Total No. of Questions :8]

SEAT No. :

P2390

[Total No. of Pages : 2

[6061]-414
S.Y.M.A./M.Sc.
MATHEMATICS
MTUTO 144 : Number Theory
(2019 Pattern) (Semester-IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***All questions are compulsory***
2) ***Figures to the right indicate full marks.***

Q1) a) Find all integers that give the remainders 1,2,3 when divided by 3,4,5 respectively. **[7]**

b) Prove that , there are infinitely many units in any real quadratic field. **[4]**

c) Find legendre symbol of $\left(\frac{38}{13}\right)$. **[3]**

Q2) a) Let K[x] denotes ring of polynomials with coefficients in a field K and $f, g \in k[x]$. If $g \neq o$ then prove that there exist polynomials $h, r \in k[x]$ such that f=hg+r, where either r=0 or $r \neq 0$ & deg (r) < deg (g). **[6]**

b) Suppose that P is a prime and that $a, b \in \mathbb{Z}$. Then $\text{ord}_p ab = \text{ord}_p a + \text{ord}_p b$ **[4]**

c) Show that square of any odd integer is of the form 8K+1. **[4]**

Q3) a) State and prove Gauss's lemma. **[8]**

b) If m=5, find quadratic residue and non-residue modulo 5. **[6]**

Q4) a) State and prove prove fermat's theorem. **[5]**

b) If P is an odd prime. The prove that $\left(\frac{a}{p}\right) = a^{\left(\frac{P-1}{2}\right)} (\bmod p)$ **[5]**

c) Prove that square of any integer is of the form 3q or 3q+1 but not of the form 3q+2. **[4]**

P.T.O.

Q5) a) If P≡3 (mod 4) then prove that, $\left(\frac{p-1}{2}\right)! \equiv \pm 1 (\text{mod}\, p)$. **[7]**

b) If f and g are relatively prime and f/gh, then prove that f/h. **[5]**

c) Find $\phi(40)$ **[2]**

Q6) a) Prove that if R is a Euclidean domain and I⊆R is an ideal then there is an element a∈R such that I=Ra={ra|r∈R} **[8]**

b) Prove that if ξ is an algebraic number of degree n, then every number in $\phi(\xi)$ can be written uniquely in form, $a_0 + a_1\xi + ... + a_{n-1}\xi^{n-1}$ where a_1 are rational numbers. **[6]**

Q7) a) Prove that, To any polynomials $f(x)$ & $g(x)$ over φ with $g(x) \equiv 0$, there correspond unique polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x). q(x) + r(x)$. where either $r(x) \equiv 0$ or $r(x)$ is of lower degree than $g(x)$. **[6]**

b) Find last two digits in the ordinary decimal representation 3^{400}. **[5]**

c) Find a least positive integer 'x' such that $13/x^2+1$. **[3]**

Q8) a) Prove that, every quadratic field is of the form $\phi\left(\sqrt{m}\right)$ where m is a square free rational integer, positive or negative but nor equal to 1 numbers of the form $a + b\sqrt{m}$ with rational integers a & b are integers of $\phi\left(\sqrt{m}\right)$. There are the only integers of $\phi\left(\sqrt{m}\right)$ if m≡2 or 3 (mod 4). If m≡1 (mod 4), the numbers $\left(a + b\sqrt{m}\right)/2$. With odd rational integers a & b are also integers of $\phi\left(\sqrt{m}\right)$ and there are no further integers. **[7]**

b) State and prove Euler's theorem. **[5]**

c) Prove that $15x \equiv 24$ (mod 35) has no solution. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P-2369

[Total No. of Pages : 4

[6061]-111
M.A./M.Sc.
MATHEMATICS
MTUT-111 : Linear Algebra
(2019 Pattern) (Semester - I) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that, {a,b} is a linearly dependent set if and only if $a = \lambda b$ or $b=\lambda' a$ for some λ or λ' in F. **[7]**

b) Let A,B,C be points in R_2 then prove that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$. **[4]**

c) The set of all $f \in C(R)$ such that $f\left(\frac{1}{2}\right)$ is a rational number is subset of C(R). Is f subspace of C(R)? Justify. **[3]**

Q2) a) Let $\{v_1,........,v_n\}$be a basis of V over F. If S and T are elements of L(V,W) such that $S(v_i)=T(v_i)$, $l\leq i\leq n$ then prove that S=T and if $w_1,.....,w_n$ be arbitrary vectors in W, then there exists one and only one linear transformation $T \in L(V,W)$ such that $T(v_i) = w_i$. **[7]**

b) Let $T \in L(V,V)$ be given by **[4]**

$$T(u_1) = u_1 - u_2$$
$$T(u_2) = 2u_2$$

where $\{u_1,u_2\}$ is a basis for V.

i) Find rank and nullity of T.

ii) Check T is invertible or not.

P.T.O.

c) Test the linear transformation $T:R_3 \rightarrow R_2$ defined by the system of equation

$$y_1 = 3x_1 - x_2 + x_3$$
$$y_2 = -x_1 + 2x_3$$

is one to one or not. **[3]**

Q3) a) Prove that, every orthonormal set of vectors is linearly independent set. **[6]**

b) State and prove Cauchy-Schwarz Inequality. **[4]**

c) Prove that, a linear transformation T Preserves distances if and only if $\|T(e_1)\| = \|T(e_2)\| = 1$ and $T(e_1) \perp T(e_2)$. **[4]**

Q4) a) Let $v_1, v_2, \ldots v_r$ be characteristics vectors belonging to distinct characteristic roots $\alpha_1, \alpha_2, \ldots, \alpha_r$ of $T \in L(V,V)$ then prove that, $\{v_1, v_2, \ldots, v_r\}$ are linearly independent. **[6]**

b) Let T be a linear transformation on a finite dimensional vector space over F and let $\alpha \in F$. Then show that, α is a characteristic root of T if and only if the determinant $D(T - \alpha, 1) = 0$, where 1 is the identity transformation on V. **[6]**

c) Find the rational canonical form of a linear transformation on a vector space over the field of rational numbers Q whose elementary divisors are $(x-1)^2$, $x^2 - x + 1$. **[2]**

Q5) a) Let $\{v_1, \ldots, v_n\}$ be a basis for V over F, and there exist linear functions $\{f_1, \ldots, f_n\}$ such that for each i, **[6]**

$$f_i(v_i) = 1, \qquad f_i(v_j) = 0, \; j \neq i$$

Then prove that, the linear functions $\{f_1, \ldots, f_n\}$ form a basis for V* over F which is dual basis to $\{v_1, \ldots, v_n\}$.

b) Let $T \in L(V,V)$ and $\{v_1, \ldots, v_n\}$ ba a basis of V and $\{f_1, \ldots, f_n\}$ the dual basis of V*. Let A be the matrix of T with respect to the basis $\{v_1, \ldots, v_n\}$. Then prove that, the matrix of T* with respect to the basis $\{f_1, \ldots, f_n\}$ is the transpose matrix ${}^tA$. **[6]**

c) If $F(x_1, x_2) = x_1^2 - 10x_1x_2 - 5x_2^2$. Find symmetric matrix A whose quadratic equation is $F(x_1, x_2)$. **[2]**

Q6) a) Let V and V' be finite dimensional vector spaces which are dual with respect to a nondegenerate bilinear form B and let V_1 and V_1' be subspaces of V and V' respectively.

Then prove that,

$$\dim V_1 + \dim V_1^{\perp} = \dim V, \ \dim \left(V_1'\right)^{\perp} + \dim V_1' = \dim V'.$$

[5]

b) Find Jordan canonical forms of the following matrices over C. Show that pairs of matrices are similar? **[6]**

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

c) Define an Unitary transformation.

Show that, $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ is unitary matrix. **[3]**

Q7) a) Find an orthonormal basis for the subspace of C(R) generated by the function $\{1, x, x^2\}$ with respect to the inner product $(f, g) = \int_0^1 f(t).g(t).dt$.

[6]

b) Let T be a normal transformation on V. Then prove that, there exist common characteristics vectors for T and T'. For such a vector V, Tv=av and $T'v = \bar{a}v$. **[5]**

c) Compute $A_1 \dot{x} B_1$ where, **[3]**

$$A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Q8) a) Let T be an orthogonal transformation on a real vector space V with an inner product then prove that V is a direct sum of irreducible invariant subspaces $\{W_1,...,W_s\}$ for $S \geq 1$. Such that vectors belonging to distinct subspaces W_i and W_j are orthogonal. **[7]**

b) Let Q be a quadratic form on V whose matrix with respect to the basis $\{e_1,....,e_n\}$ is $S = (\sigma_{ij})$. Let $\{f_1,....,f_n\}$ be another basis of V such that $f_i = \sum_{j=1}^{n} \gamma_{ji} e_j, 1 \leq i \leq n.$ Then prove that the matrix of Q with respect to the basis $\{f_1,...,f_n\}$ is given by $S' = {}^tCSC$ where $C = (\gamma_{ij})$. **[5]**

c) Define symmetric tensor. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2370

[Total No. of Pages : 3

[6061]-112
F.Y. M.A./M.Sc.
MATHEMATICS
MTUT - 112 : Real Analysis
(2019 Pattern) (CBCS) (Semester -I)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions :***
2) ***Figures to the right indicate full marks.***

Q1) a) i) Prove that any set of outer measure zero is measurable. **[4]**

ii) Let A be the set of irrational numbers in [0,1]. Prove that m*(A)=1. **[3]**

b) Let $\{E_k\}_{k=1}^{\infty}$ be the countable collection of sets in A. Prove that

$$m\left(\bigcup_{k=1}^{\infty} E_k\right) \leq \sum_{k=1}^{\infty} m(E_K)$$ **[7]**

Q2) a) i) Evaluate the four Dini's derivatives at $x = 0$ of the function f(x),

where $f(x) = \begin{cases} ax\sin^2\frac{1}{x} + bx\cos^2\frac{1}{x}, & x > 0 \\ px\sin^2\frac{1}{x} + qx\cos^2\frac{1}{x}, & x < 0 \\ 0 \quad , & x = 0 \end{cases}$

given that $a < b, p < q$. **[4]**

ii) State Little woods three principles. **[3]**

b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions on E that converges pointwise *a.e.* on E to the function f. Then prove that f is measurable. **[7]**

P.T.O.

Q3) a) i) With a suitable example. Show that absolutely continuous functions are continuous but the converse is false even for incrrasing functions. **[4]**

ii) For any set A and number y. Prove that m* (A+y) = m*(A) **[3]**

b) Prove that function f on closed bounded interval $[a , b]$ is absolutely continuous on $[a, b]$ if and only if it is an indefinite integral over $[a, b]$. **[7]**

Q4) a) Prove that any set E of real numbers with positive outer measure contains *a* subset that fails to be measurable. **[7]**

b) Show that the function f defined by $f(x)= \dfrac{1}{x^{1/3}}$ $,0 < x \leq 1$ and $f(0)= 0$ is Lebesgue integrable over [0, 1]. **[7]**

Q5) a) If f and g are finite valued and measurable then prove that **[7]**

i) The integer powers f^k, $k \geq 1$ are measurable.

ii) $f+g$ in measurable.

b) i) Let $\{f_n\}_{n=1}^{\infty}$ be an increasing sequence of continuous functions on $[a , b]$ which converges point wise on $[a, b]$ to function f on $[a, b]$. Prove that the convergence is uniform on $[a, b]$. **[4]**

ii) Prove that real valued function which is continuous on its measurable domain is measurable. **[3]**

Q6) a) i) Let f be an increasing function on the closed bounded interval [a,b].

Prove that for $\alpha > 0$ $m^*\left\{x \in (a, b) / \overline{D} f(x) \geq \alpha\right\} \leq \dfrac{1}{\alpha}[f(b) - f(a)]$

[4]

ii) If f is measurable function then show that f^+ and f^- are measurable on E. **[6]**

b) Let f be integrable over the closed bounded interval $[a, b]$. Then prove that $f(x) = 0$ for almost all $x \in [a, b]$ if and only if $\int_{x_1}^{x_2} f = 0$. for all $(x_1, x_2) \subseteq [a, b]$. **[7]**

Q7) a) State and prove Lusin theorem. **[7]**

b) Let E be a measurable set of finite outer measure. Then for each $\in > 0$ prove that there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^{\infty}$ for which if $\mathcal{O} = \bigcup_{k=1}^{n} I_k$ then $m^*(E \sim \mathcal{O}) + m^*(\mathcal{O} \sim E) < \in$. **[7]**

Q8) a) State and prove Egoroff's theorem **[7]**

b) Let E be a bounded measurable set of real numbers. Suppose there is a bounded countably infinite set of real numders $\wedge$ for which the collection of traslates of E, $(\lambda + E)_{\lambda \in \wedge}$ is disjoint.

