Total No. of Questions : 8]

SEAT No. :

PD-2973

[Total No. of Pages : 3

[6474]-11
M.A./M.Sc.
MATHEMATICS
MTUT-111 : Linear Algebra
(2019 Pattern) (CBCS) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*

Q1) a) Let $\{v_1, v_2 \,......v_n\}$ be a basis of V over F. If S and T are elements of L(V, W) such that $S(v_i) = T(v_i)$ $1 \le i \le n$ the prove that S = T and if $w_1, w_2, ..., w_n$ be arbitrary vectors in W, there exists one and only one linear transformation $T \in L(V, W)$ such that $T(v_i) = w_i$. **[7]**

b) Let $T \in L(V, W)$ be given by $T(u_1) = u_1 - u_2$, $T(u_2) = 2u_2$ where $\{u_1, u_2\}$ is a basis for V. **[4]**

i) Find rank and nullity of T

ii) Check T is invertible or not

c) Test the linear transformation $T : R_3 \to R_2$ defined by the system of equation $y_1 = 3x_1 - x_2 + x_3 \quad y_2 = -x_1 + 2x_3$ is one to one or not. **[3]**

Q2) a) Show that, $\{a, b\}$ is a linearly dependent set if and only if $a = \lambda b$ or $b = \lambda' a$ for some λ and λ' in F. **[7]**

b) Let A, B, C be points in R_2 the prove that $\overline{AB} + \overline{BC} = \overline{AC}$. **[4]**

c) The set of all $F \in C(R)$ such that $F\left(\frac{1}{2}\right)$ is a rational number is subset of C(R). Is F subspace of C(R)? Justify. **[3]**

P.T.O.

Q3) a) Let $v_1, v_2, ..., v_r$ be characteristics vectors belonging to different (distinct) characteristic roots $\alpha_1, \alpha_2......, \alpha_r$ or $T \in L(V, V)$ then prove that $\{v_1, v_2, ..., v_r\}$ are linearly independent. **[6]**

b) Find the rational canonical form of a linear transformation on a vector space over the field of rational numbers φ whose elementary divisor are $(x-1)^2, x^2 - x + 1$. **[2]**

c) Let T be a linear transformation on a finite dimensional vector space over F and Let $\alpha \in F$, then show that α is a characteristic root of T if and only if the determinant $D(T-\alpha, 1) = 0$, where 1 is the identity transformation on V. **[6]**

Q4) a) Prove that, every orthonormol set of vectors is linearly independent set. **[6]**

b) Prove that, a linear transformation T preserves distances if and only if $\|T(e_1)\| = \|T(e_2)\| = 1$ and $T(e_1) \perp T(e_2)$. **[4]**

c) State and prove Canchy-Schwarz Inequality. **[4]**

Q5) a) Let $T \in L(V, V)$ and $\{v_1, ..., v_n\}$ be a basis of V and $\{f_1, ..., f_n\}$ the dual basis of V^*. Let A be the matrix of T w.r.t. the basis $\{v_1......v_n\}$ prove that the matrix Df T^* w.r.t. basis $\{f_1, ..., f_n\}$ is transpose matrix 0 f ^tA. **[6]**

b) Let $\{v_1, ..., v_n\}$ be a basis for V over F prove that there exists linear functions $\{f_1, ..., f_n\}$ such that for each i $f_i(v_i) = 1$, $f_i(v_j) = 0, j \neq i$. **[6]**

c) If $F(x_1, x_2) = x_1^2 - 10x_1x_2 - 5x_2^2$, find symmetric matrix A whose quadratic equation is $F(x_1, x_2)$. **[2]**

Q6) a) Let v and v' be finite dimensional vector space which are dual with respect to nondegenerate bilinear form B and let v_1 and v_1^1 be subspaces of v and v' respectively. Then prove that **[5]**

$\dim v_1 + \dim v_1^1 = \dim v$

$\dim (v_1^1)^{\perp} + \dim v_1^1 = \dim v^1$

b) Find Jordan Canonical form of the following matrices over C show that pair of matrices are similar? $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. **[6]**

c) Define an unitary transformation show that, $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ is unitary matrix. **[3]**

Q7) a) Let T be an orthogonal transformation on a real vector space V with an inner product then prove that V is a direct sum of irreducible invarient subspaces $\{w_1 \ldots w_s\}$ for $S \geq 1$, such that vectors belong to distinct subspaces w_i and w_j are orthogonal. **[7]**

b) Define symmetric tensor. **[2]**

c) Let φ be a quadratic form on V whose matrix with respect to the basis $\{e_1 \ldots\ldots. e_n\}$ is $S = (F_{ij})$ Let $\{f_1 \ldots.. f_n\}$ be another basis of V such that $f_i = \sum_{j=1}^{n} v_{-ij} e_j \;\; 1 \leq -i \leq n$ then prove that the matrix of φ with respect to the basis $\{f_1,\ldots.,f_n)$ is given by $s' = csc$ where $c = (v_{-ij})$. **[5]**

Q8) a) Find an orthonormal basis for the subspace C(R) generated by the function $\{1, x, x^2\}$ w.r.t. inner product $(f \cdot g) = \int_0^1 f(t) \cdot g(t) dt$. **[6]**

b) Let T be a normal transformation on V prove that there exists common characteristics vectors for T and T' as $T_v = av$ and $T'_v = \bar{a}v$. **[5]**

c) Compute $A_1 \times B_1$ where $A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. **[3]**

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Total No. of Questions : 8]

SEAT No. :

PD-2974

[Total No. of Pages : 3

[6474]-12
M.A./M.Sc.
MATHEMATICS
MTUT-112 : Real Analysis
(2019 Pattern) (Semester - I) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of single memory, non-programmable calculator is allowed.*

Q1) a) i) Let f and g are measurable function on E. Prove that $f \cup g$ and $f \cap g$ are measurable. **[4]**

ii) Prove that countable set has outer measure zero. **[3]**

b) Prove that any set E of real numbers with positive outer measure contains a subset that fails to be measurable. **[7]**

Q2) a) i) Let f is measurable function over measurable set E then prove that $|f|$ is measurable. **[4]**

ii) State Little woods three principles. **[3]**

b) State and prove Egoroff's theorem. **[7]**

Q3) a) i) Let f be a function defined on [0,1] as

$$f(x)=\begin{cases} x\cos\left(\dfrac{\pi}{2x}\right), & \text{if } 0<x\le 1 \\ 0, & \text{if } x=0 \end{cases}$$ **[4]**

Show that f is not bounded variation on [0,1].

ii) If f is measurable then show that f^+ and f^- are measurable on E. **[3]**

P.T.O.

b) Let the function f be monotone on the closed, bounded interval [a,b]. Then prove that f is absolutely continuous on [a,b] if and only if

$$\int_a^b f' = f(b) - f(a).$$ **[7]**

Q4) a) Let $\{E_k\}_{k=1}^{\infty}$ is any countable collection of sets which are disjoint or not then prove that $m^*\left(\bigcup_{k=1}^{\infty} E_k\right) \le \sum_{k=1}^{\infty} m^*(E_k)$ **[7]**

b) Prove that the cantor set C is closed, uncountable set of measure zero. **[7]**

Q5) a) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of measurable functions on E that converges pointwise almost everywhere on E to the function f. Prove that f is measurable. **[7]**

b) Let f be an extended real valued function on E, also f is measurable on E and $f = g$ almost everywhere on E. Then prove that g is measurable on E. **[7]**

Q6) a) Let function f be continuous on closed, bounded interval [a,b]. Prove that f is absolutely continuous on [a,b] if and only if the family of divided difference function $\{\text{Di}\, ff_h\, f\}_{0<h\le 1}$ is uniformly integrable over [a,b]. **[7]**

b) If the function f is monotone on the open interval (a,b). Then prove that f is differentiable almost everywhere on (a,b). **[7]**

Q7) a) Let A and B are any two disjoint subset of $\mathbb{R}$. Show that $m^*(A \cup B) = m^*(A) + m^*(B)$. **[7]**

b) Let E be a measurable set of finite outer measure. For each $\in\, > 0$ there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^{\infty}$ if $\theta = \bigcup_{k=1}^{n} I_k$ then prove that $m^*(E \sim \theta) + m^*(\theta \sim E) > E$. **[7]**

Q8) a) Show that the function f defined by $f(x) = \frac{1}{x^{1/3}}, 0<x\leq 1$ and $f(0) = 0$ is Lebesque integrable over [0,1]. **[7]**

b) Let f be integrable over the closed bounded interval [a,b]. Then prove that $f(x) = 0$ for almost all $x \in [a,b]$ if and only if $\int_{x_1}^{x_2} f = 0$, for all $(x_1, x_2) \subseteq [a,b]$. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PD-2975

[Total No. of Pages : 3

[6474]-13
F.Y. M.A./M.Sc.
MATHEMATICS
MTUT-115 : Ordinary Differential Equations
(2019 Pattern) (CBCS) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*

Q1) Attempt the following.

a) i) Solve the differential equation $y' - 2y = 1$ **[2]**

ii) Find the solution of differential equation $y'' - 2y' - 3y = 0$, $y(0) = 0$, $y'(0) = 1$. **[5]**

b) Explain the variable separable method for first order differential equation $y' = f(x, y)$. **[7]**

Q2) Attempt the following.

a) Find all solutions of the differential equation $y' + 3y = e^{ix}$ and $y' + y = e^{x}$. **[7]**

b) Prove that the two solutions ϕ_1 and ϕ_2 of the differential equation $L(y) = y'' + a_1y' + a_2y = 0$ are linearly independent on interval I if and only if $W(\phi_1, \phi_2) \neq 0 \ \forall \ x \in I$ **[7]**

Q3) Attempt the following.

a) i) Determine whether the function $\phi_1(x) = \cos x$ and $\phi_2(x) = \sin x$ are linearly dependent or independent on $-\infty < x < \infty$. **[4]**

ii) Solve the differential equation $y'' + 4y = \cos x$. **[3]**

P.T.O.

b) Let ϕ_1 be the solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I. Then show that the second solution ϕ_2 on I of $L(y) = 0$ is given by **[7]**

$$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp.\left[-\int_0^s a_1(t)dt\right] ds$$

Q4) Attempt the following.

a) Verify that the function $\phi_1(x) = x$ satisfies the equation $x^2y'' - xy' + y = 0$, $(x > 0)$ and find the second independent solution. **[7]**

b) Explain the method of reduction of order for solving n^{th} order homogeneous equation with variable coefficients. **[7]**

Q5) Attempt the following.

a) If $\phi_1(x) = x^2$ is one solution of $y'' - \frac{2}{x^2}y = x$ on $0 < x < \infty$ then find all solution of $y'' - \frac{2}{x^2}y = x$ **[7]**

b) Show that every solution $\psi(x)$ of $L(y) = b(x)$ on I is of the form $\psi(x) = \psi_p + c_1\phi_1 + c_2\phi_2$ where b(x) is continuous on I, ψ_p is particular solution and ϕ_1, ϕ_2 are linearly independent solution of $L(y) = 0$ **[7]**

Q6) Attempt the following.

a) If M, N be two real valued functions which have continuous first partial derivatives on some rectangle R, $|x - x_0| \leq a$, $|y - y_0| \leq b$ then prove that $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R. **[7]**

b) Obtain two solutions of Bessel's equation of order zero. **[7]**

Q7) Attempt the following.

a) Show that $\phi_1(x) = |x|^i$ and $\phi_2(x) = |x|^{-i}$ are linearly independent solutions of the equation $x^2y'' + xy' + y = 0$ **[7]**

b) Determine that the equation $y' = \dfrac{3x^2 - 2xy}{x^2 - 2y}$ is exact and solve it **[7]**

Q8) Attempt the following.

a) Consider the initial value problem $y' = (3y + 1)$, $y(0) = 2$

Compute the first four approximations $\phi_0, \phi_1, \phi_2, \phi_3$ to the solution **[7]**

b) Find two linearly independent power series solution of the equation $y'' - xy' + y = 0$ **[7]**

Total No. of Questions : 8]

SEAT No. :

PD-2976

[Total No. of Pages : 3

[6474]-14
M.A./M.Sc.
MATHEMATICS
MTUT-114 : Advanced Calculus
(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***
3) ***Use of single memory, non-programmable calculator is allowed.***

Q1) a) Prove that if f is differentiable at 'a' with total derivative T_a then derivative $f'(a, y)$ exist for every y in $\mathbb{R}^n$ & $T_a(y) = f'(a, y)$. Moreover $f'(a, y)$ is a linear combination of the component of y. In fact, if $y = (y_1, y_2,.....,y_n)$, we have $f'(a,y) = \sum_{k=1}^{n} D_k f(a) y_k$. **[6]**

b) Show that for the function $f(x,y) = \dfrac{x-y}{x+y}$, if $(x + y) \neq 0$ repeated limit exist, But not equal. **[4]**

c) Find the gradient of the function $f(x, y, z) = x^2y^3z^4$. **[4]**

Q2) a) Prove that if a scaler field f is differentiable at $\bar{a}$ then f is continuous at $\bar{a}$. **[6]**

b) Evaluate the directional derivative of the scaler field $f(x,y,z) = \left(\dfrac{x}{y}\right)$ at $(-1, 1, 1)$ in the direction of $2\bar{i} + \bar{j} - \bar{k}$. **[4]**

c) Give any two Basic properties of line integral. **[4]**

P.T.O.

