

Total No. of Questions : 4]

SEAT No. :

PD-3217

[Total No. of Pages : 2

[6474]-101

M.A./M.Sc. (Part - I)

MATHEMATICS

MTS - 501 MJ : Linear Algebra

(2023 Credit Pattern) (Semester -I)

Time : 2 Hours]

[Max. Marks : 35

Instruction to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right side indicates full marks.

Q1) Attempt any three of the following : [6]

- a) Test the linear dependence of the following set of vectors in $\mathbb{R}^3$
(1,1,2), (3,1,2), (-1,0,0)
- b) Prove that if two vectors are linearly dependent then one of them is a scalar multiple of other.
- c) Is $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined as $T(x_1, x_2) = (1+x_1, x_2)$ is linear transformation? Justify.
- d) Find the range and Null space for identity transformation on a finite dimensional vector space V.

Q2) Attempt any two of the following : [10]

- a) Let T be any linear operator on a finite dimensional inner product space V then show that there exists a unique Linear operator T^* on V such that $(T\alpha|\beta) = (\alpha|T^*\beta)$ for all α, β in V.
- b) Let $B = \{(1,0,-1), (1,1,1), (2,2,0)\}$ be the basis of $\mathbb{R}^3$ find the dual Basis of B.
- c) Let W_1 and W_2 are finite dimensional subspaces of a vector space V then show that W_1+W_2 is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1+W_2)$.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Let V and W be a vector space over the field F and Let T be a linear transformation from V into W . Suppose V is the finite dimensional then prove that $\text{rank}(T) + \text{Nullity}(T) = \dim V$.
- b) Write all possible Jordan canonical form if the characteristics polynomial is $(x-1)^3 (x-2)^2$.
- c) Let T and U be the linear operators on $\mathbb{R}^2$ defined by
 $T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, 0)$ Then find
 - i) $T+U$
 - ii) TU
 - iii) UT
 - iv) T^2
 - v) U^2

- b) Write all possible Jordan canonical form if the characteristics polynomial is $(x-1)^3(x-2)^2$.

- c) Let T and V be the linear operators on $\mathbb{R}^2$ defined by

$T(x_1, x_2) = (x_2, x_1)$ and $U(x_1, x_2) = (x_1, \theta)$ Then find

- | | |
|-------------------|--------------------|
| i) T+U | ii) TU |
| iii) UT | iv) T ² |
| v) U ² | |

- iii) UT

iv) T²

- $$v) \quad U^2$$

Q4) a) Attempt any one of the following.

[5]

- i) Let U be a linear operator on an inner product space V . Then show that U is unitary if and only if the adjoint U^* of U exists and $UU^* = U^*U = I$.
- ii) Let q represent quadratic form on $\mathbb{R}^2$. Then find the symmetric bilinear form f corresponding to q where $q = 3x_1x_2 - x_2^2$.

- ii) Let q represent quadratic form on $\mathbb{R}^2$. Then find the symmetric bilinear form f corresponding to q where $q = 3x_1x_2 - x_2^2$.

b) Attempt any one of the following.

[4]

- i) Define bilinear form on vector space V . Also, describe the bilinear form on $\mathbb{R}^3$ which satisfy $f(\alpha, \beta) = -f(\beta, \alpha)$ for all α, β .
- ii) Let T be the Linear operator on $\mathbb{R}^3$ which is represented in the

- ii) Let T be the Linear operator on $\mathbb{R}^3$ which is represented in the

standard ordered basis by the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ prove that T is diagonalizable.



Total No. of Questions : 7]

SEAT No. :

PD-3218

[Total No. of Pages : 3

[6474]-102
M.A./M.Sc.
MATHEMATICS
MTS-503 MJ : Group Theory
(2023 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicates full marks.*

Q1) Attempt any five of the following :

[10]

- a) Determine the operation $*$ on $\mathbb{Z}$ defined by $a * b = a - b$, is associative.
- b) Find the order of $\sigma = (1 \ 12 \ 8 \ 10 \ 4) (2 \ 13) (6 \ 9)$.
- c) Find all subgroups of $\mathbb{Z}_{45}$.
- d) Draw the lattice of subgroups of the group $\mathbb{Z}_{12}$.
- e) State the Cayley's theorem.
- f) Let G be a group. Prove that $[y, x] = [x, y]^{-1}$, $x, y \in G$.
- g) What is order of $(3, 2)$ in $\mathbb{Z}_4 \times \mathbb{Z}_4$?