Then prove that m (E) = 0 **[7]**

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Total No. of Questions : 8]

SEAT No. :

P2371

[Total No. of Pages : 3

[6061]-113
F.Y. M.A/M.Sc.
MATHEMATICS
MT UT -115 : Ordinary Differential Equations
(CBCS) (2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt the following.

a) i) Show that the function $\phi(x)=\frac{2}{3}+e^{-3x}$ is the solution of the equation $y'+3y=2$. **[2]**

ii) Show that every solution of the equation $y'+2xy=x$ tends to $\frac{1}{2}$ as $x\to\infty$. **[5]**

b) Explain the method of solving the equation $y'+ay=0$ where a is complex constant. **[7]**

Q2) Attempt the following.

a) Solve the equation $\mathrm{L}y'+\mathrm{R}y=\mathrm{E}$ where L, R and E are constants. Also show that every solution of it tends to E/R as $x\to\infty$. **[7]**

b) Find solution of the equation $y''-y'-2y=e^{-x}$. **[7]**

P.T.O.

Q3) Attempt the following.

a) i) Find the solution of the initial value problem $y''-2y'-3y=0$ with $y=(0)=0,\ y'(0)=1.$ **[4]**

ii) Compute two linearly independent solutions of the equation $3y''+2y'=0$ **[3]**

b) Explain the method for solving non-homogeneous equation with constant coefficients of order *n*. **[7]**

Q4) Attempt the following.

a) Define Wronskian of $\phi_1,\ \phi_2$. Hence, show that two solutions ϕ_1 and ϕ_2 of $y''+a_1y'+a_2y=0$ are linearly independent on an interval I if and only if $W(\phi_1,\ \phi_2)\neq 0,\ \forall\ x\in I.$ **[7]**

b) Verify that the function $\phi_1(x)=x^2$ satisfies the equation $y''-\frac{2}{x^2}y=0,\ (0<x<\infty)$ and find the second independent solutions. **[7]**

Q5) Attempt the following.

a) Compute $W(\phi_1,\ \phi_2,\phi_3)(x)$ at a point $x=0$ for the function $\phi_1=e^x,\ \phi_2=xe^x,$ and $\phi_3=x^2e^x.$ **[7]**

b) Explain the method of reduction of order for solving n[th] order homogeneous equation with variable coefficients. **[7]**

Q6) Attempt the following.

a) Show that $\phi_1(x)=|x|^i$ and $\phi_2(x)=|x|^{-i}$ are linerly independents solutions of the equation $x^2y''+xy'+y=0$. **[7]**

b) i) If Γ is gamma function, show that $\Gamma(z+1)=z\Gamma(z)$. **[3]**

ii) Compute the indicial polynomial and its roots for the differential equation $x^2y''+(x+x^2)y'-y=0.$ **[4]**

Q7) Attempt the following.

a) Explain the variable separable method for first order differential equation $y' = F(x, y)$ **[7]**

b) Obtain two solutions of Bessel's equation of order zero, $x^2 y'' + xy' + x^2 y = 0.$ **[7]**

Q8) Attempt the following.

a) Determine that the equation $\left(3x^2 \log|x| + x^2 + y\right)dx + x \cdot dy = 0$ is exact or not and solve. **[7]**

b) Compute first five approximations $\phi_0, \phi_1, \phi_2, \phi_3, \phi_4$ of the solution of the initial value problem $y' = 1 + xy,\ y(0) = 1$ **[7]**

Total No. of Questions : 8]

SEAT No. :

P2372

[Total No. of Pages : 3

[6061]-114
M.Sc.-I
MATHEMATICS
MTUT - 114 : Advanced Calculus
(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove chainrule for derivative of scalar field. **[6]**

b) Evaluate directional derivative of scalar field $f(x, y, z) = \left(\frac{x}{y}\right)^2$ at (1, 1, 1) in the direction of $2\overline{i} + \overline{j} - \overline{k}$. **[4]**

c) Show that for the function $f(x, y) = \frac{x-y}{x+y}$ if $(x+y) \neq 0$ repeated limit exist but not equal. **[4]**

Q2) a) Prove that if f is differentiable at 'a' with total derivative T_a then derivative $f'(a, y)$ exist for every y in $\mathbb{R}^n$ and $T_a(y) = f'(a, y)$. Moreover f' (a, y) is linear combination of the component of y infact if $y = (y_1\ y_2 .... y_n)$ wehave $f'(a, y) = \sum_{k=1}^{n} D_k\, f(a)\, y_k$. **[6]**

b) Calculate the line integral of the vector field $f(x, y) = (2a - y)\ \overline{i} + x\overline{j}$ along path described by $\alpha(t) = a$ (t-sint) $\overline{i} + a(1-\cos t)\overline{j}$ $0 \leq t \leq 2\pi$. **[4]**

c) Give an example of scalar field f such that $f'(x, y) > 0$ for fixed vector y and every vector x. **[4]**

Q3) a) Prove that, if ϕ be a differentiable scalar field with continuous gradient $\nabla\phi$ on an open connected set s in $\mathbb{R}^n$. Then for any two points a and b joined by piecewise smooth path α in s we have $\int_a^b \overline{\nabla}\phi . d\overline{\alpha} = \phi(\overline{b}) - \phi(\overline{a})$. **[6]**

b) Give any two basic proportion of line integral. **[4]**

c) Prove that if $f(x, y) = y\overline{i} - x\overline{j}$ then f is not gradient. **[4]**

P.T.O.

Q4) a) Prove that if ϕ be real valued function that is continuous on an interval $[a, b]$ then graph of ϕ has content zero. **[6]**

b) Evaluate $\int_{c} \frac{(x+y)dx-(x-y)dy}{x^2+y^2}$. **[4]**

Where c is circle $x^2 + y^2 = a^2$ traversed once in counterclockwise direction.

c) Calculate the line integral of the vector field $\overline{F}; \overline{f}(x, y) = (x^2 - 2xy)\overline{i} + (y^2 - 2xy)\overline{j}$ from (–1, 1) to (1, 1) along the parabola $y = x^2$. **[4]**

Q5) a) Prove that fundamental vector product is normal to the surface. **[6]**

b) Evaluate the double integral $\iint_{Q}\left(x^3 + 3x^2 y + y^3\right)dxdy$ where $Q = [0,1]\times[0,1]$. **[4]**

c) Define Bounded set of content zero. Give any two consequences to this definition. **[4]**

Q6) a) State and prove Greens theorem. **[6]**

b) Use Greens th^m to evaluate the line integral $\int_{c}\left(5 - xy - y^2\right)dx - \left(2xy - x^2\right)dy$ where C is square with vertices (0, 0) (1, 0) (1, 1) (0, 1) traversed counter clock wise. **[4]**

c) Find the jacobian for transformation by polar co-ordinates. **[4]**

Q7) a) i) Define curl and divergence of vector field **[6]**

ii) Show that a vector field $\overline{f}(x, y) = -\frac{y}{x^2+y^2}\overline{i} + \frac{x}{x^2+y^2}\overline{j}$ has zero divergence and zero curl.

b) Evaluate $\iint_{Q}\left(x\sin y - ye^{x}\right)dx\,dy$ where $Q = [-1,1]\times[0,\pi/2]$ **[4]**

c) Show that $f(x, y) = \frac{-y}{x^2+y^2}\overline{i} + \frac{x}{x^2+y^2}\overline{j}$ is not gradient on the set $S = R^2 - \{(0, 0)\}$. **[4]**

Q8) a) State and prove stokes theorem. **[6]**

b) Define: **[4]**

i) Area of parametric surface.

ii) Surface integral.

c) Evaluate $\iiint\limits_{s}\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\right)dx\,dy\,dz$ where s is the solid bounded by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P2373

[Total No. of Pages : 2

[6061]-115

First Year M.A./M.Sc.

MATHEMATICS

MTUT-113 : Group Theory

(2019 Pattern) (Semester-I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define a subgroup let H be finite subset of group G. Then prove that H is a subgroup of G. if and only if H is closed under the operation of G. **[5]**

b) Prove that the center of a group is a normal subgroup of the group. **[5]**

c) Find the inverse of the element $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$ in $GL(2, \mathbb{Z}_{11})$. **[4]**

Q2) a) Prove that set of 2×2 matrices with determinant 1 having entries from θ forms a group under matrix multiplication. Is this group abelian? Justify. **[5]**

b) Prove that subgroup of a cyclic group is cyclic. **[5]**

c) If $\tau=(7\ 1\ 4)(5\ 2)$, $\rho=(8\ 4\ 6\ 9\ 1)(10\ \ 2\ \ 11) \in S_{11}$ .The find $\rho^{-1}\tau\rho$ **[4]**

Q3) a) Suppose G is a group that his exactly one non-trivial proper subgroup. Prove that G is isomorphic to $\mathbb{Z}p^2$ **[5]**

b) Prove that the set A_n of even permutations in S_n is a normal subgroup of S_n of order $\frac{n!}{2}$. **[5]**

c) Let K be a proper subgroup of H and H a proper subgroup of G. If O(K)=42 and O(G)=420 what are the possible orders of H. **[4]**

P.T.O.

Q4) a) State and prove fundamental theorem of cyclic groups. **[5]**

b) If N is a normal subgroup of G and $\left|G/N\right|$=m, show that $x^m \in N$ for all x in G. **[5]**

c) if G is a non-abelian group of order p^3, where p is prime and $Z(G) \neq \{e\}$, prove that O(Z(G))=p. **[4]**

Q5) a) Let G be a finite group of permutations of a set S. prove that for any $i \in S, O(G) = |orb_G(i)||\text{stub}_G(i)|$. **[5]**

b) If H and K are subgroups of group G then show that HK is a subgroup of G if and only if HK=KH. **[5]**

c) Determine all homomorphisms from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{24}$ **[4]**

Q6) a) Let H be an index 2 Subgroup of group G, prove that $a^2 \in H \ \forall a \in G$ **[5]**

b) Show that conjugacy relation is an equivalence relation. **[5]**

c) Let $\phi : G \to \overline{G}$ be a homomorphism and $g \in G$. Then prove that if |g| is finite then $|\phi(g)|$. divides |g|. **[4]**

Q7) a) Prove that every permutation of a finite set can be written as a cycle or a product of disjoint cycles. **[5]**

b) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$. **[5]**

c) Prove that a group of order 15 is not simple. **[4]**

Q8) a) State and prove sylow's Third theorem. **[5]**

b) Let G be a group and |G|=2p, where p is a prime prove that G is either isomorphic to $\mathbb{Z}_{2p}$ or D_p. **[5]**

c) Prove that on abelian group with two distinct elements of order 2 must have a subgroup of oder 4. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P-2374

[Total No. of Pages : 4

[6061]-211
F.Y. M.A./M.Sc.
MATHEMATICS
MTUT-121 : Complex Analysis
(2019 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions out of eight questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and prove the fundamental theorem of algebra in $\mathbb{C}$. **[5]**

b) Let $f : A \to \mathbb{C}$, $z_0 \in A$, $r > 0$ such that $B_r(z_0) C A$. Then prove that & is complex differentiable at z_0 if and only if $\exists \alpha \in \mathbb{C}$ and a function $\phi : B_s(0)/\{0\} \to \mathbb{C}$, $(0 < s \leq r)$ such that for all $h \in B_s(0)/\{0\}$

$$f(z_0 + h) - f(z_0) = h\alpha + h\phi(h) \; ; \; \lim_{h \to \infty} \phi(h) = 0 .$$ **[5]**

c) i) If $|z| = 1$ then compute $|1 + z|^2 + |1 - z|^2$.

ii) Find the absolute value of $\frac{3}{3+4i}$.