Q3) a) Prove that, if $\overline{\alpha}$ & $\overline{\beta}$ be equivalent piecewise smooth path. Then $\int_C \overline{f}\cdot d\overline{\alpha} = \int_C \overline{f}.d\overline{\beta}$, if $\overline{\alpha}_1$ and $\overline{\beta}$ trace out C is opposite direction, and $\int_C \overline{f}.d\overline{\alpha} = -\int_C \overline{f}.d\overline{\beta}$, if $\overline{\alpha}$ & $\overline{\beta}$ trace out C in opposite directions. **[6]**

b) Calculate the line integral of vector field

$\overline{f}(x,y,z) = x\overline{i} + y\overline{j} + (xz - y)\overline{k}$ from (0, 0, 0) to (1, 2, 4) along a line segment. **[4]**

c) Determine weather or not the vector field $f(x,y) = 3x^2 y\overline{i} + x^3 y\overline{j}$ is a gradient on any open subset of $\mathbb{R}^2$. **[4]**

Q4) a) A particle of mass m along a curve under the action of a force field $\overline{f}$. If the speed of the particle at time t is $\overline{v}(t)$. it's kinetic energy is defined to be $\frac{1}{2}m\overline{v}^2(t)$. Prove that the change in kinetic energy in any time interval is equal to the workdone by $\overline{f}$ during the time interval. **[6]**

b) Define : **[4]**

i) Line integral with respect to arc length.

ii) Open connected set.

c) Evaluate $\iint_{\varphi} (x^3 + 3x^2 y + y^3)dxdy$, where φ : [0, 1] × [0, 1] **[4]**

Q5) a) State and prove Green's theorem in the plane. **[6]**

b) Evaluate the line integral $\int_C (5 - xy - y^2)dx - (2xy - x^2)dy$ where C is the square with vertices (0, 0), (1, 0), (1, 1), (0, 1), traversed counterclockwise. **[4]**

c) Define Bounded set of content zero. Give any two consequences of this defination. **[4]**

Q6) a) Let ϕ be a real valued function that is continuous on an interval [a, b]. Then the graph of ϕ has content zero. **[6]**

b) Find the Jacobian for cylindrical transformation. **[4]**

c) Evaluate $\iiint_S xyz\,dx\,dy\,dz$, where $S = \{(x, y, z)/x^2 + y^2 + z^2 \leq 1, x \geq 0, y \geq 0, z \geq 0\}$ **[4]**

Q7) a) Prove that, fundamental vector product is normal to the surface. **[6]**

b) If $f(x,y,z) = x\bar{i} + y\bar{j} + z\bar{k}$. Prove that the Jacobian matrix D $f(x, y, z)$ is the identity matrix of order 3. **[4]**

c) For given vector field $\bar{F}$, determine the curl and divergence. **[4]**

$$\bar{F}(x,y,z) = (x^2 + yz)\bar{i} + (y^2 + xz)\bar{j} + (z^2 + xy)\bar{k}$$

Q8) a) State and prove stokes theorem. **[7]**

b) Use the Divergence theorem to evaluate $\iint_S \bar{F} \cdot \bar{n}\, dS$ where $\bar{F}(x,y,z) = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and the surface S be the unit cube, $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. **[7]**

Total No. of Questions : 8]

SEAT No. :

PD-2978

[Total No. of Pages : 3

[6474]-21
M.A./M.Sc.
MATHEMATICS
MTUT121 : Complex Analysis
(2019 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $P : \mathbb{C} \to \mathbb{C}$ be a polynomial function then prove that $|P| : \mathbb{C} \to \mathbb{R}$ attains its infimum **[5]**

b) Let $f : U \to V$ and $g : V \to W$ be such that f is differentiable at $Z_0 \in U$ and g is differentiable at $W_0 = f(z_0) \in V$ then prove that gof is differentiable at z_0 and $D(gof)_{z_0} = D(g)_{W_0} oD(f)_{z_0}$ **[5]**

c) If $Z = x + iy$ then show that $\frac{|x|+|y|}{\sqrt{2}} \leq |z|$ **[4]**

Q2) a) State and prove fundamental theorem of algebra. **[5]**

b) Let $f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

show that f is continuous and all the directional derivatives of f exists. **[4]**

c) Use polar co-ordinates to show that $z \to |z|^2$ is complex differentiable at 0. **[3]**

d) Compute the length of the circle **[2]**

$Cr : Z(\theta) = a + re^{i\theta},\ 0 \leq \theta \leq 2\pi$

P.T.O.

Q3) a) Let Ω be a simply connected domain $\mathbb{C}$ and *f* be a holomorphic function on it. Then show that for any simple closed contour γ in Ω $\int_{\gamma} f(z)dz = 0$ **[5]**

b) For all W such that $|W - a| < r$, show that $\int_{C} \frac{dW}{Z-W} = 2\pi i$ **[5]**

c) Evaluate $\int_{C} \frac{\sin z}{z+3i} dz$ where C : $|Z - 2 + 3i| = 1$ traversed in counterclockwise direction. **[4]**

Q4) a) Let *f* be a complex differentiable non-constant function on a domain Ω. Then prove that there doesnot exists any point $W \in \Omega$ Such that $|f(z)| \leq |f(w)| \ \forall \ z \in \Omega$ **[5]**

b) Let C be a circle $|z| = 3$ traced in the counterclockwise sense for any z with $z \neq 3$ let $g(z) = \int_{C} \frac{2w^2 - w - 2}{w - z} dw$, prove that $g(2) = 8\pi i$ Find g(4). **[4]**

c) Is the function $\frac{z^2+1}{z(z-1)}$ meromorphic? Justify? **[3]**

d) Find the value of the integration $\int_{C} \frac{e^{az}}{z} dz$ where C is the unit circle $|Z| = 1$ traversed in counterclockwise direction. **[2]**

Q5) a) Let $f : \mathbb{C} \to \mathbb{C}$ be a holomorphic function then prove that *f* is a rational function. **[5]**

b) Let f and g be holomorphid function on a domain Ω. Suppose $K \subset \Omega$ is such that for every $Z \in K$, f(z) = g(z) and K has a limit point of f(z). then show that f(z) = g(z) on Ω. **[5]**

c) Compute the residues of all singular points of the function $f(z) = \frac{e^z}{z^2 - 1}$ $z \neq \pm 1$ **[4]**

Q6) a) Let Ω be a holomorphic function on $A(r_1, r_2)$. Let $r_1 < \rho_1 < \rho_2 < r_2$. Then for $\rho_1 < |z| < \rho_2$, show that **[5]**

$$f(z)=\frac{1}{2\pi i}\int_{|W|=\rho_2}\frac{f(W)}{W-z}dw-\frac{1}{2\pi i}\int_{|W|=\rho_1}\frac{f(W)}{W-z}dw$$

b) Obtain Laurent series expansion for the function $f(z)=\frac{1}{1+z^2}$ for the region $A=\{z\,|z-1-i|<1\}$ **[5]**

c) Find the function f(z) to evaluate the improper integral $\int_{-\infty}^{\infty}\frac{\cos 3x}{(x^2+1)^2}dx$ by using complex method. **[4]**

Q7) a) Show that $\int_{-\infty}^{\infty}\frac{x^2}{1+x^4}dx=\frac{\pi}{\sqrt{2}}$ **[7]**

b) Show that $\int_0^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}},\ -1<a<1$ **[7]**

Q8) a) Let $f:D\to\bar{D}$ be a holomorphic function such that $f(0) = 0$ then prove that $|f(z)| < |z|$ and $|f'(0)| \leq 1$. Further prove that the following conditions are equivalent **[6]**

i) There exists Z_0 with $Z_0 \neq 0$, $|Z_0| < 1$ and $|f(Z_0)| =| Z_0|$

ii) $|f'(0)| = 1$

iii) $f(z) = cz$ for some $|c| = 1$

b) Let *f*, *g* be holomorphic in an open set C, containing closure $\bar{D}$ of a disc D and satisfying the inequality $|f(z) - g(z)| < |g(z)|\ \forall z \in \partial D$ then show that f and g have same number of zeros inside C. **[5]**

c) Find the Cauchy's principal value of $\int_{-\infty}^{\infty}\frac{e^{iax}}{1-x}dx$ for H : $|Z| < 1$ **[3]**

Total No. of Questions : 8]

SEAT No. :

PD-2979

[Total No. of Pages : 2

[6474]-22
M.A./M.Sc.
MATHEMATICS
MTUT-122 : General Topology
(2019 Pattern) (Semester - II) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that countable union of countable sets is countable. **[6]**

b) The set $\mathbb{Z}_+ \times \mathbb{Z}_+$ is countably infinite. **[4]**

c) If C is an infinite subset of $\mathbb{Z}_+$, then C is countably infinite. **[4]**

Q2) a) Show that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$ but are not comparable with one another. **[6]**

b) If $\{\tau_\alpha\}$ is a family of topologies on X, then show that $\cap\tau_\alpha$ is a topology on X. **[4]**

c) Show that Π_1 and Π_2 are open maps. **[4]**

Q3) a) Show that a subspace of a Hausdorff space is Hausdorff. **[5]**

b) Every finite point set in Hausdorff space X is closed. **[5]**

c) Define : **[4]**

i) Homeomorphism

ii) Limit point

P.T.O.

Q4) a) State and prove pasting lemma. **[5]**

b) The image of connected space under a continuous map is connected. **[5]**

c) Let X = $\mathbb{R}$ define $d(x, y) = |x - y|$ for $x, y \in$ X. Then show that d is metric on X. **[4]**

Q5) a) If the set C and D form a separation of X, and if Y is a connected subspace of X, then Y lies entirely within either C or D. **[5]**

b) Show that, the rationals are not connected. **[4]**

c) Show that every closed subspace of a compact space is compact. **[5]**

Q6) a) Show that every path connected space is connected. **[5]**

b) Define Isolated point. Is {3} is isolated point of $\mathbb{R}$? Justify **[4]**

c) Prove that, compactness implies limit point compact. **[5]**

Q7) a) Show that every metrizable space is normal. **[6]**

b) Prove that every regular space with a countable basis is normal. **[6]**

c) State Urysohn lemma. **[2]**

Q8) a) Define : **[6]**

i) Hausdorff space

ii) Regular space

iii) Normal space

b) Suppose that X has a countable basis. Then every open covering of X contains a countable subcollection covering X. **[6]**

c) Define dense set. **[2]**

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Total No. of Questions : 8]

SEAT No. :

PD-2980

[Total No. of Pages : 2

[6474]-23
M.A./M.Sc. (Part - I)
MATHEMATICS
MTUT-123 : Ring Theory
(2019 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) If R is a ring with 1 and I is a 2-sided ideal in R such that $I \neq R$ then prove that there is a 2-sided maximal ideal M such that $I \leq M$. **[7]**

b) Prove that every principal Ideal domain is unique factorisation domain. **[7]**

Q2) a) Define : **[5]**

i) Division Ring

ii) Characteristic of a ring with suitable example.

b) Prove that every Euclidean domain is principal Ideal domain. **[7]**

c) Show that $\mathbb{Z}[i\sqrt{2}]$ is a subring of $\mathbb{C}$. **[2]**

Q3) a) Let S be subring of a ring with any example show that S and R both have unities but they may not be the same. **[7]**

b) Prove that a Euclidean domain R has unity and whose group of units is given by $U(R) = \{a \in R^* \,/\, d(a) = d(1)\}$. **[7]**

P.T.O.