Q2) Attempt any two of the following :

[10]

- a) If G is a group under the operation $*$. Then prove that
 - i) The identity of G is unique
 - ii) $(a * b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$
- b) Find the orders of each element of Q_8 .
- c) Let $H = \langle x \rangle$ be a cyclic group. Prove that every subgroup of H is cyclic.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Let G be an arbitrary group, $x \in G$ and $m, n \in \mathbb{Z}$. Prove that
 - i) If $x^n = 1$ and $x^m = 1$ then $x^d = 1$, where $(m, n) = d$.
 - ii) If $x^m = 1$ then $|x|$ divides m .
- b) Prove that if A and B are subsets of G with $A \subseteq B$ then $C_G(B)$ is subgroup of $C_G(A)$.
- c) Define a map $\pi : \mathbb{R}^2 \rightarrow \mathbb{R}$ by $\pi(x, y) = x$. Prove that $\bar{\pi}$ is a homomorphism and find the kernel of π .

Q4) Attempt any two of the following :

[10]

- a) State and prove the Lagrange's theorem.
- b) Prove that quotient groups of a cyclic group are cyclic.
- c) Prove that the automorphism group of the cyclic group of order n is isomorphic to $\mathbb{Z}/n\mathbb{Z}$.

Q5) Attempt any two of the following :

[10]

- a) If G be a finite group of order $p^\alpha m$, where p is a prime not dividing m , then prove that G has a subgroup of order p^α .
- b) Find all conjugate classes and their sizes in the group Q_8 .
- c) Let $\phi: G \rightarrow H$ be a homomorphism of groups. Prove that ϕ is injective if and only if $\ker \phi = 1$.

Q6) Attempt any two of the following :

[10]

- a) Suppose G is a group with subgroups H and K such that
 - i) H and K are normal in G and
 - ii) $H \cap K = 1$. Then prove that $HK \cong H \times K$.
- b) Let p be a prime. prove that the elementary abelian group of order p^2 has exactly $(p + 1)$ subgroups of order p .
- c) If A is any nonempty collection of subgroup of G , then prove that the intersection of all members of A is also a subgroup of G .

Q7) Attempt any two of the following :

[10]

- a) Let G be a group, let $x, y \in G$ and let $H \leq G$ then prove that
 - i) $xy = yx$ if and only if $[x, y] = 1$
 - ii) $\sigma[x, y] = [\sigma(x), \sigma(y)]$, for any automorphism σ of G .
- b) Let G be any group containing a subgroup H of index 2. Prove that $H \trianglelefteq G$
- c) List the non isomorphic abelian groups of order p^5 , where p is prime.



Total No. of Questions : 7]

SEAT No. :

PD-3219

[Total No. of Pages : 3

[6474]-103
F.Y. M.A./M.Sc.
MATHEMATICS
MTS - 504 - MJ : Ordinary Differential Equations
(2023 Credit Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicates full marks.*

Q1) Attempt any Five of the following : [10]

- a) Show that e^x and e^{-x} are linearly independent solutions of the differential equation $y'' - y = 0$ on any interval.
- b) Find all solutions of the differential equation $y' + y = e^x$.
- c) Find the general solution of the differential equation $y''' - 5y'' + 6y' = 0$.
- d) Compute Wronskian of the functions $\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$.
- e) Write the general form of second order
 - i) Euler's equation
 - ii) Bessel equation of order α
- f) Determine whether the differential equation $2xy \, dx + (x^2 + 3y^2) \, dy = 0$ is exact.
- g) Define Lipschitz condition.

Q2) Attempt any two of the following : [10]

- a) Explain the method of solving $y' + ay = b(x)$ where a is constant and $b(x)$ is continuous function.
- b) Find a solution ϕ of the equation $y' + (\cos x) y = e^{-\sin x}$ which satisfies $\phi(\pi) = \pi$.
- c) Solve the equation $Ly' + Ry = E$, where L , R and E are all positive constants. Also show that every solution tends to $\frac{E}{R}$ as $x \rightarrow \infty$.

P.T.O.