[4]

Q2) a) Let $f : U \to V$ and $g : V \to \mathbb{R}$ be such that f is differentiable at $z_0 \in U$ and g is differentiable at $w_0 = f(z_0) \in v$. Then prove that gof is differentiable at z_0 and

$$\mathrm{D(gof)}_{z_0} = \mathrm{D}(g)_{w_0} 0 \, \mathrm{D}(f)_{z_0} .$$ **[5]**

b) Let $f(x,y) = \begin{cases} \dfrac{x^3}{x^2 + y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$

Then show that f is continuous and all the directional derivatives of f exists. **[4]**

P.T.O.

c) Use polar coordinates to show that $z \to |z|^2$ is complex differentiable at 0. **[3]**

d) Compute the length of the circle $c_r : z(0) = a + re^{i\theta}, 0 \leq \theta \leq 2\pi$. **[2]**

Q3) a) Let f be a continuous function in a region Ω and complex differentiable in Ω/A where A is a discrete subset of Ω. Let T be a triangle completely contained in Ω. Then prove that $\int_{\partial T} f(z)dz = 0$. **[5]**

b) Let f be a complex differentiable in a region Ω. Then prove that f has complex derivatives of all order in Ω. Moreover if D is a disc whose closure is contained in Ω and $z \in$ int D then prove that for all $n \geq 0,\ f^{(n)}(z) = \frac{n!}{2\pi i}\int_{\partial D} \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi$. **[5]**

c) Let c is the unit circle traversed counter clockwise then find the value of $\int_c \frac{z^3}{2z - i} dz$. **[4]**

Q4) a) Let U be a convex region and A be a discrete subset of U. Let f be a continuous function on U and complex differentiable on U/A. Then for any closed contour w in U, prove that $\int_w f(z)dz = 0$. **[5]**

b) Let f be a complex differentiable function on a disc $B_R(z_0)$. Then for $0 < r < R$, show that $f(z_0) = \frac{1}{2\pi}\int_0^{2\pi} f(z_0 + re^{i\theta})d\theta$. **[4]**

c) Identify the singularity and its nature for the function $f(z) = e^{1/z}, z \neq 0$. **[3]**

d) Define entire function. Give one example. **[2]**

Q5) a) Let f be a holomorphic function in $B_r(a)/\{a\}$ and let a be an essential singularity of &. Then prove that f takes values arbitrarily close to any arbitrary complex number inside arbitrary neighbourhood of a. **[5]**

b) Let Ω be a holomorphic function on $A(0, r_1, r_2)$. Then show that the function $h : r \rightarrow \int_{|z|=r} f(z)dz$ is a constant on the interval (r_1, r_2). **[5]**

c) Find the residue of the function $f(z) = \frac{\sinh z}{z^3}$ at all singular points that lies inside $|z| = 1$. **[4]**

Q6) a) Let $f : \hat{\mathbb{C}} \rightarrow \hat{\mathbb{C}}$ be a holomorphic function. Then prove that f is a rational function. **[5]**

b) Obtain the Laurent series expansion of the function $f(z) = \frac{1}{1+z^2}$ in the annulus $A = \{z : |z - 1 - i| < 1\}$. **[5]**

c) Prove that $\int_0^\pi e^{-R\sin\theta} d\theta < \frac{\pi}{R}, R > 0$. **[4]**

Q7) a) Use complex method to show that

$$\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta} = \frac{2\pi}{\sqrt{1-a^2}}, |a| < 1, a \in \mathbb{R}.$$ **[7]**

b) Find the value of $\int_\theta^\infty \frac{2x^2 - 1}{x^4 + 5x^2 + 4} dx$. **[7]**

Q8) a) Let f, g be holomorphic functions in an open set containing the closure $\bar{D}$ of a disc D and satisfy the inequality $|f(z) - g(z)| < |g(z)|\ \forall z \in \delta D$. Then show that f and g have same number of zeros inside D. Hence determine the number of zeros of the polynomial $T^7 - 4T^3 + T - 1$ inside the unit circle. **[6]**

b) Show that if $|f(z)| < 1$ for $|z| < 1$ and $f(z)$ has a zero of order n at 0, then **[5]**

i) $|f(z)| < |z|^n$ for all $|z| < 1$.

ii) $|f^{(n)}(0)| \leq n!$.

c) Find the Cauchy's principal value of $\int_{-\infty}^{\infty} \frac{e^{iax}}{1-x} dx$ for H : $|z| < 1$. **[3]**

□□□

Total No. of Questions : 8]

SEAT No. :

P2375

[Total No. of Pages : 2

[6061]-212
MATHEMATICS
F.Y. M.A./M.Sc.
MTUT - 122: General Topology
(2019 Pattern) (CBCS) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that a countable union of countable sets is countable. **[6]**

b) Show that a set $\mathbb{Z}$ of all integers is countably infinite. **[4]**

c) Show that if A is an infinite subset of $\mathbb{Z}_+$ then A is also countable. **[4]**

Q2) a) Show that the topologies of $\mathbb{R}_1$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$ but are not comparable with one another. **[6]**

b) Show that B = $\{(a,b) / a < b, \text{a and b rational}\}$ is a basis that generated the standard topology on $\mathbb{R}$. **[4]**

c) Show that π_1 and π_2 are open maps. **[4]**

Q3) a) Show that if Y is a subspace of X and A is a subset of Y then the topology A inherits as a subspace of Y is the same as the topology it inherits as a subspace of X. **[6]**

b) Prove that every finite point set in a Hausdorff space X is closed. **[4]**

c) Show that a subspace of a Hausdorff space is Hausdorff. **[4]**

Q4) a) Show that [a,b] is homeomorphic to [0,1] . **[5]**

b) State and prove pasting lemma. **[5]**

c) Let A be a connected subspace of X. If $\subset \mathrm{A} \subset \mathrm{B}\overline{\mathrm{A}}$ then show that B is also connected. **[4]**

Q5) a) Show that every path connected space is connected. **[5]**

b) Show that every closed subspace of a compact space is compact. **[5]**

c) Let $f : x \rightarrow y$ be a bijective continuous function. If x is compact and y is Hausdorff then prove that f is homeomorphism. **[4]**

P.T.O.

Q6) a) Show that metrizable space is first countable. **[5]**

b) Let A be a closed subspace of X. Show that if X is Lindel öf then A is Lindel öf. **[5]**

c) Define: **[4]**

i) Lindel öf space.

ii) Normal space.

Q7) a) Show that $\mathbb{R}_1$ is normal. **[6]**

b) Prove that every regular space with a countable basis is normal. **[6]**

c) Define : Regular space. **[2]**

Q8) a) Prove that an arbitrary product of compact spaces is compact in the product topology. **[7]**

b) Show that a connected normal space having more than one point is uncountable. **[5]**

c) State Urysohn lemma. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P-2376

[Total No. of Pages : 2

[6061]-213

First Year M.A//M.Sc.

MATHEMATICS

MTUT 123: Ring Theory

(2019 Pattern) (Semester - II) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) The natural multiplication of cosets of I, namely, $(a + \mathrm{I})(b + \mathrm{I}) = ab + \mathrm{I}$ make R/I into a ring if and only if I is a 2-sided ideal. **[6]**

b) Define : **[4]**

i) Division Ring

ii) Characteristic of a ring.

c) Define simple ring and give an example of simple ring. **[4]**

Q2) a) If R is a ring with 1 and I is a 2-sided ideal in R such that $\mathrm{I} \neq \mathrm{R}$ then prove that there is a 2-sided maximal ideal M such that $\mathrm{I} \subseteq \mathrm{M}$. **[7]**

b) Show that ring R is an integral domain if and only if $\mathrm{R} \neq (0)$, R has no non-trivial nilpotent elements and (0) is prime ideal in R. **[7]**

Q3) a) State and prove the epimorphism theorem. **[7]**

b) For $z \leq n \in \mathrm{N}$, then prove that any ring $\mathbb{Z} / n\mathbb{Z}$ is a field if and only if $\frac{\mathbb{Z}}{n\mathbb{Z}}$ is an integral domain if and only if n is a prime number. **[7]**

P.T.O.

Q4) a) Prove that every Euclidean domain is PID. **[7]**

b) Prove that ring $\mathrm{End}_K(V)$ is a simple ring if and only if V is a finite dimensional vector space over field K. **[7]**

Q5) a) Let I be an ideal in a ring R. Then I is a 2-sided ideal in R if and only if is the kernel of some homomorphism $f : R \to S$ for a suitable ring S. **[7]**

b) Prove that a vector space is a free module. **[7]**

Q6) a) Let S be subring of a ring. Give an example such that S and R both have unities but they may not be the same. **[7]**

b) Prove that a Euclidean domain R has unity and whose group of units is given by $U(R) = \{a \in R^* / d(a) = d(1)\}$. **[7]**

Q7) a) Prove that set of all nilpotent elements in a commutative ring R with 1 is intersection of all prime ideals. **[7]**

b) If d is a positive integer, then the ring $Z\left[i\sqrt{d}\right]$ is a factorization domain. **[7]**

Q8) a) Show that direct sum of free modules is a free module and give an example of a non-free module. **[7]**

b) Show that the ring Z[i] of Gaussian integers is Euclidean. **[7]**

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Total No. of Questions : 8]

SEAT No. :

P-2377

[Total No. of Pages : 4

[6061]-214

F.Y. M.A./M.Sc.

MATHEMATICS

MTUT-124 : Advanced Numerical Analysis

(2019 Pattern) (CBCS) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Derive composite Simpson's Rule with error term : **[5]**

$$\int_a^b f(x)dx = \frac{h}{3}\left[f(x_0) + 4\sum_{j=1}^{m} f(x_{2j-1}) + 2\sum_{j=1}^{m-1} f(x_{2j}) + f(x_{2m})\right]$$

$$-\frac{(b-a)h^4}{180} f^4(\xi)$$

b) Evaluate $\int_{-2}^{2} \frac{x}{5+2x} dx$ by using Trapezoidal rule by dividing interval [–2, 2] into five equal subintervals. **[5]**

c) Define : **[4]**

i) Relative error

ii) Triangular matrix

Q2) a) Apply Euler's method of approximate the solution of initial value problem $\frac{dx}{dt} = \frac{t}{x}$; $0 \le t \le 5$ $x(0) = 1$ using 4-steps. **[5]**

b) Find solution of initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}(1 \le t \le 6), x(1) = 1$ using second order Range-Kutta method with $n = 0.5$. **[5]**

P.T.O.

c) Solve the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t} (1 \le t \le 1.5), x(1) = 1$ $h = 0.25$ by using Taylor method of order N = 2. **[4]**

Q3) a) Solve the following system of equations by using Gaussian elimination with scaled partial pivoting **[5]**

$x_1 + 2x_2 - x_3 + 2x_4 = 10$

$x_1 + x_2 + 2x_3 - 3x_4 = -3$

$2x_1 - x_2 + 3x_3 - x_4 = 5$

$2x_1 - 4x_2 + 2x_3 - x_4 = -4$

b) Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}$ by using crout decomposition method. **[5]**

c) Compute the condition number K_∞ for the matrix $A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}$. **[4]**

Q4) a) Solve the following system of linear equations by SoR method with $x^{(0)} = [0\ 0\ 0]^T$ and $w = 0.9$ (perform 2-iterations) **[5]**

$5x_1 + x_2 + 2x_3 = 10$

$-3x_1 + 9x_2 + 4x_3 = -14$

$x_1 + 2x_2 - 7x_3 = -33$

b) Solve the following system of linear equations by Guass-Seidel method, start with $x^{(0)} = [0\ 0\ 0]^T$ (perform 3 iterations) **[5]**

$4x - y = 2$

$-x + 4y - z = 4$

$-y + 4z = 10$

c) Construct Householder matrix H for $w = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T$. **[4]**

Q5) a) Find the matrix $A=\begin{bmatrix} -2 & -2 & 3 \\ -10 & -1 & 6 \\ 10 & -2 & -9 \end{bmatrix}$ with initial vector $x^{(0)} = [1\ 0\ 0]^T$ perform three iteration of power method to find dominant eigenvalue and corresponding eigen vector. **[5]**

b) Solve the following system of non-linear algebraic equations by using Newtons method start with $x^{(0)} = [1\ 1\ 1]^T$ (perform 3 iterations) **[5]**

$$x_1^3 - 2x_2 - 2 = 0$$

$$x_1^3 - 5x_3^2 + 7 = 0$$

$$x_2 x_3^2 - 1 = 0$$

c) Use the QR Factorization of a symmetric tridiagonal matrix $A=\begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ find product $R^{(0)}Q^{(0)}$. **[4]**

Q6) a) Let f be a continuous function with m continuous derivatives. The equation $f(x) = 0$ has a root of multiplicity m at $x = p$ if and only if $f(p) = f'(p) = f''(p) = \ldots\ldots = f^{(m-1)}(p) = 0$ but $f^{(m)}(p) \neq 0$. **[5]**

b) Show that when Newton's method is applied to equation $x^2 - a = 0$, the resulting iteration function is $g(x)=\frac{1}{2}\left(x+\frac{a}{x}\right)$. **[5]**

c) Determine the following limit and determine the corresponding rate of convergence $\lim\limits_{n\to 0}\frac{\sin n}{n}$. **[4]**

Q7) a) The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a root on the interval (1, 2). Approximate this zero within an absolute tolerance of 5×10^{-15} using Newton's method starting with $P_0 = 1$. **[5]**

b) Prove that the order of convergence of secant method is approximately 1.618 (α = 1.618) and asymptotic error constant **[5]**

$$\lambda \approx c^{\frac{1}{\alpha}} = \left(\frac{f''(p)}{2f'(p)}\right)^{(\alpha-1)}$$

c) Define the terms : **[4]**

i) Order of convergence

ii) Orthogonal matrix

Q8) a) Derive the closed Newton cotes with $n = 3$. **[5]**

$$\int_a^b f(x)dx = \left(\frac{b-a}{8}\right)\left[f(a) + 3f(a+\Delta x) + 3f(a+2\Delta x) + f(b)\right]$$

b) Derive the second order central difference approximation for the first derivative including error term : **[5]**

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} - \frac{h^2}{6}f'''(\xi)$$

c) For the following differential equation identify the function f(t, x) and calculate $\frac{df}{dt}, \frac{d^2f}{dt^2}, \frac{d^3f}{dt^3}$ **[4]**

$x^1 + 2x^2 = t^2 - 1.$

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Total No. of Questions: 8]

SEAT No. :

P2378

[Total No. of Pages : 3

[6061]-215
M.Sc.-I
MATHEMATICS
MTUT-125 : Partial Differential Equations
(CBCS 2019 Pattern) (Semester-II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that the PDE's $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatible **[5]**

b) Attempt the following

i) Find the complete integral of the partial differential eqution $z = px + qy + p^2 + q^2$ by charpits method **[4]**

ii) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible and hence find it's solution **[5]**

Q2) a) Explain the method of second order partial differential equation Rr+Ss+Tt+F$(x, y, z, p, q) = 0$ to a cannonical form if $S^2 - 4RT > 0$ **[5]**

b) Attempt the following **[5]**

i) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$ to a cannonical Form and hence solve it

ii) Classify the PDE's **[4]**

1) $u_{xx} + 2u_{xy} + u_{yy} = 0$

2) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$

P.T.O.