Q4) a) Let $R = \mathbb{Z}[i, j, k]$ be the ring of integral quaternions. Show that the units in R from a group of order 8. **[7]**

b) State and prove Eisenstein's criterion. **[7]**

Q5) a) Let R be a commutative ring with 1 and I be an ideal in R, then prove that R/I is an integral domain if and only if I is a prime ideal in R. **[7]**

b) Let $f : R \to S$ be a homomorphism of rings. Then prove that inverse image of a prime ideal is a prime ideal in R if both R and S are commutative. **[4]**

c) In ring $Z[i]$ show that the elements $3 + 4i$ and $4 - 3i$ are associates. **[3]**

Q6) a) In a Unique factorisation Domain prove that every irreducible element is a prime. **[7]**

b) Let R be a domain. Then prove that $R[x]$ is a Unique factorisation Domain (V F D) if and only if R is Unique factorisation domain (V F D). **[7]**

Q7) a) If d is a positive integer, then prove that the ring $Z[i\sqrt{d}]$ is a factorisation domain. **[7]**

b) Show that ring R is an integral domain if and only if $R \neq (0)$, R has no non-trivial nilpotent elements and (0) is prime ideal in R. **[7]**

Q8) a) Show that vector space is free module. **[7]**

b) State and prove Schur's lemma for simple module. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PD-2981

[Total No. of Pages : 4

[6474]-24
M.A./M.Sc. (Part - I)
MATHEMATICS
MTUT124 : Advanced Numerical Analysis
(CBCS) (2019 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Determine the corresponding rate of convengence for the function $f(x)=\dfrac{2^n+3}{2^n+7}$. **[4]**

b) Prove that if $g : [a, b] \to [a, b]$ is continuous function on [a, b], differentiable on (a, b) and there exists a constant k < 1 such that, $|g'(x)| \le k < 1,\ \forall\ x \in (a, b)$ then, **[5]**

i) The sequence $\{P_n\}$ generated by $P_n = g(P_{n-1})$ converges to the fixed point P for any $P_0 \in [a, b]$.

ii) $|P_n - P| \le \dfrac{k^n}{1-k}|P_1 - P_0|$

c) Use the secant Method to determine P_5, the fifth approximation to root of $f(x) = x^7 - 3$ in (1, 2) with $P_0 = 1$ and $P_1 = 2$ **[5]**

Q2) a) The sequence listed below was obtained from false position method applied to the function $f(x) = x^3 + 2x^2 - 3x - 1$ has zero on the interval (1, 2). Applying Aitken's Δ^2 – Method to the given sequence. Find $\hat{P}_3$, $\hat{P}_4$ and $\hat{P}_5$ **[6]**

1	1.1
2	1.1517436381
3	1.1768409100
4	1.1886276733
5	1.1940789113

P.T.O.

b) Show that when Newton's Method is applied to the equation $x^2 - a = 0$, the resulting iteration function is $g(x)=\frac{1}{2}\left(x+\frac{a}{x}\right)$. **[4]**

c) Define : **[4]**

i) Orthogonal matrix

ii) Absolute Error

Q3) a) Solve the following system of equations by using Gaussian elimination with scaled partial pivoting. **[5]**

$2x_1 + 3x_2 + x_3 = -4$

$4x_1 + x_2 + 4x_3 = 9$

$3x_1 + 4x_2 + 6x_3 = 0$

b) Show that the Matrix **[4]**

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$$ has no LU Decomposition

c) Solve the following system by Jacobi Method starting with vector $x(0) = [0\ 0\ 0]$ perform two iteration. **[5]**

$3x_1 + x_2 = -1$

$-x_1 + 2x_2 + x_3 = 3$

$x_2 + 3x_3 = 4$

Q4) a) Use the QR Factorization of a symmetric Tridiagonal matrix.

$$A=\begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Find the product $R^{(0)}\ Q^{(0)}$, **[7]**

b) Define Householder Matrix and Show that it is symmetric and orthogonal. **[4]**

c) For the following differential equation, identify the function f(t, x) and calculate $\frac{df}{dt}, \frac{d^2 f}{dt^2}$

$x^1 = t^2 - 1 - 2x^2$ **[3]**

Q5) a) Use Euler's method to solve Initial Value problem

$$\frac{dx}{dt} = t^2 + x, \ 0 \le t \le 0.03, \ x(0) = 1$$ **[7]**

b) i) Solve the Initial Value Problem

$$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 1.5, \ x(1) = 1, \ h = 0.25$$

By using Taylor Method of order N = 2

ii) Solve the Initial Value Problem

$$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 2, \ x(1) = 1, \ h = 0.5$$

By using Taylor Method of order N = 4

[7]

Q6) a) Solve the following system of linear equations by Gauss-Seidal Method start with $x^{(0)} = [\ 0\ 0\ 0]^T$ (Perform 3 iterations) **[5]**

$5x_1 + x_2 + 2x_3 = 10$

$-3x_1 + 9x_2 + 4x_3 = -14$

$x_1 + 2x_2 - 7x_3 = -33$

b) Solve the following system of non-linear algebric equations by using Newton's method start with $x^{(0)} = [\ 1\ \ 1\ \ 1\ ]^T$. **[5]**

(Perform 3 iterations)

$x_1^3 - 2x_2 - 2 = 0$

$x_1^3 - 5x_3^2 + 7 = 0$

$x_2 x_3^2 - 1 = 0$

c) Find the dominance eigen value of the matrix.

$$A=\begin{bmatrix} -2 & -2 & 3 \\ -10 & -1 & 6 \\ 10 & -2 & -9 \end{bmatrix}$$

By Power Method (Take $x^{(0)} = [1\ \ 0\ \ 0]^T$ perform 2 iteration) **[4]**

Q7) a) Use Euler's Method to solve Initial Value Problem :

$$\frac{dx}{dt}=\frac{t}{x}, 0 \leq t \leq 5, x(0)=1$$ **[7]**

b) Use the Runge Kutta Method of order N = 4 to solve Initial Value Problem.

$$\frac{dx}{dt}=1+\frac{x}{t}, 1 \leq t \leq 6, x(1)=1$$ **[7]**

Q8) a) Use House Holder's Method to reduce the following symmetric matrix to tridiagonal form. **[7]**

$$A=\begin{bmatrix} -1 & -2 & 1 & 2 \\ -2 & 3 & 0 & -2 \\ 1 & 0 & 2 & 1 \\ 2 & -2 & 1 & 4 \end{bmatrix}$$

b) Derive the difference equation for the three step Adoms-Bashforth method:

$$\frac{w_{i+1}-w_i}{h}=\frac{23}{12}f(t_i , w_i)-\frac{4}{3}f(t_{i-1}, w_{i-1})+\frac{5}{12}f(t_{i-2}, w_{i-2})$$

Also, Derive the associated truncation error : **[7]**

$$\tau_i=\frac{3h^3}{8}y^{(4)}(\xi)$$

Total No. of Questions : 8]

SEAT No. :

PD-2982

[Total No. of Pages : 3

[6474]-25
M.A./M.Sc. (Part - I)
MATHEMATICS
MTUT-125 : Partial Differential Equations
(CBCS) (2019 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Explain the Charpit's method of solving non-linear partial differential $f(x,y,z,p,q)=0$. **[5]**

b) Attempt the following.

i) Solve the Partial differential equation

$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$ **[4]**

ii) Find the complete integral of the Partial differential equation $f(x, y, z, p, q) = 2(z + xp + yq) - yp^2 = 0$ by using Charpit's method. **[5]**

Q2) a) If $u_1 = \frac{\partial u}{\partial x}$, $u_2 = \frac{\partial u}{\partial y}$, $u_3 = \frac{\partial u}{\partial z}$ show that the equation

$f(x, y, z, u_1, u_2, u_3) = 0$ and $g(x, y, z, u_1, u_2, u_3) = 0$ are compatible if

$\frac{\partial(f,g)}{\partial(x,u_1)} + \frac{\partial(f,g)}{\partial(y,u_2)} + \frac{\partial(f,g)}{\partial(z,u_3)} = 0$ **[6]**

b) Attempt the following.

i) Use Jacobi's method to solve $p^2x + q^2y = z$. **[4]**

ii) Verify that the equation $z=\sqrt{2x+a} + \sqrt{2y+b}$ is solution of partial differential equation $z = \frac{1}{p} + \frac{1}{q}$ **[4]**

P.T.O.

Q3) a) Explain the method of second order partial differential equation $R_r + S_s + T_t + f(x, y, z, p, q) = 0$ to a cannonical form if $S^2 - 4RT < 0$. **[5]**

b) Attempt the following.

i) Find complete solution of $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$ **[5]**

ii) Find the characteristics equations of partial differential equation **[4]**

$u_{xx} - 2\sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$

Q4) a) If $\beta_r D' + \gamma_r$ is a factor of F(D, D') and $\phi_r(\xi)$ is an arbitrary function of a single variable ξ, then if $\beta_r \neq 0$,

$$u_r = \exp\left(\frac{-\gamma_r y}{\beta_r}\right)\phi_r(\beta_r x)$$

is a solution of the equation F(D, D')Z = 0 **[6]**

b) Attempt the following.

i) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{2\partial^4 z}{\partial x^2 \partial y^2}$ **[4]**

ii) Classify the PDE's **[4]**

I) $u_{xx} + 2u_{xy} + u_{yy} = 0$

II) $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$

Q5) a) Find the solution of two dimensional Laplace equation $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ by using separation variable method. **[6]**

b) Attempt the following :

i) Solve the boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial y}{\partial y}$ with $u(0, y) = 8e^{-3y}$ by the method of separation of variables. **[4]**

ii) Derive the diffusion equation of second order partial differential equation. **[4]**

Q6) a) Derive one dimensional wave equation. **[5]**

b) Attempt the following :

i) Find the deflection $u(x, t)$ of the vibrating string whose length is π^2 and $c^2 = 1$ corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$. **[4]**

ii) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, where $u = p_0 \cos pt$ (p_0 is a constant) when $x = l$ and $u = 0$ when $x = 0$. **[5]**

Q7) a) Solve : $u_t = k u_{xx}$, $0 < x < l, t > 0$

$u(0, t) = u(l, t) = 0 \; , t > 0$

$u(x, 0) = f(x) \; , \quad 0 \le x \le l$ **[6]**

b) Attempt the following :

i) Find the characteristics equations of differential equation $u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0$. **[4]**

ii) Prove that if the Dirichlet problem for a bounded region has a solution then it is unique. **[4]**

Q8) a) Show that $T(x,t) = \frac{1}{\sqrt{4\pi\alpha t}} \exp\left[-(x-\xi)^2/(4\alpha t)\right]$ is a solution of diffusion equation $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$, $-\infty < x < \infty, t > 0$. **[7]**

b) Attempt the following :

Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to cannonical form and hence solve it. **[7]**

Total No. of Questions : 8]

SEAT No. :

PD-2983

[Total No. of Pages : 2

[6474]-31

M.A./M.Sc.