Q3) Attempt any two of the following : [10]

- a) Explain the method for solving non-homogeneous equation with constant coefficients of order two.
- b) Find the solutions of the initial value problem $y'' - 2y' - 3y = 0$, $y(0) = 0$, $y'(0) = 1$.
- c) Find all solutions of the equation $y'' + y = \sec x$ $(-\pi/2 < x < \pi/2)$.

Q4) Attempt any two of the following : [10]

- a) Explain the method of reduction of order for solving the n^{th} order homogeneous linear differential equation with variable coefficients.
- b) Find two linearly independent solutions of the differential equation $y'' + \frac{1}{x} y' - \frac{1}{x^2} y = 0$, for $x > 0$. Also prove that the two solutions are linearly independent.
- c) Let ϕ_1 and ϕ_2 be the two solutions of the differential equation $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I containing a point x_0 .

Then prove that. $W(\phi_1, \phi_2)(x) = \exp \left[- \int_{x_0}^x a_1(t) dt \right] W(\phi_1, \phi_2)(x_0)$.

Q5) Attempt any two of the following : [10]

- a) State Euler's equation and find its solution.
- b) Find two linearly independent power series solutions in powers of x of the equation $y'' + y = 0$.
- c) Compute the indicial polynomial and its roots for the differential equation $x^2 y'' + (x + x^2) y' - y = 0$.

Q6) Attempt any two of the following : [10]

- a) Explain the variable separable method for the first order differential equation $y' = f(x, y)$.
- b) Compute the first four approximations ϕ_0, ϕ_1, ϕ_2 and ϕ_3 to the solution of the initial value problem $y' = 3y + 1$, $y(0) = 2$.
- c) Show that the function $\phi(x) = \frac{y_0}{1 - y_0(x - x_0)}$ which passes through the point (x_0, y_0) is a solution of the equation $y' = y^2$.

Q7) Attempt any two of the following :

[10]

- a) Show that the function $f(x,y) = xy^2$
- i) Satisfies the Lipschitz condition on the rectangle $R : |x| \leq 1, |y| \leq 1$.
 - ii) Does not satisfy the Lipschitz condition on the strip $S : |x| \leq 1, |y| < \infty$.
- b) Let ϕ_1 and ϕ_2 be the two solutions of $\overline{x}' = A(t)\overline{x}$ where $A(t)$ is a 2×2 matrix, continuous on an interval I . Then prove that their Wronskian $W(t)$

is given by $W(t) = W(t_0) \cdot \exp \left[\int_{t_0}^t (a_{11}(s) + a_{22}(s)) ds \right]$.

- c) Find the general solution of $\overline{x}' = A \overline{x}$ Where $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.



[6474]-104

M.A./M.Sc. (Part - I)

MATHEMATICS

MTS - 506(A) MJ : Advanced Numerical Analysis

(2023 Credit Pattern) (Semester -I)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Use of single, non-programmable scientific calculator is allowed.
- 3) Figures to the right indicates full marks.

Q1) Attempt any THREE of the following : [6]

- a) Compute the limit and determine the corresponding rate of

$$\text{convergence } \lim_{n \rightarrow \infty} \left(\frac{n-1}{n^3+2} \right)$$

- b) Find the vector valued function F associated with the following system and compute the Jacobian of F $5 \cos x + 6 \cos y - 10 = 0$, $5 \sin x + 6 \sin y - 4 = 0$.
- c) If $f(x) = \ln(x)$, find $f'(2)$ for $h = 0.1, 0.01$.
- d) Show that the Householder matrix is symmetric and orthogonal.

Q2) Attempt any TWO of the following : [10]

- a) Verify that the equation
- $x^3 - 18x^2 + 45 = 0$
- has a root on the interval (1,2). Perform 3 iterations by Newton's method starting with
- $P_0 = 1$
- .

- b) Derive the following forward difference approximation for the second

$$\text{derivative. } f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}$$

- c) Construct the House holder matrix H for
- $w = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix}^T$
- .

P.T.O.

Q3) Attempt any TWO of the following.

[10]

- Evaluate $\int_1^2 \frac{1}{x} dx$, by using Trapezoidal rule by dividing the interval [1,2] into five equal subintervals.
- Solve the following system by Jacobi method starting with vector $X^{(0)} = [0 \ 0 \ 0]^T$. perform 3 iterations.
 $2x_1 - x_2 = -1$, $-x_1 + 4x_2 + 2x_3 = 3$, $2x_2 + 6x_3 = 5$.
- Consider the function $f(x) = e^x$. Construct the Lagrange's form of the interpolating polynomial for f passing through the points $(-1, e^{-1})$, $(0, e^0)$ and $(1, e^1)$.