Q3) a) Derive the diffusion equation of second order differential equation **[5]**

b) Attempt the following **[6]**

i) Find by method of separation of variables the solution $u(x,t)$ of the Boundry value Problem

$u_t = 3u_{xx}, \quad t > 0, 0 < x < 2$

$u(0,\text{t}) = 0, u(2,t) = 0, t > 0$

$u(x,0) = x \ ; 0 < x < 2$

ii) Solve $z + 2u_x - (u_x + u_y)^2 = 0$ by Jacobi's method **[3]**

Q4) a) Derive a poisson Equation of second order partial differential equation **[5]**

b) Attempt the following

i) Prove that if the Dirichlet problem for a bounded region has a solution then it is unique **[4]**

ii) Solve by the method of Separation of variables $\frac{\partial^2 u}{\partial x^2} - \frac{2\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ **[5]**

Q5) a) Find the steady State temperature distribution in a thin rectangular plate bounded by lines $x = 0, x = a, y = 0, y = b.$ The edges $x = 0, x = a, y = a$ are kept at temperature zero while the edge $y = b$ is kept at 100°C. **[8]**

b) A uniform rod 20 cm is insulated over its sides. Its ends are kept at 0°C its initial temperature is sin $\left(\frac{\pi x}{20}\right)$ at a distance '*x*' from an end find temperature $u(x,t)$ at time *t*, given that $\frac{\partial u}{\partial t} = a^2\left(\frac{\partial^2 u}{\partial x^2}\right)$ **[6]**

Q6) a) Prove that if $\alpha_r D+\beta_r D'+\gamma_r$ is a factor of $F(D,D')$ and $\phi_r(\xi)$ is an arbitrary Function of the single variable ξ, then $u_r = exp\left(\frac{-\gamma_r x}{\alpha_r}\right)\phi_r(\beta_r x-\alpha_r y)$ for $\alpha_r \neq 0$ **[6]**

b) Attempt the following

i) Find the complementary function of the partial differential equation

$$\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}-\frac{\partial^3 z}{\partial x\partial y^2}+\frac{\partial^3 z}{\partial y^3}=e^{x+y}$$ **[4]**

ii) Solve $(D+D'-1)\ (D+2D'-3)\ z=0$ **[4]**

Q7) a) Solve $\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial y^3}=0$ **[4]**

b) Attempt the following

i) Find the general integral of partial differential equation **[5]**

$(y^2+z^2-x^2)P-2xyq+2zx=0$

ii) Find complete solution of $px+qy=pq$ by charpits method **[5]**

Q8) a) Explain the charpits method of solving non linear partial differential equation $f(x,y,z,p,q)=0$ **[5]**

b) Attempt the following **[5]**

i) Show that the equations $xp=yq$ and $z(xp+yq)=2xy$ are compatible and Solve them

ii) Find the complete integral of $p^3+q^3=27z$ **[4]**

Total No. of Questions : 8]

SEAT No. :

P-2379

[Total No. of Pages : 3

[6061]-311

MA/M.Sc.

MATHEMATICS

MTUT-131 : Functional Analysis

(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five of the following questions.***
2) ***Figures to the right side indicate full marks.***
3) ***Symbols have their usual meanings.***

Q1) a) i) Define : Normed linear space. **[2]**

ii) Let N be a normed linear space. For any $x, y \in$ N, show that **[3]**

$$\big|\|x\| - \|y\|\big| \le \|x - y\|$$

b) Let N and N′ be normed linear spaces and β(N, N′) be the set of all continuous linear transformations from N into N′. Prove that β(N, N′) is a normed linear space. Also, Prove that if N′ is a complete then β(N, N′) is a Banach space. **[7]**

c) Give a statement of the open mapping theorem. **[2]**

Q2) a) Let M be a linear subspace of normed linear space N, and let F be a functional on M if $x_0 \notin$ M and M = M + [x_0] then prove that f can be extended to a linear functional f_0 such that $\|f_0\| = \|f\|$. **[7]**

b) Let B be a Banach space and N be a normed linear space. If {T_i} is a non-empty set of continuous linear transformations of B into N with the property that {Ti(x)} is a bounded subset of N for each vector x in B, then show that {$\|$ Ti $\|$} is a bounded set of numbers that is {Ti} is bounded as a subset of $\mathbb{B}$(B, N) **[7]**

P.T.O.

Q3) a) Let B and B′ be two Banach spaces and T is a continuous linear transformation of B onto B′. Then prove that image of each open sphere centred at origin in B contains an open sphere centred at origin in B′. **[7]**

b) Show that any two finite dimensional vector spaces with same dimension over same field are isomorphic. **[3]**

c) True or False? Justify your answer "IF N is reflexive, then N is complete." **[4]**

Q4) a) If x and y are any two vectors in a Hilbert space. Prove that : **[5]**

i) $\|x + y\|^2 + \|x + y\|^2 = 2\,\|x\|^2 + 2\,\|y\|^2$

ii) $\|x + y\|^2 - \|x - y\|^2 + i\|x - iy\|^2 - i\|x - iy\|^2 = 4(x, y)$

b) Prove : A closed convex subset C of a Hilbert space H contains a unique vector of a smallest norm. **[5]**

c) Show that Hilbert space is uniformly convex. **[4]**

Q5) a) Let M be a proper closed linear subspace of a Hilbert space H. Prove that there exists a non-zero vector Z_0 in H such that $Z_0 \perp M$. **[5]**

b) Let $\{ei\}$ be the orthonormal set in a Hilbert space H. If x is any vector in H then prove that : **[5]**

$\Sigma \mid (x, e_i) \mid^2 \leq \| x \|^2$

c) If H is finite dimensional Hilbert space, show that every isometric isomorphism of H into itself is unitary. **[4]**

Q6) a) Let P be a projection on Hilbert space H with it's range M and null space N. **[6]**

Then Prove that :

$M \perp N$ if and only if P is self-adjoint. Also prove $N = M^{\perp}$.

b) Let M be closed linear subspace of H. Prove that M is invarient under T if and only if $M^{\perp}$ is invarient under T*. **[4]**

c) Let H be a Hilbert space and let F be an arbitrary functional in H*. Prove that there exists a unique vector y in H such that $f(x) = (x, y)$ **[4]**

Q7) a) If N_1 and N_2 are normal operator on H such that $N_1 o N_2^* = N_2^* o N_1$ and $N_2 o N_1^* = N_1^* o N_2$ then prove that $N_1 + N_2$ and $N_1 N_2$ are normal operator. **[5]**

b) Let T be a normal operator on a Hilbert space H. Prove that x is an eigen vector of T with eigen value λ if and only if x is an eigen vector of T* with eigen value $\bar{\lambda}$. **[5]**

c) Let T be any arbitrary operator on H and N be a normal operator. Prove that if T commutes with N then T commutes with N*. **[4]**

Q8) a) Let T be an operator on H. Prove that T is singular if and only if O is a eigen value of T. **[5]**

b) Let T be a normal operator on H. Then prove that each eigen space of T reduces T. **[5]**

c) Let $T : R^3 \rightarrow R^2$ be a linear transformation defined by **[4]**

$T(x, y, z) = (2x + y - z, 3x - 2y + 4z)$. Find the matrix of T with respect to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$

Total No. of Questions: 8]

SEAT No. :

P2380

[Total No. of Pages : 3

[6061]-312

S.Y.M.Sc.

MATHEMATICS

MTUT-132 : Field Theory

(2019 CBCS Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions of the following.*
2) *Figures to the right indicates full marks.*

Q1) a) Let $F \subseteq E \subseteq K$ be fields. [K:E] $< \infty$ and [E:F] $< \infty$ then prove that

i) $[K:F] < \infty$

ii) $[K:F] = [K:E]\,[E:F]$ **[7]**

b) Define algebraically closed field. Prove that a field K is algebraically closed if every irreducible polynomial in K[x] is of degree 1. **[5]**

c) Show that $x^3 - x - 1 \in \mathbb{Q}[x]$ is irreducible over $\mathbb{Q}$. **[2]**

Q2) a) Show that $f(x) = x^5 - 9x + 3$ is not solvable by radicals over $\mathbb{Q}$. **[7]**

b) If E is a finite extension of F then prove that E is an algebraic extension of F. **[5]**

c) Express the polynomial $x_1^3 + x_2^3 + x_3^3$ as rational function of elementry symmetric function **[2]**

Q3) a) Let F be a field then prove that there exists an algebraically closed field K containing F as a subfield. **[7]**

b) Which of the following are normal extension over $\mathbb{Q}$. **[5]**

i) $\mathbb{Q}\left(\sqrt{2}\right)$

ii) $\mathbb{Q}\left(\sqrt{-1}\right)$

iii) $\mathbb{Q}\left(5\sqrt{7}\right)$

iv) $\mathbb{Q}(\alpha)$

Where α is not algebraic over $\mathbb{Q}$

P.T.O.

c) If $f(x)$ is an irreducible polynomial over F then prove that $f(x)$ has multiple root if and only if $f'(x)=0$. **[2]**

Q4) a) Show that the splitting field of $f(x)=x^4-2$ over $\mathbb{Q}$ is $\mathbb{Q}(2^{1/4}, i)$. **[7]**

b) If E is a finite separable extension of a field F then prove that E is a simple extension of F. **[7]**

Q5) a) Let $E = \mathbb{Q}\left(\sqrt[3]{2}, w\right)$ where $w^3=1, w\neq 1$ & $G=\{\sigma_1,\sigma_2\}$

where $\sigma_1 : \begin{cases} \sqrt[3]{2} \to \sqrt[3]{2} \\ w \to w \end{cases}$ & $\sigma_2 : \begin{cases} \sqrt[3]{2} \to w\sqrt[3]{2} \\ w \to w^2 \end{cases}$

then show that $E_G = \mathbb{Q}\left(\sqrt[3]{2}\, w^2\right)$. **[7]**

b) State fundamental theorem of Galoi's theory. **[5]**

c) Find the minimal polynomial of $\alpha=\sqrt{2}+5$ over $\mathbb{Q}$. **[2]**

Q6) a) If $f(x) \in F(x)$ has r distinct roots in it's spliting field E over F then prove that Galoi's group G(E/F) of $f(x)$ is a subgroup of the symmetric group S_r. **[7]**

b) Give an example to show that an extension of a field F is algebraic but not finite. **[7]**

Q7) a) Show that Cyclotomic polynomial $\Phi_n(x)$ is irreducible polynomial of degree $\phi(n)$ in $\mathbb{Z}[x]$. **[7]**

b) Find the smallest extension of $\mathbb{Q}$ having a root of $x^4-2 \in \mathbb{Q}[x]$. **[5]**

c) Let F be a field of characteristic $\neq 2$ then show that $x^2-a \in F[x]$ is separable over F. **[2]**

Q8) a) Show that a regular n-gon is constructible if and only if $\phi(n)$ is a power of 2 **[7]**

b) Let E be the splitting field of a polynomial of degree n over a field F then show that $[\mathrm{E}:\mathrm{F}] \leq n!$ **[5]**

c) Show that $x^p - x - 1$ is irreducible over $\mathbb{Z}_p$. **[2]**

Total No. of Questions : 4]

SEAT No. :

P2381

[Total No. of Pages : 2

[6061]-313

M.Sc. (Part-II)

MATHEMATICS

MTUT-133 : Programming With Python

(2019 CBCS Pattern) (Semester-III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***Figures to the right indicate full marks.***
2) ***Question 1 is compulsory.***
3) ***Attempt any 2 questions from Q.2, Q.3 and Q.4.***

Q1) Attempt the following.

a) Explain the chronology of the development of Python. **[4]**

b) Write any 03 features of Python. **[3]**

Q2) Attempt the following.

a) Write a note on file hadling in python. **[7]**

b) Write a note on for loop in python. **[7]**

Q3) Attempt the following.

a) Explain the difference between logical and Relational operators in python with an example. **[4]**

b) Write Python code which accept name and age of the student using dot operator. **[5]**

c) What is operator overloding in Python? **[5]**

P.T.O.

Q4) Attempt the following.

a) Write a python program which generate the following pattern. **[7]**

1				
2	2			
3	3	3		
4	4	4	4	
5	5	5	5	5

b) Write a python program which accept positive integer and check wheather it is prime or not **[7]**

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Total No. of Questions : 8]

SEAT No. :

P-2382

[Total No. of Pages : 3

[6061]-314

M.A./M.Sc.

MATHEMATICS

MTUTO134 : Discrete Mathematics

(2019 Pattern) (Semester - III) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show with a generating functions that every positive integer can be written as unique sum of a distinct powers of 2. **[5]**

b) Prove that the isomorphism relation defined on set of simple graphs is an equivalance relation. **[5]**

c) Find generating function for the number of ways to write the integer r as a sum of positive integers in which no integer appears more than three times. **[4]**

Q2) a) Prove that a graph G is an Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **[7]**

b) Solve the following recurrence relation. **[7]**

$a_n=3a_{n-1}-3a_{n-2}+a_{n-3}$, $a_0=1$, $a_1= 1$, $a_1=1$, $a_2 = 2$

Q3) a) How many arrangemets of the letters in Mathematics are there in which TH appear together but the TH is not immediately followed by an E (not THE)? **[5]**

b) Let T be a tree with average vertex degree a. Determine n(T) in terms of a. **[5]**

c) If G is a graph then prove that $\sum_{v\in V(G)} d(v) = 2e(G)$ **[4]**

P.T.O.