MATHEMATICS

MTUT - 131 : Functional Analysis

(2019 Pattern) (Semester -III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

3) *Symbols have their usual meanings.*

Q1) a) If $\langle \cdot , \cdot \rangle$ is a semi-linear product on H then prove that $|\langle x, y\rangle|^2 \leq \langle x, x\rangle\langle y, y\rangle$ for all x and y in H. Moreover, equality occurs if and only if there are scalars α and β both not zero such that $\langle \beta x+\alpha y, \beta x+\alpha y \rangle = 0$ **[7]**

b) State and prove Bessel's Inequality. **[7]**

Q2) a) Show that i+T is non- singular then T^* is also non-singular, and that in this case $(T^*)^+ = (T^+)^*$ **[3]**

b) If T is an operator on a Hilbert space H then prove that T Ps self adjoint if and only if $\langle Tx, x\rangle$ is real for all $x \in$H. **[4]**

c) If M is a proper closed linear subspace of a Hilbert space H then prove that there exists a non-zero vector Zo in H such that Zo$\perp$ m. **[7]**

Q3) a) i) Define Banach space. **[2]**

ii) Show that set of real numbers $\mathbb{R}$ is a banach space under the norm defined by $\|x\|=|x|, x\in \mathbb{R}$ **[5]**

b) Let M be any closed linear subspace of a normed linear space N. Define norm on N/M by $\|x+M\| = \text{in}\, f\left\{\|x+m\| / x\in M\right\}$ show that N/M is normed linear space, also prove that if N is Banach space then N/M is Banach space. **[7]**

P.T.O.

Q4) a) State and prove the open mapping theorem. **[7]**

b) Define Normed linear space with suitable example which is not Banach space. **[4]**

c) For $1 \le P < \infty$, show that the space l^P is separable. **[3]**

Q5) a) State and prove parallelogram law. **[3]**

b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then prove that the linear subspace M+N is also closed. **[4]**

c) Let $T : H_1 \to H_2$ be a bounded linear operator, where H_1 and H_2 are Hilbert spaces then, $T^* : H_2 \to H_1$ is said to Hilbert adjoint operator, $\exists x \in H_1, y \in H_2$ such that $\langle Tx, y \rangle = \langle x_1 T^* y \rangle$ prove that Hilbert adjoint operator T* of T exist and it is unique and $\|T^*\| = \|T\|$. **[7]**

Q6) a) If B = {G} is a basis for H, then prove that mapping $T \to [T]$, which assigns to each operator T its matrix relative to B, is an isomorphism of the algebra $\beta(H)$ onto the total matrix algebra A_n. **[7]**

b) If T is normal then prove that the Mi's Span H.
Where Mi's - pairwise orthogonal
H - Hilbert space. **[7]**

Q7) a) If N_1 and N_2 are normal operators on H with the property that either commutes with the adjoint of the other then prove that N_1+N_2 and $N_1.N_2$ are normal. **[5]**

b) Prove that an operator T on a Hilbert space H is unitary if and only if T is isometric isomorphism of H on to H. **[7]**

c) Let H be a Hilbert space and M be closed subspace of H. Show that $(M\perp)^\perp = M$. **[2]**

Q8) a) Prove that P is projection on closed linear subspace M of H if and only if I-P is projection on $M^\perp$. **[7]**

b) If $P_1, P_2, \ldots, P_n$ are the projections on closed linear subspaces $M_1, M_2, \ldots, M_n$ respectively of a Hilbert space H then prove that $P = P_1 + P_2 + \ldots + P_n$ is projection on H if and only if $P_i P_j = 0$ for $i \neq j$ and in this case P is projection on M where $M = M_1 + M_2 + \ldots + M_n$. **[7]**

Total No. of Questions : 8]

SEAT No. :

PD-2984

[Total No. of Pages : 2

[6474]-32

M.A./M.Sc.

MATHEMATICS

MTUT - 132 : Field Theory

(2019 Pattern) (CBCS) (Semester -III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***

2) ***Figures to the right indicates full marks.***

Q1) a) Prove that the degree of extension of the splitting field of $x^3 - 2 \in Q[x]$ is 6. **[7]**

b) If $P(x)$ is an irreducible polynomial in $F[x]$. Then prove that there exist an extension E of F in which $P(x)$ has a root. **[5]**

c) Express the polynomial $x_1^2 + x_2^2 + x_3^2$ as rational function of elementary symmetric function. **[2]**

Q2) a) State and prove Eisenstein criterion. **[7]**

b) Find total number of subfields of field with field of cardinality 5^{12}. **[5]**

c) Define Galois group. **[2]**

Q3) a) Prove that distinct embeddings are linearly independent of F over E. **[7]**

b) Define Normal extension. Is $Q\left(\sqrt{3}\right)\backslash Q$ a normal extension? Justify your answer. **[5]**

c) Find the minimal polynomial of $\alpha = 1+i$ over Q. **[2]**

P.T.O.

Q4) a) Prove that every finite extension is algebraic. Is the converse true? Justify. **[7]**

b) Find $G\left(Q\left(\sqrt{2}\right)\backslash Q\right)$. **[5]**

c) State ancient geometric problems. **[2]**

Q5) a) Prove that prime field of a field F is either isomorphic to Q or $\mathbb{Z}_p$ (p-prime). **[7]**

b) If K is algebraically closed field then prove that every irreducible polynomial in K[x] is of degree 1. **[5]**

c) Prove that x^2–7 is separable polynomial over Q. **[2]**

Q6) a) Find Galois group of x^4–2 over Q. **[7]**

b) Show that $P(x)=x^2-x-1\in\mathbb{Z}_3[x]$ is irreducible over $\mathbb{Z}_3$. **[5]**

c) Find cyclotomic polynomials $\Phi_2(x),\Phi_3(x)$. **[2]**

Q7) a) State and prove fundamental theorem of algebra. **[7]**

b) If $f(x)\in$ F[x] is irreducible over F then prove that all roots of $f(x)$ have the same multiplicity. **[7]**

Q8) a) Show that the polynomial $x^7-10x^5+15x+5$ is not solvable by radicals over Q. **[7]**

b) If F* is the multiplicative group of non-zero elements of field F then prove that F* is cyclic if and only if F is finite field. **[7]**

Total No. of Questions : 4]

SEAT No. :

PD-2985

[Total No. of Page : 1

[6474]-33

M.A./M.Sc.

MATHEMATICS

MTUT - 133 : Programming With Python

(2019 Pattern) (Semester - III) (CBCS)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***Question 1 is compulsory.***
2) ***Figures to the right side indicate full marks.***
3) ***Attempt any two questions from Q.2, 3 and 4.***

Q1) Attempt the following:

a) Explain in brief applications of Python. **[3]**

b) Give any four data types of python. **[2]**

c) Name a few editors for Python. **[2]**

Q2) Attempt the following:

a) Write a program to generate the following pattern in Python. **[4]**

1
2 2
3 3 3
4 4 4 4

b) Distinguish between for loop and while loop. **[4]**

c) Write a note on conditional statement in Python with suitable example.**[6]**

Q3) Attempt the following:

a) Explain logical operators in python. **[5]**

b) Write a note on file handling in Python. **[6]**

c) Write a Python program which accept positive integer and display wheather it is odd or not. **[3]**

Q4) Attempt the following:

a) Write a note on operator overloading in python. **[7]**

b) Explain ceil and floor function using python with suitable examples. **[7]**

Total No. of Questions : 8]

SEAT No. :

PD-2986

[Total No. of Pages : 2

[6474]-34
M.A./M.Sc.
MATHEMATICS
MTUTO - 134 : Discrete Mathematics
(2019 Pattern) (CBCS) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that every u,v - walk contains a u,v - path. **[5]**

b) What is the probability of randomly choosing a permutation of the 10 digits 0,1,2, ..., 9 in which an odd digit is in the first position and 1,2,3,4 or 5 is in the last position? **[5]**

c) Show by a combinatorial argument that $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$. **[4]**

Q2) a) Prove that number of selections with repetition of r objects chosen from n types of objects is c(r + n – 1, r). **[5]**

b) If two vertices are nonadjacent in the petersen graph then prove that they have exactly one common neighbour. **[5]**

c) Find the connectivity and edge connectivity of k_4. **[4]**

Q3) a) Prove that an edge is a cut - edge if and only if it belongs to no cycle. **[7]**

b) Find the number of ways to select 10 balls from a large pile of red, white and blue balls if the selection has at most two red balls. **[5]**

c) List all partitions of the integer 5. **[2]**

Q4) a) Prove that X,Y - bi graph G has a matching that saturates X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. **[7]**

b) State and prove Inclusion - Exclusion formula. **[7]**

P.T.O.

Q5) a) Prove that a graph G is Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **[7]**

b) Find the recurrence relation for the number of sequences of 1's, 3's and 5's whose terms sum to n. **[5]**

c) Find the coefficient of x^{10} in $(1 + x + x^2 + ...)^n$. **[2]**

Q6) a) Prove that the center of a tree is a vertex or an edge. **[6]**

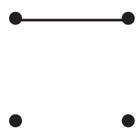
b) Prove that k-regular graph with n vertics has $\frac{nk}{2}$ edges. **[5]**

c) How many integer solutions are there to the equation $x_1 + x_2 + x_3 = 24$ with $x_i \geq i$ (i = 1, 2,3). **[3]**

Q7) a) Prove that a graph is bipartite if and only if it has no odd cycle. **[7]**

b) If the recurrence relation $a_n = C_1 a_{n-1} + C_2 a_{n-2}$ has a general solution $a_n = A_1 3^n + A_2 6^n$ then find C_1 and C_2. **[5]**

c) Find the complement of the following graph. **[2]**



Q8) a) Let G be an n-vertex graph with $n \geq 1$. Prove that the following statements are equivalent. **[7]**

i) G is connected and has no cycles.

ii) G has no loops and for each $u,v \in V(G)$ there exists exactly one u,v-path.

b) Suppose that campus telephone numbers cosists of any four digits. **[7]**

i) What is the probability that the 6 appears at least twice in campus telephone number?

ii) What is the probability that a campus telephone number contains exactly two different digits?

♦ ♦ ♦

Total No. of Questions : 8]

SEAT No. :

PD2987

[Total No. of Pages : 2

[6474]-35
S.Y.M.A./M.Sc.
MATHEMATICS
MTUTO-135 : Mechanics
(2019 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that the curve is catenary for which the area of surface of revolution is minimum when revolved about y - axis. **[5]**

b) Find the extremal of the functional $I=\int_0^{\pi/2}(y'^2-y^2+2xy)dx$ subject to the condition that $y(0)=0,\ y\left(\pi/2\right)=0$. **[5]**

c) Show that the geodesic on a right circular cylinder is a helix. **[4]**

Q2) a) Find Euler –Lagrange differential equation satisfied by twice differentiable function $y(x)$ which extremizes the functional. **[7]**

$I=(y(x))=\int_{x_1}^{x_2} f(x,y,y')\,dx$ Where y is prescribed by end points.

b) Find the minimum of the functional $I(y(x))=\int_0^1\left(\frac{1}{2}y'^2+yy'+y'+y\right)dx$ if the values of end points not given. **[7]**

Q3) a) State and explain the conservation theorem of angular momentum of the system of particles. **[5]**

b) A particle is constrained to more in a circle in vertical plane xy. Apply D' Alemberts principle to show that for equilibrium we must have $\ddot{x}y-\ddot{y}x-gx=0$. **[5]**

c) Prove that if force acting on a particle is conservative then total energy is conserved. **[4]**

P.T.O.