Q4) a) Attempt any ONE of the following.

[4]

- For matrix $A = \begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$ with initial vector $x^{(10)} = [1 \ 0 \ 0]^T$

perform two iterations of power method to find dominant eigen value and corresponding eigen vector.

- Construct the divided difference table for the following data set and write the Newton form of the interpolating polynomial

x	-7	-5	-4	-1
y	10	5	2	10

b) Attempt any ONE of the following.

[5]

- Prove that the order of convergence of secant method is approximately $\alpha = 1.618$ and asymptotic error constant

$$\lambda \approx C^{\frac{1}{\alpha}} = \left[\frac{f''(p)}{2f'(p)} \right]^{\alpha-1}$$

- Apply modified Euler's method to approximate solution for the initial value problem $\frac{dx}{dt} = t^2 - 2x^2 - 1$, $0 \leq t \leq 1$, $x(0) = 0$. perform 2 steps.



Total No. of Questions : 4]

SEAT No. :

PD-3221

[Total No. of Pages : 2

[6474]-105

M.A./M.Sc.

MATHEMATICS

MTS-506-(B)-MJ : Number Theory

(2023 Credit Pattern) (Semester - I)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Symbols and notations have their usual meanings.

Q1) Attempt any Three of the following :

[6]

- a) Prove that if x and y are odd integer's then $x^2 + y^2$ is even but not divisible by 4.
- b) Find the minimal polynomial of $\sqrt[3]{7}$.
- c) For any positive integer m , show that $(ma, mb) = m(a, b)$.
- d) For any two real number x and y show that $[x] + [y] \leq [x+y]$.

Q2) Attempt any Two of the following :

[10]

- a) Prove that for any interger n , $n^3 - n$ is divisible by 3.
- b) If 4 lemon is of ruppee 1, 1Guava is of ruppee 1 and 1 coconut is of ruppee 5 then how many of each fruit buy so that 100 fruits can buy in 100 ruppees.
- c) State and prove Gauss lemma.

P.T.O.

Q3) Attempt any Two of the following :

[10]

- a) Prove that the product of two primitive polynomial is primitive.
- b) State and prove mobius inversion formula.
- c) Let p be a prime. Prove that $x^2 \equiv 1 \pmod{p}$ if and only if $x \equiv \pm 1 \pmod{p}$

Q4) a) Attempt any One of the following :

[4]

- i) How many zero's are there at the end of $100!$ in the decimal representation of $100!$
- ii) If α is algebraic number then prove that there is rational number b such that $\alpha.b$ is algebraic integer.

b) Attempt any one of the following :

[5]

- i) If ξ is algebraic number of degree n then prove that every number in $Q(\xi)$ can be written uniquely in the form
$$a_0 + a_1.\xi + a_2.\xi^2 + \dots + a_{n-1}.\xi^{n-1} \text{ where } a_i \in Q$$
- ii) Find the smallest positive integer that gives remainder 1,2,3 when divided by 3,4,5 respectively.



Total No. of Questions : 4]

SEAT No. :

PD-3222

[Total No. of Pages : 2

[6474]-106

F.Y. M.A./M.Sc.

MATHEMATICS

MTS-506 (C) MJ : Combinatorics

(2023 Credit Pattern) (Semester - I)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any Three of the following : [6]

- a) Find two different chessboard (not row or column rearrangements of one another) that have the same rook polynomial. Also, write the rook polynomial.
- b) Solve the recurrence relation $a_n = 16 a_{n-2} + 5n$ (assuming that n is power of 2)
- c) Find a generating function for the number of integers between 0 and 9,99,999 whose sum of digits is r.
- d) How many r-digit ternary sequence are there with an even number of zero's?

Q2) Attempt any two of the following : [10]

- a) How many arrangements of 1,1,1,1,2,3,3, are there with 2 not beside either 3?
- b) Using generating functions, solve the recurrence relation $a_n = a_{n-1} + n(n-1)$, $a_0 = 1$
- c) i) How many even five digit numbers (leading zeros not allowed) are there?
ii) How many five digit numbers are there with exactly one 3?
iii) How many five digit numbers are there that are the same when the order of their digits is inverted (e.g.15251).?