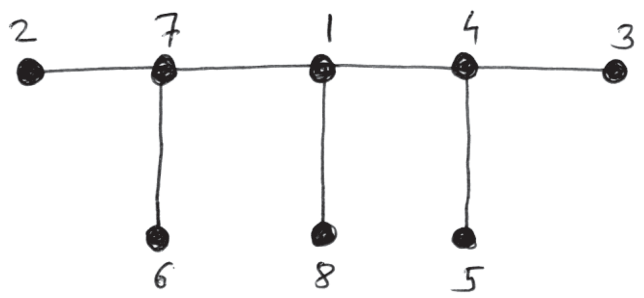
Q4) a) Prove that a graph is bipartite if and only if it has no odd cycle. **[7]**

b) What is the probability that n people randomly reach into dark closet to retrieve their hats, no person will pick his own hat? **[7]**

Q5) a) Find a recurrence relation for the number of sequences of 1s, 3s, and 5s whose terms sum to n. **[5]**

b) Prove that G is a tree if and only if G is connected and every edge is a cut edge. **[5]**

c) Find Prüfer code for the following tree. **[4]**



Q6) a) Prove that for k>0, every k-regular bipartite graph has perfect matching. **[5]**

b) Show by a combinatorial argument that **[5]**

$$\binom{n}{0}^2+\binom{n}{1}+\binom{n}{2}^2+---+\binom{n}{n}^2=\binom{2n}{n}$$

c) How many ways are there to distribute 27 indentical jelly beans among three childrent with each child getting at least one bean? **[4]**

Q7) a) Prove that deleting a leaf from an n-vertex tree produces a tree with n-1 vertices. **[5]**

b) Let X be a set of size n. Prove that the number of simple graphs with vertex set x is $2^{\binom{n}{2}}$. **[5]**

c) Find the coefficient of x^7 in $(1+x^2+x^4)\,(1+x)^{m}$. **[4]**

Q8) a) Prove that every graph with n vertices and k edges has at least n-k components. **[5]**

b) Show that $\sum_{k=0}^{n} k(k-1)\binom{n}{k} = n(n-1)2^{n-2}$, where n is positive integer. **[5]**

c) Let $A_1, A_2, \ldots, A_n$ be sets in universe U of N elements. Then prove that, $N(A_1 \cup A_2 \ldots \cup A_n) = S_1 - S_2 + \ldots + (-1)^{k-1} S_k + \ldots + (-1)^{n-1} S_n$ where S_k denote the sum of the sizes of all k-tuple intersections of the A_iS. **[4]**

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Total No. of Questions: 8]

SEAT No. :

P2383

[Total No. of Pages : 3

[6061]-315

S.Y.M.A./M.Sc.

MATHEMATICS

MTUTO135 : Mechanics

(2019 Pattern) (Semester-III)

Time :3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and explain the conservation theorem of linear momentum of the system of particles **[5]**

b) A particle is constrained to move in a circle in a vertical plane xy. Apply the D' Alembert's principle to show that for equilibrium we must have $\ddot{x}y - \ddot{y}x - gx = 0$ **[5]**

c) Show that the Lagrange's equation **[4]**

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}j}\right) - \frac{\partial T}{\partial qj} = Q\ j$$

Can also be written in the form

$$\frac{\partial \dot{T}}{\partial \dot{q}j} - z\frac{\partial T}{\partial qj} = Qj.$$

Q2) a) Show that the total energy of a particle moving in a conservative force field remains constant, if the potential energy is not an exlicit function of time. **[5]**

b) Explain Atwood machine and discuss its motion. **[5]**

c) The length of a simple pendulum changes with time such that $l=a+bt$, where a and b are constants. Find the Lagrangian equation of motion. **[4]**

Q3) a) Show that the geodesic (Shortest distance between two points) in a Euclidian plane is a straight line. **[5]**

P.T.O.

b) Find the curve for which the functional $I[y(x)] = \int_0^{\frac{\pi}{4}} (y^2 - y'^2)\,dx$ can have extrema, given that $y(0) = 0$, while the Eight-hand end point can vary along the line $x = \frac{\pi}{4}$. **[5]**

c) Find the extremal of the functional $I = \int_0^{\frac{\pi}{2}} (y'^2 - y^2 + 2xy)\,dx$ subject to the conditions that $y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 0.$ **[4]**

Q4) a) Find the geodesic on a given surface $G(x, y, z) = 0$ that gives a stationary value to an integral of the form $I = \int_{t_o}^{t_1} f(\dot{x}, \dot{y}, \dot{z})\,dt.$ **[7]**

b) Find the solid of revolution with a given surface area and maximum volume **[7]**

Q5) a) Show that the Hamilton's principle $\delta \int_{t_o}^{t_1} L\,dt = 0$ also holds for the non conservative system. **[5]**

b) Deduce Newton's second Law of motion from Hamilton's Principle. **[5]**

c) Prove that a co-ordinate which is cyclic in the Lagrangian is also cyclic in the Hamiltonian. **[4]**

Q6) a) Among all curves whose end points lie on two given vertical lines $x = a$ and $x = b$, find the curve for which the functional $I(y(x)) = \int_a^b f(x, y, y^1)\,dx$ has an extremum. **[7]**

b) Show that the Lagrange's equations are necessary conditions for the action to have a stationary value. **[7]**

Q7) a) Prove the Kepler's third Law of planetary motion. **[7]**

b) A particle of mass m is moving under the inverse square law of attractive force. Set up the Lagrangian and equation of motion obtain the first integral of motion. Show also that it is constant of total energy. **[7]**

Q8) a) If the particles attract each other according to inverse square law of force, Prove that $\overline{2T+V} = 0$, where T is the total Kinetic energy of the particles and V the potential energy. **[5]**

b) Use Hamiltons's Procedure to find the differential equation for planetary motion under inverse square law of force. **[5]**

c) A particle moves in a circular orbit in a Force field $f(r) = -\frac{k}{r^2}$ suddenly k bocomes $\frac{k}{2}$ without change in velocity of the particle. Show that the orbit becomes parabola. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P-2384

[Total No. of Pages : 3

[6061]-316
S.Y. M.A./M.Sc.
MATHEMATICS
MTUTO - 136 : Advanced Complex Analysis
(2019 Pattern) (Semester - III) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Suppose f and g are holomorphic in a region Ω and $f(z) = g(z)$ for all z in some sequence of distinct point with limit point in Ω then prove that $f(z) = g(z)$ throughout Ω. **[4]**

b) Prove that every polynomial $p(z) = anz^n + ... + a_0$ of degree $n \geq 1$ has precisely n roots in $\mathbb{C}$. **[5]**

c) Show that $\int_0^\infty \frac{\sin x}{x} dx = \pi/2$. **[5]**

Q2) a) Suppose that f is a holomorphic function in Ω^+ that extends continuously on I and such that f is real - valued on I. Prove that there exists a function F holomorphic in all of Ω such that F = f on Ω^+. **[5]**

b) If $f : U \to V$ is holomorphic and injective, then prove that $f'(z) \neq 0$ for all $z \in U$. **[5]**

c) Prove that the map $f(z) = (1 + z)/(1 - z)$ takes the upper half-disc $\{z = x + iy : |z| < 1 \text{ and } y > 0\}$ conformally to the first quadrant $\{w = u + iv : u > 0 \text{ and } v > 0\}$. **[4]**

P.T.O.

Q3) a) Let V and U be open sets in $\mathbb{C}$ and $F : V \to U$ a holomorphic function. If $u : U \to \mathbb{C}$ is a harmonic function, then prove that $u o F$ is harmonic on V. **[5]**

b) Show that the map $f(z) = \sin z$ takes the upper half plane conformally onto the half-strip $\{w = x + iy : -\pi/2 < x < \pi/2, y > 0\}$. **[5]**

c) If f is an automorphism of the disc, then prove that there exists $\theta \in \mathbb{R}$ and $\alpha \in \mathbb{D}$ such that $f(z) = e^{i\theta}\left(\dfrac{\alpha - z}{1 - \bar{\alpha} z}\right)$. **[4]**

Q4) a) Let $f : \mathbb{D} \to \mathbb{D}$ be holomorphic with $f(0) = 0$. Then prove the following :

i) $|f(z)| \leq |z|$ for all $z \in \mathbb{D}$.

ii) If for some $z_0 \neq 0$ we have $|f(z_0)| = |z_0|$, then f is a rotation.

iii) $|f'(0)| \leq 1$ and if equality holds, then f is a rotation.

[7]

b) Prove that the only automorphisms of the unit disc that fix the origin are the rotations. **[5]**

c) Define automorphism and give one example. **[2]**

Q5) a) State and prove Montel's theorem. **[7]**

b) If Ω is a connected open subset of $\mathbb{C}$ and $\{f_n\}$ a sequence of injective holomorphic functions on Ω that converges uniformly on every compact subset of Ω to a holomorphic function f, then prove that f is either injective or constant. **[7]**

Q6) a) Define the general schwarz - christoffel integral. **[2]**

b) Let z_0 be a point on the unit circle. Then prove that F(z) tends to a limit as z approaches z_0 within the unit disc where $F : \mathbb{D} \to P$ is a conformal map. **[7]**

c) Show that the conformal map F extends to a continuous function from the closure of the disc to the closure of the polygon. **[5]**

Q7) a) Suppose F(z) is holomorphic near $z = z_0$ and $F(z_0) = F'(z_0) = 0$, while $F''(z_0) \neq 0$. Show that there are two curves Γ_1 and Γ_2 that pass through z_0, are orthogonal at z_0 and so that F restricted to Γ_1 is real and has a minimum at z_0, while F restricted to Γ_2 is also real but has a maximum at z_0. **[5]**

b) Prove that the total number of poles of an elliptic function in P_0 is always ≥ 2. **[7]**

c) Does there exist a holomorphic surjection from the unit disc to $\mathbb{C}$? **[2]**

Q8) a) Show that two series

$$\sum_{(n,m)\neq(0,0)} \frac{1}{(|n|+|m|)^r} \text{ and } \sum_{n+mT\in A^*} \frac{1}{|n+mT|^r}$$

Where A* denote the lattice minus the origin, that is $A^* = A - \{(0, 0)\}$, converges if $r > 2$. **[7]**

b) Show that the weierstrass ρ function is an elliptic function that has periods 1 and τ and double poles at the lattice points. **[7]**

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Total No. of Questions : 8]

SEAT No. :

P2385

[Total No. of Pages : 3

[6061]-317
S.Y.M.A./M.Sc.
MATHEMATICS
MTUTO 137 : Integral Equations
(2019 Pattern) (Semester -III) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*
3) *Symbols have their usual meanings.*

Q1) a) If u(x) = x^2+x^3 is a solution of integral equation

$$u(x) = x^3 - x^2 - 2x + \alpha\int_{-1}^{1}\left(xt^2 + x^2t\right) u\,(t)\,\mathrm{dt}$$

then find α **[4]**

b) Explain the method of Adomian decomposition to solve fredholm integral equation. **[5]**

c) Using series solution method, solve the following integral equation

$$u\,(x) = 1 + \int_{o}^{x}(t-x)\,u\left(t\right)\,\mathrm{dt}.$$ **[5]**

Q2) a) Using direct computation method, solve the following integral equation

$$u\,(x) = -8x + 6x^2 + \int_{0}^{1}(20xt^2 + 12x^2t)\,u\,(t)\,\mathrm{dt}$$ **[7]**

b) Using succesive approximation method, solve the following integral equation $u\,(x) = \dfrac{11}{12}\,x + \dfrac{1}{4}\int_{0}^{1} xt\,u\,(t)\,\mathrm{dt}$ **[7]**

Q3) a) Using noise term phenomenon, solve the following integral equation

$$u(x) = 6x + 3x^2 - \int_0^x u(t)\ dt$$ **[4]**

b) Explain succesive substitution method, to solve volterra integral equation. **[5]**

c) Solve the following integro-differential equation **[5]**

$$u^1(x) = 1 - \frac{1}{3}x + x\int_0^1 t.u(t)\,\text{dt, u (0) = 0}$$

Q4) a) Define the Abel's singular integral equation and give two examples of Abel's singular integral equation. **[4]**

b) Solve the following Abel's integral equation

$$\pi = \int_0^x \frac{1}{\sqrt{x-t}}\ .\ \text{u (t). dt}$$ **[4]**

c) Using Adomian decomposition method, solve the following integral equation $u(x) = e^x - 1 + \int_0^1 t.u(t)\ \text{dt}$ **[6]**