Q4) a) A particle is constrained to move on surface of cylinder of fixed radius. Obtain the Lagranges equation of motion. **[5]**

b) Set up the Lagrangian and the Lagranges equation of motion for the compound pendulum. **[5]**

c) Prove that the generalised momentum corresponding to cyclic co-ordinate is conserved. **[4]**

Q5) a) Prove the kepler's third Law of planetary motion. **[7]**

b) A bead is sliding on wire in the form of cycloid $x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta), 0 \le \theta \le 2\pi$ Find Lagrangian and equation of motion. **[7]**

Q6) a) Show that the Lagranges equations are necessary conditions for the action to have stationary value. **[5]**

b) A particle moves on a smooth surface under gravity. Use Hamilton's principle to find equation of motion. **[5]**

c) Find central force under the action of which a particle will follow $r = a\ (1 + \cos\theta)$ orbit. **[4]**

Q7) a) A particle of mass m is moving under the inverse square law of attractive force. Setup Lagrangian and equation of motion. **[7]**

b) Derive the Hamilton's canonical equations of motion from Hamiltonian function. **[7]**

Q8) a) Prove that a co-ordinate which is cyclic in the Lagrangian is also cyclic in the Hamiltonian. **[5]**

b) Explain the principle of least action. **[5]**

c) Write Hamilton's equations of motion for a compound pendulum. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PD-2988

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[6474]-36
S.Y. M.A./M.Sc.
MATHEMATICS
MTUTO - 136 : Advanced Complex Analysis
(2019 Pattern) (CBCS) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let $F(z,s)$ be defined for $(z,s) \in \Omega \times [0,1]$ where Ω is an open set in $\mathbb{C}$. Suppose F satisfies the following properties. **[5]**

i) $F(z,s)$ is holomorphic in z for each s.

ii) F is continous on $\Omega \times [0,1]$

Then prove that the function f defined on Ω by $f(z) = \int_0^1 F(z,s)ds$ is holomorphic.

b) Prove that every polynomial $P(z) = a_n z^n + ... + a_0$ of degree $n \geq 1$ has precisely n roots in $\mathbb{C}$. **[5]**

c) Show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ **[4]**

Q2) a) Suppose f is a holomorphic function in a region Ω that vanishes on a sequence of distinct points in Ω. Then prove that f is identically 0. **[7]**

b) Suppose that f is a holomorphic function in Ω^+ that extends continuously to I and such that f is real-valued on I. Prove that there exists a function F holomorphic in all of Ω such that $F = f$ on Ω^+. **[5]**

c) State Symmetry principle. **[2]**

Q3) a) Prove that, the map $\mathrm{F} : \mathrm{H} \to \mathrm{D}$ is a conformal map with inverse $\mathrm{G} : \mathrm{D} \to \mathrm{H}$. **[5]**

b) Prove that the function $f(z) = -\frac{1}{2}\left(z + \frac{1}{z}\right)$ is a conformal map from the half-disc $\{z = x + iy : |z| < 1, y > 0\}$ to the upper half-plane. **[5]**

c) If $f : U \to V$ is holomorphic and injective, then prove that $f'(z) \neq 0$ for all $z \in U$. **[4]**

P.T.O.

Q4) a) Let $f : D \to D$ be holomorphic with $f(0) = 0$. Then prove the following : **[7]**

i) $|f(z)| < |z|$ for all $z \in D$.

ii) If for some $z_0 \neq 0$ we have $|f(z_0)|=|z_0|$, then f is a rotation.

iii) $|f'(0)| < 1$ and if equality holds, then f is a rotation

b) State and prove Morera's theorem. **[5]**

c) Give an example of an automorphism of unit disc. **[2]**

Q5) a) State and prove Montel's theorem. **[7]**

b) Prove that any two proper simply connected open subsets in $\mathbb{C}$ are conformally equivalent. **[5]**

c) State the Riemann mapping theorem. **[2]**

Q6) a) If Ω is a connected open subset of $\mathbb{C}$ and $\{f_n\}$ is a sequence of injective holomorphic functions on Ω that converges uniformly on every compact subset of Ω to a holomorphic function f, then prove that f is either injective or constant. **[7]**

b) For each $0 < r < \frac{1}{2}$, let C_r denote the circle centered at z_0 of radius r. Suppose that for all sufficiently small r we are given two points z_r and z'_r in the unit disc that also lie on C_r. If we let $\rho(r) = |f(z_r)| - f(z'_r)|$, then prove that there exists a sequence $\{r_n\}$ of radii that tends to zero, and such that $\lim_{x\to\infty} \rho(r_n) = 0$. **[7]**

Q7) a) If $F(z) = \int_1^z \frac{d\zeta}{(1-\zeta^n)^{2/n}}$, then show that F maps the unit disc conformally onto the interior of a regular polygon with n sides and perimeter $2^{\frac{n-2}{n}} \int_0^\pi (\text{Sin } \theta)^{-2/n} \, d\theta \cdot$ **[5]**

b) Prove that, if $F : D \to P$ is the conformal map , then F extends to a continuous bijection from the closure $\overline{D}$ of the disc to the closure $\overline{P}$ of the polygonal region. **[5]**

c) Let Ω be an open subset of $\mathbb{C}$. **[4]**

i) Define a uniformly bounded family F on compact subsets of Ω.

ii) Define an equicontinuous family F on a compact set $K \subset \Omega$.

Q8) a) Suppose $F(z)$ is holomorphic near $z = z_0$ and $F'(z_0) = F(z_0) = 0$, while $F''(z_0) \neq 0$. Show that there are two curves Γ_1 and Γ_2 that pass through z_0, are orthogonal at z_0, and so that F restricted to Γ_1 is real and has a minimum at z_0, while F restricted to Γ_2 is also real but has a maximum at z_0. **[7]**

b) Show that an entire doubly periodic function is constant. **[5]**

c) Define the general Schwarz - Christoffel Integral. **[2]**

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Total No. of Questions : 8]

SEAT No. :

PD2989

[Total No. of Pages : 3

[6474]-37

M.A./M.Sc. - II

MATHEMATICS

MTUTO-137 : Integral Equations

(2019 Pattern) (Semester - III)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the method of successive approximations to find the solution of Fredholm integral equation. **[5]**

b) Derive an equivalent Fredholm integral equation from the following boundary value problem. $y'' + 2xy = 1, 0 < x < 1, y(0) = y(1) = 1.$ **[5]**

c) Show that $u(x) = x$ is a solution of the following Fredholm integro-differential equation $u'(x) = \frac{2}{3} + \int_0^1 tu(t)dt.$ **[4]**

Q2) a) Find the solution $u(x)$ of the following Fredholm integral equation using Adomian Decomposition method. **[5]**

$$u(x) = \frac{9}{10}x^2 + \int_0^1 \frac{1}{2}x^2t^2u(t)\,dt$$

b) Derive an initial value problem from the following Volterra integral equation. **[5]**

$$u(x) = x^3 + \int_0^x (x-t)^2 u(t)\,dt$$

c) Find the solution of following Fredholm integral equation by using direct computation method. **[4]**

$$u(x) = x\sin x - x + \int_0^{\pi/2} xu(t)dt$$

Q3) a) Explain the series solution method to find the solution of Volterra integral equation. **[5]**

b) Solve the following Fredholm integral equation by using the method of successive substitutions. **[5]**

$$u(x) = \frac{9}{10}x^3 + \frac{1}{2}\int_0^1 x^3 t u(t)dt$$

c) Solve the following Volterra integral equation by using modified decomposition method. **[4]**

$$u(x) = \cosh x + \frac{x}{2}(1 - e^{\sinh x}) + \frac{x}{2}\int_0^x e^{\sinh t} u(t)dt$$

Q4) a) Find the solution of following Volterra integral equation by using successive approximations method. **[5]**

$$u(x) = 2 - \int_0^x (x-t)u(t)dt$$

b) Solve the following Fredholm integro-differential equation by using Adomian decomposition method. **[5]**

$$u'(x) = \cos x + \frac{1}{4}x - \frac{1}{4}\int_0^{\pi/2} x t u(t)dt, u(0) = 0$$

c) Classify each of the following equation. **[4]**

i) $u(x) = 1 + x^2 + \int_0^x (x-t)u(t)dt$

ii) $u(x) = \int_0^1 (x-t)^2 u(t)dt$

iii) $x + 1 - \frac{\pi}{2} = \int_0^{\pi/2} (x-t)u(t)dt$

iv) $u'(x) = 1 - \frac{x^3}{3} + \int_0^x t u(t)dt$

Q5) a) Solve the following Volterra integro-differential equation by using series solution method. **[5]**

$$u''(x) = \cosh x + \frac{1}{4} - \frac{1}{4}\cosh 2x + \int_0^x \sinh t u(t)dt, u(0) = 1, u'(0) = 0$$

b) Solve the following Fredholm integral equation by using direct computation method. **[5]**

$$u(x) = x^2 - \frac{1}{6}x - \frac{1}{24} + \frac{1}{2}\int_0^1 (1 + x - t)u(t)dt$$

c) Find the value of α in the following Volterra integral equation if the solution is $u(x) = e^{-x^3}$ of the integral equation. **[4]**

$$u(x) = 1 - \alpha\int_0^x 3^2 u(t)dt$$

Q6) a) Explain the method of Adomian decomposition to find the solution of Volterra integro-differential equation. **[5]**

b) Convert the following initial value problem to equivalent Volterra integral equation. **[5]**

$y'' + 5y' + 6y = 0, y(0) = 1, y'(0) = 1$

c) Solve the following homogeneous Fredholm integral equation **[4]**

$$u(x) = \lambda \int_0^1 xu(t)dt$$

Q7) a) Solve the following Volterra integral equation by converting it to an equivalent initial value problem. **[7]**

$$u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t-x)u(t)dt$$

b) Solve the following Fredholm integral equation **[7]**

$u(x) = \frac{5}{6}x + \frac{1}{2}\int_0^1 xtu(t)dt$ by using

i) direct computation method

ii) Successive substitutions method

Q8) a) Solve the following second order Fredholm integro-differential equation by using the direct computation method. **[7]**

$$u''(x) = 2 - \frac{2}{3}x - \frac{16}{15}x^2 + \int_{-1}^1 (xt + x^2t^2)u(t)dt, u(0) = 1,\ u'(0) = 1$$

b) Explain series solution method converting Volterra integro-differential equation to standars Volterra integral equation. Hence find the solution of equation **[7]**

$$u'(x) = 2 - \frac{1}{4}x^2 + \frac{1}{4}\int_0^x u(t)dt, u(0) = 0$$

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Total No. of Questions : 8]

SEAT No. :

PD2990

[Total No. of Pages : 3

[6474]-38

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO 138 : Differential Manifolds

(2019 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let W be a linear subspace of $\mathbb{R}^n$ of dimension k. Then prove that there is an orthonormal basis for $\mathbb{R}^n$ whose first k elements forms a basis for W. **[5]**

b) Let U be open in H^k but not in $\mathbb{R}^k$; let $\alpha : U \to \mathbb{R}^n$ be of class C^r. Let $\beta : U' \to \mathbb{R}^n$ be a C^r extension of α defined on an open set U' of $\mathbb{R}^k$. Then prove that for $x \in U$, the derivative $D\beta(x)$ depends only on the function α and is independent of the extension β. **[5]**

c) Show that $I = [0,1]$ is a 1-manifold in $\mathbb{R}'$. **[4]**

Q2) a) Let M be k-manifold in $\mathbb{R}^n$, of class C^r. If ∂M is non empty, then prove that ∂M is k–1 manifold without boundary in $\mathbb{R}^n$ of class C^r. **[5]**

b) Show that n-ball $B^n(a)$ is an n-manifold in $\mathbb{R}^n$ of class C^∞ and $S^{n-1}(a) = \partial B^n(a)$ **[5]**

c) Prove that if the support of f can be covered by a single co-ordinate patch, the integral $\int_M f dv$ is well defined, independent of the choice of coordinate path. **[4]**

P.T.O.

