1: Theory of Games **[15 Hrs]**

1.1 Decision under uncertainty
1.2 Characteristics of Game Theory
1.3 Two-person zero-sum game
1.4 Solution of game with saddle point
1.5 Solution of games without saddle point
a) Graphical Method b)Dominance Principle

UNIT 2: Network Problems **[15 Hrs]**

2.1 Basic steps in PERT.CPM techniques
2.2 Basic difference between PERT and CPM
2.3 Network diagram representation
2.4 Rules for drawing network diagram
2.5 Critical Path analysis
a) Forward pass computation b) Backward pass computation c) Floats and slack time
2.6 Determination of critical path
2.7 Project evaluation and Review Technique
2.8 Estimation of project completion time

UNIT 3: Classical Optimization techniques **[15Hrs]**

3.1 Unconstrained problems of maxima and minima
3.2 Constrained problems with equality constraints
a) Direct substitution method
b) Lagrangian method
3.3 Constrained problems with inequality constraints
a) Kuhn-Tucker necessary conditions
b) Kuhn-Tucker sufficient condition

UNIT 4: Deterministic inventory control models **[15 Hrs]**

4.1 Introduction
4.2 Direct and indirect inventories
4.3 Reasons for carrying inventory

4.4 Inventory model building
4.5 Single item inventory control models without shortages

Recommended Books:

1. S. D. Sharma, Operations Research, Kedar Nath Ram Nath and Company, Thirteenth edition 2001. **(Unit 1, 2, 3)**
2. J. K. Sharma, Operations Research, (Third Edition, Macmillan India Ltd.), 2008. **(Unit 4)**

Reference Books:

1. Hamdy A. Taha, Operations Research, (Eighth Edition, Prentice Hall of India), 2008.
2. P. K. Gupta and D. S. Hira, Operations Research, (Fifth Edition, S. Chand), 2014.

****

MTS 681 MJP: Research Project **[06 Credits]**

****