Q5) a) Find the initial value problem which is equivalent to volterra integral equation $u(x) = x^3 + \int_0^x (x-t)^2\, u(t)\ \text{dt}$ **[4]**

b) Convert the following initial value problem into volterra integral equation $y'' + y = \cos x,\ y(0) = 0,\ y'(0) = 1$ **[5]**

c) Find the eigenvalue and corresponding eigen function of following integral equation $u(x) = \lambda \int_0^1 x.u(t)\ \text{dt}$ **[5]**

Q6) a) Solve the following volterra integral equation by converting it into equivalent initial value problem $u(x) = e^x + \int_0^x (t-x)\, u(t)\,\text{dt}$ **[7]**

b) Solve the following first kind volterra integral equation by converting it into second kind volterra integral equation

$$5x^2 + x^3 = \int_0^x (5 + 3x - 3t)\, u(t)\, dt$$ **[7]**

Q7) a) Using Leibniz rule of differentiation find $u'(x)$ and **[4]**

$$u''(x) \text{ if } u(x) = x^3 + \int_0^x (x-t)^2\, u(t)\, dt.$$

b) Solve the following integro-differential equation **[6]**

$$u'''(x) = \text{Sin}\, x - x - \int_0^{\pi/2} x.t.u'(t)\, dt$$

c) Classify the following integral equation as fredholm or volterra, Linear or non linear and homogenious or non homogenious **[4]**

i) $u(x) = e^x + \int_0^x t.u^2(x)\, dt$

ii) $u(x) = \int_0^1 (x-t)^2 . u(t)\, dt$

Q8) a) Solve the following Abel's integral equation **[4]**

$$\pi . x + \pi = \int_0^x \frac{1}{\sqrt{x-t}} . u(t)\, dt$$

b) Show that $u(x) = x + e^x$ is a solution of **[4]**

$$u''(x) = e^x - \frac{4}{3}x + \int_0^1 x.t\, u(t)\, dt$$

c) Solve the following integral equation by direct Computation method.**[6]**

$$u(x) = 1 + g.x + 2x^2 + x^3 - \int_0^1 (20xt + 10x^2t^2)\, u(t)\, dt$$

Total No. of Questions : 8]

SEAT No. :

P2386

[Total No. of Pages : 3

[6061]-318

S.Y.M.A./M.Sc.

MATHEMATICS

MTUTO - 138 : Differential Manifolds

(2019 Pattern) (CBCS) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let $g : A \to B$ be a diffeomorphism of open sets in R^k. Let $\beta : B \to \mathbb{R}^n$ be map of class C^r; Let $Y = \beta(B)$. Let $\alpha = \beta \circ g$ then $\alpha : A \to R^n$ and $Y = \alpha(A)$. If $f : Y \to R$ is continuous function, then prove that f is integrable over Y_β iff it is integrable over Y_α; in this case $\int_{Y_\alpha} f\,dV = \int_{Y_\beta} f\,dV$. In particular $V(Y_\alpha) = V(Y_\beta)$. **[7]**

b) Let A be open in R^k; Let $f : A \to R$ be class of C^r; Let Y be the graph of f in R^{k+1} parameterized by the function $\alpha : A \to \mathbb{R}^{k+1}$ given by $\alpha(x) = (x, f(x))$. Express $V(Y_\alpha)$ as integral. **[5]**

c) Is $I \times I$ a 2-manifold in $\mathbb{R}^2$? Where I is a closed in $\mathbb{R}$. **[2]**

Q2) a) Let W be linear subspace of R^n of dimension K. Then prove that there is an orthonormal basis for R^n whose first K elements form a basis for W. **[5]**

b) Let $\alpha : R \to \mathbb{R}^2$ be the map $\alpha(x) = (x, x^2)$; Let M be the image set of α. Show that M is 1-manifold in R^2 covered by single co-ordinate patch α. **[5]**

c) Let $h : R^n \to R^n$ be the function $h(x) = \lambda x$. If P is k-dimensional parallelepiped in R^n, Find the volume of h (P) in terms of volume of P. **[4]**

P.T.O.

Q3) a) Let O be open in R^n; Let $f : O \to R$ be of class C^r, Let M be the set of points x for which f (x)= 0; Let N be the set of points for which f (x) ≥ 0. Suppose M is non empty and Df(x) has rank 1 at each point of M. Then prove that N is n-manifold in R^n and ∂ N=M. **[7]**

b) Let A be open in $\mathbb{R}^2$, $\alpha : A \to \mathbb{R}^3$ be of class C^r; Let Y = α (A). Find 2 - dimensional volume of Y_α as an integral. **[5]**

c) Define n-manifold in $\mathbb{R}^{n+1}$. **[2]**

Q4) a) Let V be a vector space with basis $a_1, a_2, ...,a_n$ Let $I = (i_1, i_2 ,...., i_k)$ be a k-tuple of integers from the set {1, 2,......,n}. Then prove that there is a unique k-tensor ϕ_I on V such that, for every k-tuple $J = (j_1, j_2,...j_k)$ from the set {1, 2, ...,n}, $\phi_I (a_{j1}, a_{j2}, ..,a_{jk}) = \begin{cases} 0 & \text{if} \quad I \neq J \\ 1 & \text{if} \quad I = J \end{cases}$ The tensors ϕ_I from a basis for $L^k(V)$. **[5]**

b) Let f and g be following tensors on $\mathbb{R}^4$ $f (x, y, z) = 2x_1 \, y_2 \, z_2 - x_2 y_3 z_1$ $g = \phi_{2,1} - 5 \, \phi_{3,1}$ **[5]**

i) Express $f \otimes$ g as a linear combination of elementary 5-tensors.

ii) Express $f \otimes$ g (x, y, z, u, v) as a function.

c) Let $a_1, a_2,..a_n$ be basis for V. If f, g are alternating k-tensor on V, and if $f (a_{i1}, a_{i2},..., a_{ik}) = g (a_{i1}, a_{i2},.... a_{ik})$ for every ascending k-tuple of integers $I = (i_1,i_2,..i_k)$ from the set $\{1,2,...,n\}$, then show that f = g. **[4]**

Q5) a) Let T : V $\to$ W be linear transformation If f is an alternating tensor on W, then prove that T * f is an alternating tensor on V, where T * is adjoint of T. **[5]**

b) Let V & W be vector space. f and g are alternating tensor of order k and l on W I & T: V $\to$ W is linear transformation then show that T* $(f \wedge g) = T^* f \wedge T^* g$. **[5]**

c) If $w = xyz \, dx + 2 \cos xy^2z \, dy + z \, e^y dz$ find dw. **[4]**

Q6) a) Let A be open in R^k or H^k; Let $\alpha : A \to R^m$ be of class C^r. Let B be open set of $\mathbb{R}^m$ or H^m containing α (A); Then show that $(\beta \circ \alpha)_* = \beta_* \circ \alpha_*$. **[5]**

b) Let A be open set in R^n. There exist a unique linear transformation d: $\Omega^k(A) \to \Omega^{k+1}$ (A). If ω & η are forms of order k and l respectively then show that $d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$. **[7]**

c) State Stoke's theorem. **[2]**

Q7) a) Let $g : A \to B$ be differeomorphism of open sets in R^k. Assume det Dg does not change sign on A. Let $\beta : B \to R^n$ be a map of class C^∞; Let $Y = \beta$ (B). Let $\alpha = \beta \circ g$; then $\alpha : A \to R^n$ and $Y = \alpha$ (A). If ω is k - form defined on open set of R^n containing Y. Then prove that ω is integrable over Y_β iff it is integrable over Y_α; in this case $\int_{Y_\alpha} \omega = \pm \int_{Y_\beta} \omega$.

[7]

b) T:V→W be a linear transformation; Let $T^*: L^k(W) \to L^k(V)$ be the dual transformation. Then prove that **[7]**

i) T* is linear.

ii) $T^*(f \otimes g) = T^* f \otimes T^* g$

iii) If $S : W \to X$ is linear transformation then $(S \circ T)^* f = T^* (S^* f)$.

Q8) a) Let M be a compact oriented 1-manifold in $\mathbb{R}^n$; give ∂M the induced orientation if ∂M is non empty. Let f be a o-form defined in an open set of $\mathbb{R}^n$ containing M then show that $\int_{\partial M} f = \int_{M} df$ If ∂M is non empty; and $\int_{M} df = 0$ *if* ∂M is empty. **[7]**

b) Let k>1. If M is an orientable k-manifold with non empty boundary then prove that ∂M is orientable. **[7]**

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Total No. of Questions : 8]

SEAT No. :

P-2387

[Total No. of Pages : 4

[6061]-411

S.Y.M.A./M.Sc.

MATHEMATICS

MTUT-141 : Fourier Series and Boundary Value Problems

(2019 Pattern) (Semester - IV) (CBCS)

Time : 3 Hour] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let f denote a function such that **[7]**

i) f is contineous on the internal $-\pi \le x \le \pi$.

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is picewise contineous on the internal $-\pi < x < \pi$.

If $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$ and

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$ are fourier

Coefficient's for f, then prove that the series $\sum_{n=1}^{\infty}\sqrt{a_n^2 + b_n^2}$ converges.

b) Find the fourier series on the internal $-\pi < x < \pi$ that corresponds to the

function $f(x) = \begin{Bmatrix} \frac{2}{\pi}x + 2, -\pi < x \le 0 \\ 2,\ 0 \le x < \pi \end{Bmatrix}$ **[5]**

c) Find fourier sine series for the function $f(x) = x$ $(0 < x < 1)$ **[2]**

P.T.O.

Q2) a) Let f denote a function that is piece wise contineous on the internal $-\pi < x < \pi$ and periodic with period 2π on entire x-axis. Then prove that fourier series converges to the mean value $\frac{f(x,+)+f(x,-)}{2}$ of the one sided limit's of each point x $(-\infty < x < \infty)$ where both of the one - sided derivative $f_R'(x)$ and $f_L'(x)$ exists. **[7]**

b) Find the fourier sine series for the function $f(x) = x^3$ on the interval $0 < x < \pi$. **[5]**

c) If $f(x) = \sqrt[3]{x}$ $(-\pi < x < \pi)$, then show that $f(x)$ is picewise contineous on the interval $-\pi < x < \pi$ but $f'(0,+)$ and $f'(0,-)$ does not exist. **[2]**

Q3) a) Let f denote a function such that

i) f is contineous on the interval $-\pi \le x \le \pi$.

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is picewise contineous on the interval $-\pi < x < \pi$.

prove that the fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ for f, with coefficient's $an = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$ & $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$ converges absolutely and uniformally to $f(x)$ on the interval $-\pi \le x \le \pi$. **[7]**

b) Find the fourier cosine series on the interval $0 < x < \pi$ that corresponds to the function f defined by $f(x) = \begin{cases} 1, & \text{when } 0 < x < \frac{\pi}{2} \\ 0, & \text{when } \frac{\pi}{2} < x < \pi \end{cases}$ **[5]**

c) Let f denote the contineous function defined by the equation

$$f(x) = \begin{bmatrix} x^2 & \text{, when } x \le 0 \\ \sin x, & \text{when } x > 0 \end{bmatrix}$$

then find $f_R^1(0)$ and $f_L^1(0)$. **[2]**

Q4) a) Solve the following linear boundary value problem. **[7]**

$$u_t(x,t) = k\,u_{xx}(x,t) \quad (0 < x < c, t > 0)$$
$$u_x(0,t) = 0, u_x(c,t) = 0 \qquad (t > 0)$$
$$u(x,0) = f(x) \qquad (0 < x < c)$$

b) Solve the following boundary value problem. **[7]**

$$y_{tt}(x,t) = a^2 y_{xx}(x,t) \qquad (0 < x < c, t > 0)$$
$$y(0,t) = 0, y(c,t) = 0,\ y_t(x,0) = 0$$
$$y(x,0) = f(x)$$

Q5) a) Solve the following boundary value problem. **[7]**

$$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_\rho(\rho,\phi) + u_{\phi\phi}(\rho,\phi) = 0 (0 < \rho < a, 0 < \phi < \pi)$$

satisfying homogeneous condition's

$u_\phi(\rho,0) = 0, u_\phi(\rho,\pi) = 0 \quad (0 < \rho < a)$ nonhomogeneous condition

$u(a,\phi) = f(\phi)\ (0 < \phi < \pi)$.

b) Solve the following boundary value problem. **[7]**

$$u_{xx}(x,y) + u_{yy}(x,y) = 0 \quad (0 \le x \le a, 0 \le y \le b)$$
$$u(0,y) = 0, u(a,y) = 0 \quad (0 < y < b)$$
$$u(x,0) = f(x),\ u(x,b) = 0\ (0 < x < a)$$

Q6) a) If x and y are eigen function's corresponding to the same eigen value of a regular sturm-liouville problem. then prove that $y(x) = C\times(x)$. Where C is non zero constant. **[7]**

b) Show that the function $\psi_1(x) = 1$ and $\psi_2(x) = x$ are orthogonal on the interval $-1 < x < 1$ and determine constant's A and B such that the function $\psi_3(x) = 1 + Ax + Bx^2$ is orthogonal to both ψ_1 and ψ_2 on the interval. **[5]**

c) If m and n are positive integer's then show that

$$\int_0^{\pi} \sin mx \sin nx \, dx = \begin{cases} 0 & , \text{ when } m \neq n \\ \pi/2 & , \text{ when } m = n \end{cases}$$ **[2]**