Q3) a) Let V be a vector space with basis $a_1, a_2, .. a_n$. Let I=(i_1,.. i_k) be a k-tuple of integers from the set {1,2, .. n}. Then show that there is a unique k-tensor ϕ_I on V such that, for every k-tuple J = ($j_1, j_2, .. j_k$) from the set {1,2, .. n}.

$$\phi_I\left(a_{j_1}, a_{j_2}, .a_{j_k}\right) = \begin{cases} 0 & if\ \mathrm{I} \neq \mathrm{J} \\ 1 & if\ \mathrm{I} = \mathrm{J} \end{cases}$$

The tensor ϕ_I form a basis for $L^k(\nu)$. **[7]**

b) Let T : V→W be a linear transformation. If f is an alternating tensor on W, then prove that T^*f is an alternating tensor on V. **[5]**

c) Show that if f, g : V^k→R are multilinear so is $af+bg$. **[2]**

Q4) a) Let w be a k-form on the open set A of $\mathbb{R}^n$. Then show that w is of class C^r if and only if its component function b_I are of class C^r on A. **[5]**

b) If $w = x^2zdx + 2ydy + ye^{-z}dz$ and $\eta = \cos xzdx + yzdy + xdz$, then find $d(w \wedge \eta)$ **[5]**

c) Let A be open set in $\mathbb{R}^n$; let f : A→$\mathbb{R}$ be of class C^r. Then show that $df = (D_1 f)dx_1 + \text{---} + (D_n f)dx_n$. **[4]**

Q5) a) Let M be a compact k-manifold in $\mathbb{R}^n$, of class C^r. Let f,g : M→R be continuous. Then show that.

$$\int_M (af + bg)d\mathrm{V} = a\int_M fd\mathrm{V} + b\int_M gd\mathrm{V}$$ **[5]**

b) Let M be a k-manifold in $\mathbb{R}^n$; Let α: U→V be a coordinate patch about the point P of M. If U is open in H^k and p = α(x_0) for $x_0 \in \mathbb{R}^{k-1}$ X0 then P is a boundary point of M. **[5]**

c) Show that the function α:[0,1)→S′ given by α(t) = (cos 2πt, sin 2πt) is not co-ordinate patch on S′. **[4]**

Q6) a) If T:V→W be linear transformation and if f and g are alternating tensor on W, then prove that $T^*(f \wedge g) = T^* f \wedge T^* g$. **[5]**

b) Let A be open in $\mathbb{R}^k$; Let α: A→$\mathbb{R}^n$ be a C^∞ map. Let x denote the general point of $\mathbb{R}^k$; Let y denote the general point of $\mathbb{R}^n$. Then dx_i and dy_i denote the elementory 1-forms in $\mathbb{R}^k$ and $\mathbb{R}^n$ respectively. Prove that $\alpha^*(dy_i) = d\alpha_i$ **[5]**

c) Let A = $\mathbb{R}^2 - 0$, consider the 1-form in A defined by equation $w = \dfrac{xdx + ydy}{x^2 + y^2}$ Show that w is exact on A. **[4]**

Q7) a) Let M be a compact n-manifold in $\mathbb{R}^n$. Let N be the unit normal vector field to ∂M that points outwards from M. If G is a vector field defined in an open set of $\mathbb{R}^n$ containing M, then show that

$$\int_M (div\ G)\, dV = \int_{\partial M} (G, N)\, dV$$ **[5]**

b) Suppose there is an n–1 form η defined in $\mathbb{R}^n - 0$ such that dη = 0 and $\int_{S^{n-1}} \eta \neq 0$. Show that η is not exact. **[5]**

c) Let M be a compact 1-manifold in $\mathbb{R}^n$;let T be unit tangent vector field to M. Let f be C^∞ function defined in an open set about M. If ∂M is empty then show that $\int_M \langle grad\ f, T\rangle ds = 0$. **[4]**

Q8) a) Let M be a 1-manifold in $\mathbb{R}^n$; let f be a 0-form defined in an open set about M. If M is an arc with initial point p and final point q, then show that $\int_M df = f(q) - f(p)$. **[5]**

b) Show that Möbius band is non-orientable 2-manifold in $\mathbb{R}^3$. **[5]**

c) Let α : $(0, 1)^2 \to \mathbb{R}^3$ given by $\alpha(u,v) = (u+v, uv, u^2 - v^2 + 1)$. Let Y be the image set of α.

Evaluate $\int_Y x_2 dx_1 \wedge dx_3 + x_2 x_3 dx_1 \wedge dx_3$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PD-2991

[Total No. of Pages : 4

[6474]-41

M.A./M.Sc.

MATHEMATICS

MTUT - 141 : Fourier Series and Boundary Value Problems

(2019 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let f denote a function such that

i) f is continuous on the interval $-\pi \le x \le \pi$.

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is piecewise continuous on the interval $-\pi < x < \pi$.

If $a_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\cos nx\,dx$ and

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

are fourier coefficients for f, then prove that the series $\displaystyle\sum_{n=1}^{\infty}\sqrt{a_n^2 + b_n^2}$ converges. **[7]**

b) Find the Fourier series on the interval $-\pi < x < \pi$ that corresponds to the function **[5]**

$$f(x) = \begin{cases} -\pi/2, & \text{when } -\pi < x < 0 \\ \pi/2, & \text{when } 0 < x < \pi \end{cases}$$

c) Find the Fourier Sine series for the function $f(x) = x$, $(0 < x < 1)$. **[2]**

P.T.O.

Q2) a) If $f \in C_p(0,\pi)$, then prove that the Fourier sine series coefficient b_n tends to zero as n tends to infinity. **[7]**

b) Find the Fourier cosine series for the function

$f(x) = \sin x \;,\; 0 < x < \pi.$ **[5]**

c) Find the Fourier sine series for the function

$f(x) = 1 \quad , \quad (0 < x < \pi)$ **[2]**

Q3) a) Suppose that a function $g(u)$ is piecewise continuous on the interval $0 < u < \pi$ and that the right-hand derivative $g_+^r(0)$ exists. Then prove that

$$\lim_{N\to\infty}\int_0^{\pi} g(u)D_N(u)du = \frac{\pi}{2}g(0^+),$$

where $D_N(u)$ is the Dirichlet kernel. **[7]**

b) Find the Fourier sine series of the function

$f(x) = x(\pi - x) \quad , \quad (0 < x < \pi)$ **[5]**

c) Let f be the continuous function defined by

$$f(x) = \begin{cases} x^2 \;, & \text{when} \quad x \le 0 \\ \sin x \;, & \text{when} \quad x > 0 \end{cases}$$

Find $f_+^r(0)$ and $f_-^r(0)$. **[2]**

Q4) a) Solve the following Boundry Value Problem.

$$u_t(x,t) = ku_{xx}(x,t), \;\; (0 < x < c, t > 0)$$
$$u_x(0,t) = 0, u_x(c,t) = 0, (t > 0)$$
$$u(x,0) = f(x)$$

[7]

b) Solve the following Boundry Value Problem.

$$y_t(x,t) = a^2 y_{xx}(x,t) \;\; (0 < x < c, t > 0)$$
$$y(0,t) = 0, \; y(c,t) = 0, \; y_t(x,0) = 0,$$
$$y(x,0) = f(x)$$

[7]

Q5) a) Solve the following Boundry Value Problem.

$$u_t(x,t) = ku_{xx}(x,t) + q(t), (0 < x < \pi, t > 0)$$
$$u(0,t) = 0, u(\pi,t) = 0$$
$$u(x,0) = f(x)$$

[7]

b) Solve the following Boundry Value Problem.

$$u_{xx}(x,y) + u_{yy}(x,y) = 0, (0 < x < a, 0 < y < b)$$

with the conditions

$$u(0,y) = 0, u(a,y) = 0, \ (0 < y < b),$$
$$u(x,0) = 0, u(x,b) = f(x), \ (0 < x < a)$$

[7]

Q6) a) Solve the following Boundry Value Problem.

$$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_\rho(\rho,\phi) + u_{\phi\phi}(\rho,\phi) = 0 \quad (1 < \rho < b, 0 < \phi < \pi)$$
$$u(\rho,0) = 0, u(\rho,\pi) = 0, (1 < \rho < b)$$
$$u(1,\phi) = 0, u(b,\phi) = u_0 \ (0 < \phi < \pi)$$

[7]

b) The Boundry Value Problem

$$y_{tt}(x,t) = y_{xx}(x,t) + A\sin wt \ (0 < x < 1, t > 0)$$

when A is constant

$$y(0,t) = 0, y(1,t) = 0, \ y(x,0) = 0, \ y_t(x,0) = 0$$

describes transeverse displacement in a stretched string. Show that resonance occurs when w has one of the values.

$w_n = (2n - 1) \ (n = 1, 2, \ldots)$ **[7]**

Q7) a) If X(x) and Y(x) are eigen Functions corresponding to the same eigenvalue of a regular sturmliouville problem. Then prove that Y(x) = CX(x) where C is non-zero constant. **[7]**

b) If the functions $\phi_0(x)=\frac{1}{\sqrt{\pi}}, \phi_n(x)=\frac{2}{\pi}\cos nx, n=1, 2, ...$

then show that the set

$\{\phi_n(x)\}(n=1,2,....)$ is orthonormal on the interval $0 < x < \pi$. **[5]**

c) If $f(x)=\frac{e^x-1}{x}, x \neq 0$ then find $f(0,t)$ *and* $f'_R(0)$ **[2]**

Q8) a) Find eigenvalues and normalized eigenfunction of Sturm-Liouville problem. $x''+\lambda x=0, x'(0)=0, x(c)=0$. **[5]**

b) Verify that each of the function $u_0=y$ and $u_n=\sinh ny.\cos nx$

(n = 1, 2, 3, ...) satisfies laplace equation $u_{xx}(x,y)+u_{yy}(x,y)=0$ on ($0 < x < \pi$, $0 < y < 2$). **[2]**

c) If C_n (n = 1, 2, 3, ...) be the fourier constants for a function f in $C_p(a, b)$ with respect to an orthogonal set $\{\phi_n(x)\}$ (n = 1, 2, 3, ...) in the space. Then prove that all possible linear combination's of the function $\phi_1(x), \phi_2(x), \ldots, \phi_n(x)$ the combination $C_1\phi_1(x)+C_2\phi_2(x)+\ldots+C_n\phi_n(x)$ is the best approximation in the mean to $f(x)$ on the fundamental interval $a < x < b$. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PD-2992

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[6474]-42

S.Y. M.A./M.Sc.

MATHEMATICS

MTUT - 142 : Differential Geometry

(2019 Pattern) (Credit System) (Semester -IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) Define an integral curve. Give an example of a parametrized curve α(t) which is not an integral curve. **[4]**

b) Explain why an integral curve cannot cross itself as does the parametrized curve. **[5]**

c) Define complete vector field and check wheather the vector field $X(x_1,x_2) = (x_1,x_2,1,0)$ is complete or not. **[5]**

Q2) a) Show that the set S of all unit vectors at all points of $\mathbb{R}^2$ forms a 3-surface in $\mathbb{R}^4$. **[4]**

b) If U is an open set in $\mathbb{R}^{n+1}$ and X is smooth vector field on U. Suppose, $\alpha : I \to U$ is an integral curve of X with $\alpha(0)=\alpha(t_0)$ for some $t_0 \varepsilon I, t_0 \neq 0$. Then show that α (t) is periodic on I. **[5]**

c) Show that the graph of any function $f : \mathbb{R}^n \to \mathbb{R}$ is a level set for some function $F : \mathbb{R}^{n+1} \to \mathbb{R}$. **[5]**

Q3) a) State and prove Lagrange's Multiplier Theorem. **[7]**

b) If U is an open set in $\mathbb{R}^{n+1}$ and $f : U \to \mathbb{R}$ is smooth function. Let PεV be a regular point of f and let *f(p)* = c. Then show that the set of all vectors tangent to f^{-1} (c) at 'p' is equal to $\left[\nabla f(p)\right]^1$. **[7]**

P.T.O.