Q7) a) If C_n(n=1,2,3,....) be the fourier constants for a function f in C_p(a,b) with respect to an orthogonal set $\{\phi_n(x)\}(n=1,2,3....)$ in that space. Then prove that all possible linear combination's of the function $\phi_1(x)$, $\phi_2(x)$.....$\phi_n(x)$ the combination $C_1\phi_1(x)+C_2\phi_2(x)+......+C_n\phi_n(x)$ is the best approximation in the mean to $f(x)$ on the fundamental interval $a < x < b$. **[7]**

b) If $\phi_0 = \frac{1}{\sqrt{2\pi}}, \phi_{2n-1}(x) = \frac{1}{\sqrt{\pi}} \cos nx$, $\phi_{2n}(x) = \frac{1}{\sqrt{\pi}} \sin nx (n=1,2,3,.....)$

then show that the set $\{\phi_n(x)\}(n=0,1,2,.....)$ is orthonormal on the integral $-\pi < x < \pi$. **[5]**

c) Prove or disprove : Every fourier series differentiable. **[2]**

Q8) a) If λ_m and λ_n are distinct eigen values of the sturm-liouville problem $[r(x)x'(x)]' + [g(x) + \lambda p(x)] \times (x) = 0 \, (a < x < b)$ under the condition $a_1 \times (a) + a_2 \times' (a) = 0$, $b_1 \times (b) + b_2 \times' (b) = 0$, then prove that corresponding eigen function's $x_m(x)$ and $x_n(x)$ are orthogonal with respect to weight function $p(x)$ on the interval $a < x < b$. **[7]**

b) Find the eigen values and normalized eigen function of sturm-liouville problem $x^{11}+\lambda x=0$, $x(0)=0$, $x(1)-x'(1)=0$. **[5]**

c) If L is the linear operator $L = k\frac{\partial^2}{\partial x^2} - \frac{\partial}{\partial t}$ and

$y_n = \exp\left(\frac{-n^2\pi^2 ut}{c^2}\right)\cos\left(\frac{n\pi x}{c}\right)$ then show that $Ly_n = 0$ (n=1,2,3,....) **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2388

[Total No. of Pages : 3

[6061]-412

M.A./M.Sc.-II

MATHEMATICS

MTUT-142 : Differential Geometry

(2019 Pattern) (Semester-IV) (Credit system)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Answer any five questions out of eight questions.***
2) ***Figures to the right side indicate full marks.***

Q1) a) Show that the parallel transport $P_{\alpha}: S_p \to S_q$ where S is an n-surface in $\mathbb{R}^{n+1}, p, q \in S$ and α is a piecewise smooth, parametrized curve from *p* to *q*, is are to are, on to linear map which preserves the dot product. **[5]**

b) Find the integral curve through p=(a,b) of the vecter field X $(x_1,x_2) = (x_2 - x_1)$ **[5]**

c) Define weingarten map and show that $L_p : S_p \to S_p$ is linear. **[4]**

Q2) a) Show that every connected n-surface in $\mathbb{R}^{n+1}$ has exactly two orientation. **[5]**

b) Sketch the level set $f^{-1}(-1)$, $f^{-1}(0)$ and $f^{-1}(1)$ for n=1 of the function $f(x_1, x_2,...,x_{n+1}) = x_1^2 + x_2^2 + ... + x_n^2 - x_{n+1}$ which points, p∘f these sets fail to have tangent space equal to $[\nabla f(p)]^{\perp}$? **[5]**

c) Prove that the velocity vecter field along a parametrized curve α in an n-surface S is parallel if and only if α is geodesic. **[4]**

Q3) a) Let S denote the cylinder $x_1^2 + x_2^2 = r^2$ of radius r>0 in $\mathbb{R}^3$. Show that α. is a geodisic of S if and only if α is of the form
α(*t*) = (*r* cos (*at* + *b*), *r* sin (*at* + *b*), *ct* + *d*) for some $a, b, c, d \in \mathbb{R}$ **[7]**

b) Find the spherical image of the cylinder $x_2^2 + x_3^2 + ... + x_{n=1}^2 = 1$ for n=1 and n=2 **[5]**

c) Explain why an integral curve of a vector field can not cross itself as does the parametrized curve. **[2]**

P.T.O.

Q4) a) Find global parametrization of C $=f^{-1}(r^2)$ where $f(x_1, x_2) = (x_1-a)^2+(x_2-b)^2$. Also find curvature K for C oriented by outward normal. **[7]**

b) Show that the two orientations on n-sphere $x_1^2 + x_2^2 + \ldots + x_{n+1}^2 = r^2$ of radius $r > 0$ are given by $\mathrm{N}_1(p) = \left(p, \frac{p}{r}\right)$ and $\mathrm{N}_2(p) = \left(p, -\frac{p}{r}\right)$ **[5]**

c) Find the length of the parametrized curve $\alpha : \mathrm{I} \to \mathbb{R}^{n+1}$ given by $\alpha(t) = (\cos 3t, \sin 3t, 4t)$, I = [–1,1], n = 2 **[2]**

Q5) a) Show that the weingarten map is a self-adjoint linear operator. **[7]**

b) Let $a,b,c \in \mathbb{R}$ be such that $ac-b^2 > 0$ show that the maximum and minimum values of the function $g(x_1, x_2) = ax_1^2 + 2b\,x_1x_2 + cx_2^2$ on the unit circle $x_1^2 + x_2^2 = 1$ are the eigenvalues of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ **[5]**

c) Find the velocity and speed of the parametrized curve $\alpha(t)=(\cos t, \sin t, t)$. **[2]**

Q6) a) Let f : $f : \mathrm{U} \to \mathbb{R}$ be a smooth function and let $\alpha : \mathrm{I} \to \mathrm{U}$ be an integral curve of ∇f. **[7]**

i) Show that $\frac{d}{dt}(f \circ \alpha)(t) = \| \nabla f(\alpha(t)) \|^2$ for all $t \in \mathrm{I}$.

ii) Show that if $\beta : \tilde{\mathrm{I}} \to \mathrm{U}$ is such that $\beta(\mathrm{S}_o)=\alpha(\mathrm{t}_o)$ for some $\mathrm{S}_o \in \tilde{\mathrm{I}}$ and each to $\in$ I, and $\| \dot{\beta}(\mathrm{S}_o) \| = \| \dot{\alpha}(\mathrm{t}_0) \|$ then $\frac{d}{dt}(f \circ \alpha)(t_o) \geq \frac{d}{dt}(f \circ \beta)(\mathrm{S}_0)$

b) Show that the unit n-sphere is an n-surface. **[5]**

c) Let X and Y be smooth vecter fields tangent to an n-surface S in $\mathbb{R}^{n+1}$ along a parametrized curve $\alpha : \mathrm{I} \to \mathrm{S}$. Show that (X+Y)'=X'+Y' **[2]**

Q7) a) State and prove Langrange's multiplier theorem. **[7]**

b) Let $\eta = \frac{-x^2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$ be the 1-form on $\mathbb{R}_1^2\{0\}$ and C denotes the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ oriented by its inward normal and $\alpha : [0.2\pi] \to C$ defined by $\alpha(t)$ = (a cost, b sint) be a parametric curve whose restriction to [0,2π] is a one-to-one global parametrization of C. Find $\int_\alpha \eta$ is η exact? **[7]**

Q8) a) Let S be an oriented n-surface in $\mathbb{R}^{n+1}$ and let v be a unit vector in S_p, p∈ S. Then show that there exists an open set $V \subset \mathbb{R}^{n+1}$ containing p such that $S \cap N(v) \cap v$ is a plane curve. Also show that the curvature at p of this curve is equal to normal curvature k(v). **[7]**

b) Calculate the Gaussian curvature of the ellipsoid $S : \frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ oriented by the outward normal. **[7]**

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Total No. of Questions : 4]

SEAT No. :

P-2389

[Total No. Of Pages : 1

[6061]-413
M.A./M.Sc.
MATHEMATICS
MTUT-143: Introduction to Data Science
(2019 Pattern) (Semester-IV) (Credit System)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates :

1) ***Figures to the right indicate full marks.***
2) ***Question 1 is compulsory.***
3) ***Attempt any two questions from Q.2,3 and 4.***

Q1) State all the forms of data. Explain any Two forms of data. **[7]**

Q2) a) What is machine learning? Where it is used in data science process? **[5]**

b) Describe data transformation process. **[5]**

c) State the phases involved in the modeling of data science process. **[4]**

Q3) a) Distinguish between supervised and unsupervised machine learning techniques. **[5]**

b) State any Five techniques used to handle the missing data. **[5]**

c) State the characteristics of big data. **[4]**

Q4) a) Explain in detail concept of Hadoop and its components. **[5]**

b) Give packages in python which are used fortext mining. **[5]**

c) Write all steps involved in the process of data science. **[4]**

Total No. of Questions : 8]

SEAT No. :

P2391

[Total No. of Pages : 2

[6061]-415

S.Y. M.Sc.

MATHEMATICS

MTUTO - 145 : Algebraic Topology

(2019 Pattern) (CBCS) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that the homotopy relation is an equivalence relation. **[5]**

b) A space X is said to be contractible if the identity map $I_x : X \to X$ is nulhomotopic, then show that I and $\mathbb{R}$ contractible. **[5]**

c) Given spaces X and Y. Let [X, Y] denote the set of homotopy classes of Maps of X into Y. Let I = [0, 1], show that for any X, the [X, I] has single element. **[4]**

Q2) a) Let α be a path in X from α_0 to x, define a map $\hat{\alpha} : \pi_1(X, x) \to \pi_1(X, x_1)$ by $\hat{\alpha}([F]) = [\bar{\alpha}] * [F] * [\alpha]$ then show that $\hat{\alpha}$ is a group homomorphism. **[5]**

b) Let $q : X \to Y$ and $r : Y \to Z$ be covering map. Let P = *roq*. Show that $r^{-1}(Z)$ is finite for each $Z \in Z$, then P is covering map. **[5]**

c) Prove that there is no retraction of B^2 onto S'. **[4]**

Q3) a) Show that a retract of a Hausdorft space is a closed subset. **[5]**

b) Let $A = \{(x, y) \in \mathbb{R}^2 \mid x = 0, -1 \le y \le 1\}$ and

$B = \{(x, y) \in \mathbb{R}^2 \mid 0 < x \le 1, y = \cos \pi_1^{\circ} \cdot x\}$ Show that $F = A \cup B$ is connected but not path connected. **[5]**

c) Show that $\pi_1(X, x_0)$ is a group. **[4]**

P.T.O.

Q4) a) Show that the Fundamental group of real projective plane is cyclic group of order two. **[5]**

b) Using algebric topology techniques, prove that every non constant complex polynomial has a root. **[5]**

c) Find the Fundamental groups of $\mathbb{R}^2$, $\mathbb{R}^2 - O$ and $S' \times \mathbb{R}$. **[4]**

Q5) a) Let X be connected and has the same homotopy type as Y. Show that Y is connected. **[5]**

b) Prove that a non empty open connected subset of $\mathbb{R}^n$ is path connected. **[4]**

c) Prove that the fundamental group of S′ is isomorphic to the additive group $\mathbb{Z}$. **[5]**

Q6) a) Prove that $\pi : \mathbb{R} \to \mathbb{R}/\mathbb{Z}$ is a covering map. **[5]**

b) Define a covering map. Prove that covering map is a local homoeomorphism but the converse is not true. **[5]**

c) Show that if *f* is any path then $f * \overline{f}$ and $\overline{f} * f$ are null homotopic. **[4]**

Q7) a) Prove that the circle is not a retract of disc B^2. **[5]**

b) Find a continous map of torus into S′ that is not nulhomotopic. **[4]**

c) Let $\pi : E \to X$ be a closed quotient map. If E is normal then so is X. **[5]**

Q8) a) State seifert-Van Kampen theorem. **[4]**

b) Define the following term: **[4]**

i) Free group

ii) Wedge of the circle

c) Let $P : E \to B$ and $P' : E' \to B'$ be covering map. Let $P(e_0) = P'(e_0') = b_0$. There is an equivalence $h : E \to E'$ such that $h(e_0) = e_0'$ if and only if the groups $H_0 = P_*(\pi_1(E, e_0))$ and $H_0' = P_*'(\pi_1(E', e_0'))$ are equal. If his exist, it is unique. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P-2392

[Total No. of Pages : 3

[6061]-416
M.A./M.Sc.
MATHEMATICS
MTUTO-146 : Representation Theory of Finite Groups
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator is allowed.*

Q1) a) Let V be an inner product space and $W \subseteq V$, then prove that there is direct sum decomposition $V = W \oplus W^{\perp}$. **[5]**

b) Prove that a matrix $A \subset M_n(C)$ is diagonalizable if and only if $m_A(x)$ has no repeated roots. **[4]**

c) Prove that Tr.(AB) = Tr.(BA). **[5]**

Q2) a) Define following terms : **[4]**

i) Representation

ii) Equivalence

b) Prove that $\phi : \frac{z}{2z} \to C^*$ given by $\phi(m) = (-1)^m$ is a representation. **[5]**

c) Define $\phi : \frac{z}{nz} \to GL_2(c)$ by $\phi[m] = \begin{bmatrix} \cos\left(\frac{2\pi m}{n}\right) & -\sin\left(\frac{2\pi m}{n}\right) \\ \sin\left(\frac{2\pi m}{n}\right) & \cos\left(\frac{2\pi m}{n}\right) \end{bmatrix}$ and

$\psi : \frac{z}{nz} \to GL_2(C)$ by $\psi[m] = \begin{bmatrix} e^{\left(\frac{2\pi mi}{n}\right)} & 0 \\ 0 & e^{-\frac{2\pi mi}{n}} \end{bmatrix}$. Prove that $\phi \sim \psi$ **[5]**

P.T.O.

Q3) a) Prove that every representation of a finite group is equivalent to a unitary representation. **[7]**

b) Let $\rho : S_3 \rightarrow GL_2(C)$ be specified on the generators (1, 2) and (1, 2, 3) by $\rho(1,2) = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$, $\rho(1,2,3) = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$ show that the representation is unique. **[7]**

Q4) a) Prove that every representation of finite group is completely reducible. **[6]**

b) Let $\phi : G \rightarrow GL(V)$ and $\rho : G \rightarrow GL(W)$ be representations. Prove that $Hom_G(\phi, e)$ is a subspace of Hom(V, W). **[4]**

c) Define following terms : **[4]**

i) Character

ii) Multiplicity

Q5) a) Let $\phi : G \rightarrow GL(V)$, $\rho : G \rightarrow GL(W)$ be irreducible representations of G. Let $T : V \rightarrow W$ be a linear map. Prove the following statements **[7]**

i) If $\phi \nsim \rho$ then $T^{\#} = 0$

ii) If $\phi = \rho$ then $T^{\#} = \dfrac{Tr(T)}{\deg(\phi)} I$.

b) Let ϕ be a representation of G, then prove that $\chi_\phi(1) = \deg\phi$. **[3]**

c) Prove that ZL(G) is a subspace of L(G). **[4]**

Q6) a) Prove that regular representation is a unitary of G. **[4]**

b) Let G_1, G_2 be abelian groups and suppose that $\chi_1, \chi_2 \ldots\ldots\ldots \chi_m$ and $\phi_1, \phi_2, \ldots\ldots\ldots \phi_n$ are irreducible representations of G_1 and G_2 respectively, then show that the functions $\alpha_{ij} : G_1 \times G_2 \rightarrow C^*$, with $1 \leq i \leq m$, $1 \leq j \leq n$ given by $\alpha_{ij}(g_1, g_2) = \chi_i(g_1)\, \phi_j(g_2)$ forms a complete set of irreducible representations. **[7]**

c) Let $g, h \in G$. Prove that $\delta_g * \delta_h = \delta_{gh}$. **[3]**

Q7) a) Define following terms : **[4]**

i) dual group

ii) fourier inversion

b) Prove that the set L(G) is a ring with addition taken pointwise and convolution as multiplication. Further prove that δ_1 is the multiplicative identify. **[7]**

c) If $\chi,\theta\in\widehat{G}$ then show that $\widehat{\chi}=|G|\delta^*$. **[3]**

Q8) a) The Fourier transform satisfies $\widehat{a*b}=\hat{a}\bullet\hat{b}$. **[7]**

b) Show that | G | = $d_1^2 + d_2^2 + \ldots\ldots\ldots\ldots d_s^2$ holds. **[5]**

c) State Cayley - Hamilton theorem. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2393

[Total No. of Pages : 3

[6061]-417

S.Y.M.Sc.

MATHEMATICS

MTUTO - 147 : CODING THEORY

(2019 Pattern) (CBCS) (Semester -IV)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) *Attempt any five questions*
2) *Figures to the right indicate full marks.*

Q1) a) Consider a memoryless binary channel with channel probabilities

P (0 received | 0 sent) = 0.7

P (1 received | 1 sent) = 0.8

If code words from the binary code {000, 100, 111} are being sent over this channel, use the maximum likelihood decoding rule to decode the word 011. **[5]**

b) For the ternary code c = {00122, 12201, 20110, 22000}, use the neareest neighbour decoding rule to decode the word 01122. **[5]**

c) Construct the incomplete maximum likelihood decoding table for the binary code c = {000, 001, 010, 011}. **[4]**

Q2) a) Let C be a linear code of length n over F_q. Then show that $\left(C^{\perp}\right)^{\perp} = C$. **[4]**

b) Let C be an [n, k] linear code over F_q with generator matrix G. Then show that V$\in F_q^n$ belongs to $C^{\perp}$ if and only if v if orthogonal to every row of G.Also, show that (n- k) × n matrix H is a parity- check matrix for C if and only if the rows of H are linearly independent and $HG^T = 0$. **[8]**

c) Prove that there is no selp - dual binary code of parametors $[10, 5, 4]$. **[2]**

Q3) a) Find a generator matrix and a parity - check matrix for the linear code generated by the set S {1000, 0110, 0010, 0001, 1001}, take q= 2. Also give the parameters [n,k,d]. **[6]**

P.T.O.

b) Let H = $\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$ be a parity - check matrix for the binary linear code c. Find a generator matrix G in standard form for a binary linear code equivalent to C. **[6]**

c) Determine the number of binary linear codes with parameters [n, n-1, 2] for n$\geq$ 2. **[2]**

Q4) a) Assign messages to the words in F_2^3 as follows. **[5]**

000	100	010	001	110	101	011	111
A	C	D	E	G	I	N	O

Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$ use G to encode the message ENCODING.

b) Let C be the binary linear code with parity- check matrix **[5]**

$$H = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}.$$ Write down a generator matrix for C and list all the codewords in C. Also Decode the word 011011.

c) Let S= {1020,0201, 2001}. Find the F_3 - linear span<S> and its orthogonal complement $S^{\perp}$. **[4]**

Q5) a) Let q=2 and C= {0000, 1011, 0101, 1110}. Find a parity - check martrix for C. Construct a sundrome look- up table for C and decode the word W = 1111. **[6]**

b) For an integer q>1 and integers n, d such that $1 \leq d \leq n$ show that

$$\frac{q^n}{\sum_{i=0}^{d-1} \binom{n}{i} (q-1)^i} \leq A_q(\text{n,d})$$ **[8]**

Q6) a) Let n, k and d be integers satisfying $2 \le d \le n$ and $1 \le k \le n$. If $\sum_{i=0}^{d-2} \binom{n-1}{i} (q-i)^i < q^{n-k}$ then show that there exists an [n, k] - linear code over F_q with minimum distance at least d. **[8]**

b) Prove that $A_2(5, 4) = 2$ **[4]**

c) Define the extended ternary Golay code. **[2]**

Q7) a) Let g(x) be the generator polynomial of a q - ary [n, k] - cyclic code c. Put $h(x) = \dfrac{x^n - 1}{g(x)}$.Show that $h_0^{-1} h_R(x)$ is the generator polynomial of $C^{\perp}$, where h is the constant term of $h(x)$ *and* $h_R(x) = x^k h\left(\dfrac{1}{x}\right)$. **[10]**

b) Let C be the binary [7, 4]- cyclic code with generator polynomial $g(x) = 1 + x^2 + x^3$ find the generator matrix in standard form for C. **[4]**

Q8) a) Let C be the binary [7, 4, 3] - Hamming code with the generator polynomial $g(x) = 1 + x^2 + x^3$ Find a parity - check matrix for C and decode the word 1011100. **[6]**

b) Let $\alpha \in F_8$ be a root of $1 + x + x^3$.find the parameters for a narrow - sense binary BCH code of length 7 corresponding to α over F_8, generated by $lcm\left(m^{(1)}(x), m^{(2)}(x)\right)$. **[6]**

c) Find the dimension of a narrow - sense binary BCH code of length 63 with designed distance 5. **[2]**

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P2394

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M.A./M.Sc.

MATHEMATICS

MTUTO - 148 : Probability and Statistics

(2019 Pattern) (Semester-IV) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any FIVE questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of scientific calculator is allowed.***

Q1) a) Define following terms. **[4]**

i) Permutation

ii) Conditional probability of B given A.

b) Given the joint density function **[5]**

$$f(x,y)=\begin{cases}\dfrac{x(1+3y^2)}{4} & ,\ 0<x<2, 0<y<1\\ 0 & ,\ \text{elsewhere}\end{cases}$$

Find $g(x)$, $h(y)$, $f(x/y)$ and evaluate $P\left(\frac{1}{4}<X<\frac{1}{2}/Y=\frac{1}{3}\right)$.

c) A shipment of 20 similar laptop computers to a retail outlet contains 3 that are detective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives. **[5]**

Q2) a) The expected value of the sum or difference of two or more functions of a random variable. X is the sum or difference of the expected values of the functions. That is, show that **[4]**

$E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)]$

P.T.O.

b) In business it is important to plan and carry out research in order to anticipate what will occur at the end of the year. Research suggest that the profit (loss) spectrum for a company, with corresponding probabilities, is as follows. **[5]**

Profit	Probability
₹–15,000	0.05
₹0	0.15
₹15,000	0.15
₹25,000	0.30
₹40,000	0.15
₹50,000	0.10
₹1,00,000	0.05
₹1,50,000	0.03
₹2,00,000	0.02

i) What is the expected profit?

ii) Give the standerd deviation of the profit.

c) Let X be a random variable with probability distribution as follows. **[5]**

x	0	1	2	3
$f(x)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

Find the expected value of $y = (x-1)^2$.

Q3) a) Show that mean and varience of the hypergeometric distribution h(x; N,n,k) are $\mu = \frac{nk}{N}$ and $\sigma^2 = \frac{N-n}{N-1}.n.\frac{k}{N}\left(1-\frac{k}{N}\right)$ **[4]**

b) Ten is the average number of oil tankers arriving each day at a certain port. The facilities at port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away? **[5]**

c) In a manufacturing process where glass products are made, defects or bubbles occurs, occasionally rendering the piece undesirable for marketing. It is known that, on average, 1 in every 1,000 of these items produced has one or more bubbles. What is the probability that a random sample of 8,000 will yield fewer than 7 items possessing bubbles? **[5]**

Q4) a) Show that mean and variance of the gamma distribution are **[4]**

$\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$

b) The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that fewer than 30 survive? **[5]**

c) Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$, find the probability that X assumes a value greater than 362. **[5]**

Q5) a) Show that an unbiased estimate of σ^2 is,

$$s^2 = \frac{\text{SSE}}{n-2} = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n-2} = \frac{\text{S}_{yy} - b_1\text{S}_{xy}}{n-2}$$ **[4]**

b) The following data were collected to determine the relationship between pressure and the corresponding scale reading for the purpose of calibration. **[5]**

Pressure, x (lb/sq.in.)	Scade reading, y
10	13
10	18
10	16
10	15
10	20
50	86
50	90
50	88
50	88
50	92

i) Find the equation of the regression line.

ii) The purpose of calibration in the application is to estimate pressure from an observed scale reading. Estimate the pressure for a scale reading of 54 using $\hat{x} = \frac{54 - b_0}{b_1}$

c) Using the estimated value $b_1 = 0.903643$, test the hypothesis that $\beta_1 = 1.0$ against the alternative that $\beta_1 < 1.0$. **[5]**

Q6) a) Prove that $M_{ax}(t) = e^{at} M_x(t)$ **[4]**

b) The following data represent the chemistry grades for a random sample of 12 freshmen at a certain college along with their score on an intelligence test administered while they were still seniors in high school. **[6]**

Student	Test Score , x	Chemistry Grade, y
1	65	85
2	50	74
3	55	76
4	65	90
5	55	85
6	70	87
7	65	94
8	70	98
9	55	81
10	70	91
11	50	76
12	55	74

Compute and interpret the sample correlation coefficient.

c) If $S=\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A=\{0, 2, 4, 6, 8\}$, $B=\{1, 3, 5, 7, 9\}$, $C=\{2, 3, 4, 5\}$ and $D=\{1, 6, 7\}$ list. the elements of The sets corresponding to following events: **[4]**

i) $A \cup C$

ii) $A \cap B$

iii) C'

iv) $(C' \cap D) \cup B$

Q7) a) Find mean and Variance of poisson distribution. **[4]**

b) Let X denote the number of times a certain numerical controle machine will malfunction : 1,2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as. **[5]**

$f(x, y)$		x		
		1	2	3
	1	0.05	0.05	0.10
y	3	0.05	0.10	0.35
	5	0.00	0.20	0.10

i) Evaluate the marginal distribution of X

ii) Evaluate the marginal distribution of Y

iii) Find P(Y=3 | X=2).

c) Consider the situation, **[5]**

The distribution of the number of imperfections per 10 meters of synthetic failure is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Find the varience and standerd deviation of X.

Q8) a) Let X and Y be two independant random variables. Then, E (XY) =E (X) E (Y). **[4]**

b) A certain type of storage battery lasts, on average, 3.0 years with a standerd deriation of 0.5 year. Assuming that battery life is normally distributed, find the probability that a given battery will last less than 2.3 years. **[5]**

c) Calculate karla pearsons coeffiaient of correlation for the following data.

x	6	8	12	15	18	20	24	18	31
y	10	12	15	15	18	25	22	26	28

[5]

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