Q4) a) If $f: U \to \mathbb{R}$ is a smooth function and $\alpha: I \to U$ is an integral curve of ∇f. Then show that, $\frac{d}{dt}(f_0\alpha)(t) = \|\nabla f(\alpha(t))\|^2, \forall t \varepsilon I$. **[4]**

b) Show that if an n-surface S is represented both as $f^{-1}(c)$ and $g^{-1}(d)$ where $\nabla f(p) \neq 0$ and $\nabla g(p) \neq 0$ for all PεS. Then show that, for each $P \varepsilon S, \nabla f(p) = \lambda \nabla g(P).(\lambda \neq 0)$. **[5]**

c) Define terms **[5]**

i) Gauss Map

ii) Weingarten Map

Q5) a) If S is any connected n-surface in $\mathbb{R}^{n+1}$. Then show that, there exist exactly two smooth unit normal vector fields N_1 and N_2 with N_2 (P) = $-N_1$ (P) for all PεS. **[7]**

b) If C is connected oriented plane curve and $\beta: I \to C$ is unit speed global parametrization of C. Then show that, β is either one to one or periodic Moreover, β is periodic if and only if C is compact. **[7]**

Q6) a) Show that the unit n-sphere $x_1^2 + x_2^2 + ... + x_{n+1}^2 = 1$ is connected for n>1. **[5]**

b) Define Geodesics and hence show that geodesics have constant speed. **[5]**

c) Explain the difference between local and global parametrization of curve C. **[4]**

Q7) a) If S is a compact connected oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f: \mathbb{R}^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0$ for all PεS. Then show that, Gauss map, maps S on to the unit sphere S^n. **[7]**

b) Show that the velocity vector field along a parametrized curve α(t) in an n-surface S is parallel if and only if α (t) is geodesic. **[7]**

Q8) a) If $g: I \to \mathbb{R}$ is a smooth function and C denote the graph of g(t). Then show that, the curvature of C at the point (t,g(t)) is $g''(t)/\left(1+\left(g'(t)\right)^2\right)^{3/2}$ for any appropriate choice of orientation. **[5]**

b) Show that Möbius band is an unorientable 2-surface. **[5]**

c) State and prove any two properties of Levi-civita parallelism. **[4]**

Total No. of Questions : 4]

SEAT No. :

PD-2993

[Total No. of Page : 1

[6474]-43

M.A./M.Sc.

MATHEMATICS

MTUT - 143 : Introduction to Data Science

(2019 Pattern) (Semester - IV) (Credit System)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***Question 1 is compulsory.***
2) ***Attempt any two questions from Q.2, 3 and 4.***
3) ***Figures to the right side indicate full marks.***

Q1) Define the term big data and mention all steps involved in the process of data science. **[7]**

Q2) a) Describe data transformation process. **[5]**

b) Distinguish between supervised and unsupervised machine learning techniques. **[5]**

c) What is machine learning? Give python tools used in machine learning. **[4]**

Q3) a) Explain any Five forms of data. **[5]**

b) What is Spark? Give it's four components. **[5]**

c) Explain the process of data cleansing in detail. **[4]**

Q4) a) Write a short note on text mining and handling techniques to it. **[5]**

b) Describe Hadoop and its components. **[5]**

c) Give packages in python which are used for text mining. **[4]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

PD-2994

[Total No. of Pages : 3

[6474]-44

M.A./M.Sc.

MATHEMATICS

MTUTO - 144 : Number Theory

(2019 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that every non-constant polynomial is the product of irreducible polynomials. **[5]**

b) Show that $\mathbb{Z}[i]$ is a Euclidean Domain. **[5]**

c) Show that the following : **[4]**

i) $(a, a + k) \mid k$, where $a, k \in \mathbb{Z}$

ii) For all odd integers n, $8 \mid n^2 - 1$

Q2) a) Let $a, b, m \in \mathbb{Z}$ and $m \neq 0$. Then prove the following : **[5]**

i) If $a \equiv b(\text{mod } m)$ and $b \equiv c(\text{mod } m)$ then $a \equiv c(\text{mod } m)$

ii) If $a \equiv b(\text{mod } m)$ then $ac \equiv bc(\text{mod } mc)$ for $c > 0$

b) Prove that $a^{\phi(m)} \equiv 1(\text{mod } m)$ when $(a, m) = 1$. **[5]**

c) Find the last two digits in the ordinary decimal representation of 3^{400}. **[4]**

P.T.O.

Q3) a) Show that there is no x for which both $x \equiv 29 \pmod{52}$ and $x \equiv 19 \pmod{72}$. **[5]**

b) For any odd prime p let $(a, p) = 1$. Consider the integers $a, 2a, 3a, ..., \{(p-1)/2\}a$ and their least positive residues modulo p. If n denotes the number of these residues that exceed $p/2$, then show that $\left(\frac{a}{p}\right) = (-1)^n$. **[7]**

c) Show that if p is a prime and $a^2 \equiv b^2 \pmod{p}$ then $p/(a+b)$ or $p|(a-b)$. **[2]**

Q4) a) If p and q are distinct odd primes then prove that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left(\frac{p-1}{2}\right)\left(\frac{q-1}{2}\right)}$. **[7]**

b) If p is an odd prime then prove the following : **[4]**

i) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$

ii) If $(a, p) = 1$ then $\left(\frac{a^2}{p}\right) = 1$

c) Determine whether the congruence $x^2 \equiv 5 \pmod{227}$ is solvable or not. **[3]**

Q5) a) Find the highest power of 6 dividing 500!. Also find the highest power of 70 dividing 500! **[5]**

b) If p is prime, then prove that the largest exponent e such that $p^e/n!$ is $e = \sum_{i=1}^{\infty}\left[\frac{n}{p^i}\right]$. **[5]**

c) Prove that $\left[x + \frac{1}{2}\right]$ is the nearest integer to x and if two integers are equally near to x then it is the larger of the two. **[4]**

Q6) a) If $f(n)$ be a multiplicative function and let $f(n) = \sum_{d/n} f(d)$. Then show that $F(n)$ is multiplicative. **[7]**

b) For every positive integer n, show that $\sigma(n) = \prod_{p^{\alpha}//n} \left(\frac{p^{\alpha+1} - 1}{p - 1} \right)$. **[5]**

c) Find the smallest integer x for which $\phi(x) = 6$. **[2]**

Q7) a) If $F(n) = \sum_{d/n} f(d)$ for every positive integer n then prove that $f(n) = \sum_{d/n} \mu(d) F(n/d)$. **[5]**

b) Prove that the product of two primitive polynomials is primitive. **[7]**

c) Find the minimal polynomial of the algebraic number $\frac{1+\sqrt[3]{7}}{2}$. **[2]**

Q8) a) If α is any algebraic number, then prove that there is a rational integer b such that $b\alpha$ is an algebraic integer. **[5]**

b) If an irreducible polynomial $p(x)$ divides a product $f(x)\, g(x)$ then prove that $p(x)$ divides at least one of the polynomials $f(x)$ and $g(x)$. **[5]**

c) Prove that $\mu(n)\, \mu(n+1)\, \mu(n+2)\, \mu(n+3) = 0$ if n is a positive integer. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PD-2995

[Total No. of Pages : 2

[6474]-45
S.Y. M.A./M.Sc.
MATHEMATICS
MTUTO-145 : Algebraic Topology
(2019 Pattern) (CBCS) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that path homotopy relation is an equivalence relation. **[5]**

b) A space X is said to be contractible if the identity map $i_x : X \to X$ is nulhomotopic. Show that a contractible space is path connected. **[5]**

c) Show that if $h, h' : X \to Y$ are homotopic and $k, k' : Y \to Z$ are homotopic, then $k \circ h$ and $k' \circ h'$ are homotopic. **[4]**

Q2) a) Show that in a simply connected space X, any two paths having the same initial and final points are path homotopic. **[5]**

b) Let α be a path in X from x_0 to x_1 and β be a path in X from x_1 to x_2. Show that if $\gamma = \alpha * \beta$, then $\hat{\gamma} = \hat{\beta} o \hat{\alpha}$. **[5]**

c) Define : **[4]**

i) simply connected space. ii) star convex set.

Q3) a) If $P : E \to B$ and $P' : E' \to B'$ are covering maps, then prove that $P \times P' : E \times E' \to B \times B'$ is a covering map. **[5]**

b) Let $q : X \to Y$ and $r : Y \to Z$ be covering maps. Let $p = r \circ q$. Show that if $r^{-1}(z)$ is finite for each $z \in Z$, then p is a covering map. **[5]**

c) Define retraction. Prove that there is no retraction of B^2 onto S^1. **[4]**

P.T.O.

Q4) a) State and prove the fundamental theorem of algebra. **[10]**

b) Give an example of a non-identity covering map from S^1 onto S^1. Justify your answer. **[4]**

Q5) a) Show that if $G = G_1 \oplus G_2$, where G_1 and G_2 are cyclic of orders m and n respectively, then m and n are not uniquely determined by G in general. **[5]**

b) Let $f : X \rightarrow Y$ be continuous and let $f(x_0) = y_0$. If f is a homotopy equivalence, prove that $f_* : \pi_1(X, x_0) \rightarrow \pi_1(Y, y_0)$ is an isomorphism. **[5]**

c) State and prove the Borsuk-Ulam Theorem for S^2. **[4]**

Q6) a) Let p, q and r be continuous maps with $p = r \circ q$ as shown in the following diagram. **[7]**

$$\begin{tikzcd} X \arrow[d, "p"'] \arrow[dr, "q"] & \\ Z & Y \arrow[l, "r"] \end{tikzcd}$$

Prove that, if p and r are covering maps, so is q.

b) Prove that the fundamental group of the n-fold dunce cap is a cyclic group of order n. **[7]**

Q7) a) Prove that the fundamental group of the torus has a presentation consisting of two generators α, β and a single relation $\alpha\beta\alpha^{-1}\beta^{-1}$. **[5]**

b) Let X be the wedge of the circles S_α for $\alpha \in J$. Let p be the common point of these circles, then prove that $\pi_1(X, p)$ is a free group. **[5]**

c) State the Seifert-van Kampen theorem. **[4]**

Q8) a) Let $p : E \rightarrow B$ be a covering map, with E simply connected. Show that, given any covering map $r : Y \rightarrow B$, there is a covering map $q : E \rightarrow Y$ such that $r \circ q = p$. **[5]**

b) Show that if $n > 1$, every continuous map $f : S^n \rightarrow S^1$ is nulhomotopic. **[5]**

c) Find a continuous map of the torus into S^1 that is not nulhomotopic. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PD-2996

[Total No. of Pages : 3

[6474]-46
M.A./M.Sc.
MATHEMATICS
MTUTO - 146 : Representation Theory of Finite Groups
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If V is an inner product space and W is subspace of V then show that $V = W \oplus W^{\perp}$. **[5]**

b) For a finite set X, the set $\mathbb{C}^X = \{f : X \to \mathbb{C}$ is a fuction$\}$ is a vector space with pointwise operations. Give its orthonormal basis with the natural inner product on it. **[5]**

c) Show that the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over $\mathbb{C}$. **[4]**

Q2) a) Define the degree of the representation of a group and equivalent representations of a group. Show that the representations $\phi : \mathbb{Z}_n \to GL_2(\mathbb{C})$ *and* $\psi : \mathbb{Z}_n \to GL_2(\mathbb{C})$ are equivalent, where

$$\phi_{\overline{m}} = \begin{bmatrix} \cos\left(\frac{2\pi m}{n}\right) & -\sin\left(\frac{2\pi m}{n}\right) \\ \sin\left(\frac{2\pi m}{n}\right) & \cos\left(\frac{2\pi m}{n}\right) \end{bmatrix} \text{ and } \psi_{\overline{m}} = \begin{bmatrix} e^{\left(\frac{2\pi m}{n}\right)} & 0 \\ 0 & e^{-\left(\frac{2\pi m}{n}\right)} \end{bmatrix}.$$ **[5]**

b) Give the standard representation $\phi : S_3 \to GL_3(\mathbb{C})$ and give an S_3- invariant subspace of $GL_3(\mathbb{C})$. **[5]**

c) Let ρ : $S_3 \to GL_2(\mathbb{C})$ be a representation defined on generators (12) and (123) by $\rho_{(12)} = \begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}$ and $\rho_{(123)} = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$. Show that ρ is irreducible. **[4]**

P.T.O.

Q3) a) Let $\phi : G \to GL(V)$ be equivalent to a decomposable representation. Then show that ϕ is decomposable. **[5]**

b) Give a irreducible representation $\rho : D_4 \to GL_2(\mathbb{C})$. **[5]**

c) Define a completely reducible representation and describe its diagonalizability. **[4]**

Q4) a) Let $\phi : G \to GL(V)$ be a unitary representation of a group. Then show that ϕ is either irreducible or decomposable. **[5]**

b) Define Unitary representation. Show that $\phi : \mathbb{R} \to S^1$ defined by $\phi(t)=e^{2\pi it}$. Then show that it is a unitary representation of $\mathbb{R}$. **[5]**

c) Give an example of a group representation which is indecomposable but not irreducible. **[4]**

Q5) a) Show that every representation of a finite group is equivalent to a unitary representation. **[5]**

b) Show that every representation of a finite group is completely reducible. **[5]**

c) Let $\phi : G \to GL_n(C)$ be a group representation and $\psi : G \to GL_n(\mathbb{C})$ is defined by $\psi_g = \overline{\phi}_g = \left(\overline{\phi_{ij}(g)}\right)$. Give an example where ϕ and ψ are not equivalent. **[4]**

Q6) a) Define a homomorphism of group representations. Let $\phi : V \to GL(V)$ and $\rho : G \to GL(W)$ be two group representations. Let $T : V \to W$ *be a* linear transformation in $Hom_G(\phi, \rho)$. Show that $Ker(T)$ and $\mathrm{Im}(T)$ are G – invariant subspace of V and W respectively. Also show that $Hom_G(\phi,\rho)$ is subspace of $Hom(V,W)$. **[5]**

b) Let ϕ,ρ be irreducible representations of a group G and $T \in Hom_G(\phi,\rho)$. Then show that either T is ivertible or $T = 0$. Consequently show that (a) If not $\phi \sim \rho$ then $\mathrm{Hom}_G(\phi,\rho) = 0$, (b) If $\phi \sim \rho$ *then* $T = \lambda I$ with $\lambda \in \mathbb{C}$. **[5]**

c) Let G be an abelian group. Then show that any irreducible representation of G has degree one. **[4]**

Q7) a) Let G be a finite abelian group and $\phi : G \to GL_n(\mathbb{C})$ a representation. Then show that there is an invertible matrix T such that $T^{-1}\phi_g T$ is diagonal for all $g \in G$. **[5]**

b) Let $A \in GL_m(\mathbb{C})$ be a matrix of finite order. Then show that A is diagonalizable. Moreover, if $A^n = I$, then show that the eigenvalues of A are n^{th}-roots of unity. **[5]**

c) Let $\phi : G \to GL(V)$ be a representation. Define the character $\chi_\phi : G \to \mathbb{C}$ of ϕ. Show that $\chi_\phi(1) = \deg(\phi)$. Also show that if ϕ, ρ are equivalent representations then $\chi_\phi = \chi_\rho$. **[4]**

Q8) a) Let G be a group. Define a class function $f : G \to \mathbb{C}$ and show that space of all class functions $ZL(G)$ is subspace of $L(G)$. Also show that $\dim(ZL(G)) = |Cl(G)|$, where $C1(G)$ is the set of all conjugacy classes of G. **[5]**

b) Show that irreducible characters of a finite group G form an orthonormal set of class functions. **[5]**

c) Compute the character table of S_3. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PD-2997

[Total No. of Pages : 3

[6474]-47

M.A./M.Sc.

MATHEMATICS

MTUTO 147 : Coding Theory

(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that code C is v-error-correcting if and only if $d(C) \geq 2v + 1$. **[5]**

b) Construct the IMLD (incomplete maximum likelihood decoding) table for code C = {000, 001, 010, 011}. **[5]**

c) Determine the number of binary codes with parameters $(n, 2, n)$ for $n \geq 2$. **[4]**

Q2) a) Show that dimension of a self-orthogonal code of length n must be $\leq n/2$, and the dimension of a self-dual code of length n is $n/2$. **[5]**

b) Find a generator matrix and a parity-check matrix for the linear code generated by set S = {1000, 0110, 0010, 0001, 1001}, $q = 2$ and give the parameters $[n, k, d]$. **[6]**

c) Let C = {0000, 1000, 0100, 1100} be binary linear code. Find $d(C)$ and $Wt(C)$. **[3]**

P.T.O.

Q3) a) Let $g(x)$ be the generator polynomial of an ideal of $\frac{Fq[x]}{(x^n - 1)}$. Then show that the corresponding cyclic code has dimension k if the degree of $g(x)$ is $n - k$. **[5]**

b) Determine the generator polynomial and the dimension of the smallest cyclic code containing word 1000111 $\in F_2^7$. **[5]**

c) Let $g(x) = (1+x)(1+x+x^3) \in F_2[x]$ be the generator polynomial of binary [7, 3] - Cyclic code C. Write down a generator matrix. **[4]**

Q4) a) Prove that BCH code with designed distance δ has minimum distance at least δ. **[7]**

b) Suppose we have three nonzero polynomials $f_1(x)$, $f_2(x)$ and $f_3(x)$. Show that $lcm(f_1(x), f_2(x), f_3(x)) = lcm(lcm(f_1(x), f_2(x), f_3(x))$. **[5]**

c) Find dim (C) where C = {0000, 1000, 0100, 1100} be binary linear code.**[2]**

Q5) a) Prove that minimal polynomial of an element of $F_q m$ with respect to F_q exists and is unique. It is also irreducible over F_q. **[5]**

b) Find all cyclobomic cosets of 2 mod 63. **[5]**

c) Factorize the following polynomials. **[4]**

i) $x^7 - 1$ over F_2

ii) $x^8 - 1$ over F_3

Q6) a) Prove that for an integer $q > 1$ and integers n, d such that $1 \leq d \leq n$, we have $\frac{q^n}{\sum_{l=0}^{d-1} \binom{n}{i}(q-1)^i} \leq A_q(n,d)$. **[5]**

b) Find an optimal code with $n = 3$ and $d = 2$. **[5]**

c) Determine whether the a binary linear code with parameters (8, 8, 5) exists. **[4]**

Q7) a) Prove that for any integers $q > 1$, any positive integer n and any integer d such that $1 \le d \le n$, we have $A_q(n, d) \le q^{n-d+1}$. **[7]**

b) Let $q \ge 2$ and $n \ge 2$ be any integers. Show that $A_q(n, 2) = q^{n-1}$. **[5]**

c) Define relative minimum distance of code C. **[2]**

Q8) a) If $G = (I_k \mid X)$ is the standard form generator matrix of an $[n, k]$ - code C, then prove that parity - check matrix for C is $H = (-X^T \mid I_{n-k})$. **[5]**

b) Find generator and parity - check matrix for the binary linear code $C = \langle S \rangle$, where $S = \{11101, 10110, 01011, 11010\}$. **[6]**

c) Find dim (C) and all distinct basis for binary linear code $C = \{000, 001, 100, 101\}$. **[3]**

Total No. of Questions : 8]

SEAT No. :

PD-2998

[Total No. of Pages : 4

[6474]-48

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-148 : Probability and Statistics

(2019 Pattern) (Creidt System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator is allowed.*

Q1) a) The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that : **[5]**

i) Exactly 5 survive and

ii) at most 9 survive?

b) A shipment of 8 similar micro-computers to a retail outlet contains 3 micro-computers that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives. **[5]**

c) Prove that : The expected value of the sum or difference of two or more functions of a random variable X is the sum or difference of the expected values of the functions. That is, $E[g(x) \pm h(x)] = E[g(x)] \pm E[h(x)]$ **[4]**

Q2) a) Estimate the regression line for the pollution data given below : **[5]**

x	2	4	6	8	9	11
y	2	2	4	4	8	10

b) If $X_1, X_2, \ldots\ldots, X_n$ are mutually independent random variables that have respectively Chi squared distributions with $V_1, V_2, \ldots\ldots, V_n$ degrees of freedom then prove that the random variable $Y = X_1 + X_2 + \ldots\ldots + X_n$ has a Chi squared distribution with $V = V_1 + V_2 + \ldots. + V_n$ degrees of freedom. **[5]**

P.T.O.

c) i) Define the term probability density function.

ii) If A and A' are complementary events, then show that P(A) + P(A') = 1 **[4]**

Q3) a) A random variable X has a mean, $\mu = 10$ and a variance, $\sigma^2 = 4$ Using Chebyshev's theorem, find. **[5]**

i) $P(|X - 10| < 3)$ ii) $P(10 < X < 15)$

b) Prove that mean and variance of the discrete uniform distribution $f(x\,;k)$ are $\mu = \frac{1}{k}\sum_{i=1}^{k} x_i$ and $\sigma^2 = \frac{1}{k}\sum_{i=1}^{k}(x_i - \mu)^2$ respectively. **[5]**

c) Ten is the average number of oil tankers arriving each day at a certain port city. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away? **[4]**

Q4) a) Suppose that the shelf life, in years, of a certain perishable food product packaged in cardboard containers is a random variable whose probability density function is given by **[5]**

$$f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & elsewhere \end{cases}$$

Let X_1, X_2 and X_3 represent the shelf lives for three of these containers selected independently.

find $P(X_1 < 2, 1 < X_2 < 3, X_3 > 2)$

b) Let x be random variable with moment generating function $M_x(t)$. Then show that **[5]**

i) $M_{x+a}(t) = e^{at} M_x(t)$ ii) $M_{ax}(t) = M_x(at)$

c) Let X be the random variable that denotes the life in hours of a certain electronic device. The probability density function is **[4]**

$$f(x) = \begin{cases} \dfrac{20000}{x^3} & x \geq 100 \\ 0 & elsewhere \end{cases}$$

Find the expected life of this type of device

Q5) a) Define normal distribution. Also show that mean of $n(x\,;\,\mu, \sigma)$ is μ. **[5]**

b) Calculate the variance of $g(x) = 2x + 3$, where x is a random variable with probability distribution. **[5]**

x	0	1	2	3
$f(x)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

c) Suppose that the error in the reaction temperature, in °C, for a controlled laboratory experiment is a continuous random variable x having the probability density function **[4]**

$$f(x) = \begin{cases} \dfrac{x^2}{3}, & -1 < x < 2 \\ 0\ , & elsewhere \end{cases}$$

i) Find $p(0 < x \leq 1)$

ii) Verify that $\int_{-\infty}^{\infty} f(x)dx = 1$

Q6) a) In a certain assembly plant, three machines, B_1, B_2 and B_3 make 30%, 45% and 25% of the products respectively. It is known from past experience that 2%, 3% and 2% of the products made by each machine are defective respectively. Now, suppose that a finished product is randomly selected. What is the probability that it is defective? **[5]**

b) Given a normal distribution with mean, $\mu = 40$ and $\sigma = 6$, find the value of x that has **[5]**

i) 45% of the area to the left

ii) 14% of the area to the right

c) i) State Baye's theorem

ii) Define the moment generating function of the random variable X.

[4]

Q7) a) The fraction X of male runners and the fraction Y of female runners who compete in marathon races are described by the joint density function.

$$f(x,y)=\begin{cases}8xy & 0\le y\le x\le 1\\ 0 & elsewhere\end{cases}$$

Find the covariance of X and Y. **[6]**

b) Prove that if X and Y are random variables with joint probability distribution $f(x, y)$, then $\sigma^2_{ax+by}=a^2\sigma^2_x+b^2\sigma^2_y+2ab\sigma_{xy}$. **[5]**

c) A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If A be the event that an even number turns up, then find the probability of the event A. **[3]**

Q8) a) Determine the values of c so that the following functions represent joint probability distributions of the random variables X and Y. **[5]**

i) $f(x, y) = cxy$ for $x = 1, 2, 3$ and $y = 1, 2$

ii) $f(x, y) = c|x - y|$ for $x = -2, 0, 2$ and $y = -2, 3$

b) A group of 10 individuals is used for a biological case study. The group contains 3 people with blood type O, 4 with blood type A and 3 with blood type B. What is the probability that a random sample of 5 will contain 1 person with blood type O, 2 people with blood type A and 2 people with blood type B? **[5]**

c) In how many ways can 8 people be seated at a round table if **[4]**

i) they can sit anywhere

ii) two particular people must not sit next to each other?

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