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Find ordinary generating function whose coefficient a_r equals $3r^2$. Hence, evaluate the sum $0+3+12+....+ 3n^2$
- b) How many ways are there to collect \$24 from 4 children and 6 adults if each person gives at least \$1, but each child can give at most \$4 and each adult at most \$7?
- c) How many arrangements of the letters in 'MISSISSIPPI' in which.
 - i) The M is immediately followed by an I?
 - ii) The M is beside an I, that is, an I is just before or just after the M.

Q4) a) Attempt any one of the following :

[5]

- i) How many ways are there to distribute eight different toys among four children if the first child gets at least two toys?
- ii) Suppose a bookcase has 200 books, 70 in French and 100 about Mathematics. How many non-French books not about mathematics are there if.
 - I) There are 30 French mathematics books?
 - II) There are 60 French non mathematics books?

b) Attempt any one of the following :

[4]

- i) State and prove Inclusion - Exclusion formula.
- ii) How many ways are there to distribute 25 identical balls into six distinct boxes with at most 6 balls in any of the first boxes?



Total No. of Questions : 4]

SEAT No. :

PD-3223

[Total No. of Pages : 2

[6474]-107

M.A./M.Sc. - I

MATHEMATICS

MTS-506(D)-MJ : Lattice Theory
(2023 Credit Pattern) (Semester - I)

Time : 2 Hours]

[Max. Marks : 35

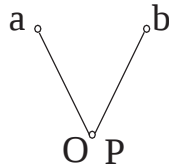
Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Symbol and notations have their usual meanings.

Q1) Attempt any Three of the following :

[6]

- a) Show that every chain is a lattice.
- b) Define ideal of lattice L .
- c) Prove that $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$ for all $x, y, z \in L$
- d) Let $H(P)$ denotes the set of all hereditary subset of poset P . Find $H(P)$ where P is following.



Q2) Attempt any two of the following :

[10]

- a) Prove that a modular lattice L is distributive if and only if it does not contain a diamond.
- b) Show that a lattice is distributive if and only if it is isomorphic to a ring of sets.
- c) Let L be a finite distributive lattice. Show that map $\phi : a \mapsto r(a)$ is an isomorphism between L and $H(J(L))$.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Draw Hasse diagram for following
 - i) Natural divisors of 20 and
 - ii) Powers set of X , where $X = \{a, b, c\}$ with inclusion
- b) Let $(P, \leq)$ be a poset. Prove that $(P, \leq)$ is a lattice if and only if $\inf(H)$ and $\sup(H)$ exists for any non-empty finite subset H of P .
- c) Show that I is a prime ideal of lattice L if and only if there is a homomorphism ϕ of L onto C_2 such that,
$$I = \phi^{-1}(0) = \{x \in L / \phi(x) = 0\}$$

Q4) a) Attempt any one of the following :

[4]

- i) Prove that every distributive algebraic lattice is a pseudocomplemented.
- ii) Define Stone algebra and also give any two examples of pseudo complemented lattice L .

b) Attempt any one of the following :

[5]

- i) Prove that every maximal chain C of finite distributive lattice L is of Length $|J(L)|$.
- ii) Prove that a modular lattice satisfies upper covering condition and the lower covering condition.



Total No. of Questions : 4]

SEAT No. :

PD-3224

[Total No. of Pages : 2

[6474]-108

F.Y. M.A./M.Sc.

MATHEMATICS

**MTS-508 MJ : Research Methodology
(2023 Credit Pattern) (Semester - I)**

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicates full marks.*

Q1) Attempt any three of the following : **[6]**

- a) Write short note on ZMATH.
- b) Explain the concept of case studies.
- c) Write atleast four qualities of good Research.
- d) Write difference between primary source data and secondary source data.

Q2) Attempt any two of the following : **[10]**

- a) Explain the concept of questionnaires in Research.
- b) Write a short note MathsciNet.
- c) Write short note on ethical committees in Research.

Q3) Attempt any two of the following : **[10]**

- a) Write a note on Development of Research plan.
- b) Explain the h-index and it's importance in research.
- c) Write short note on Intellectual property Rights.

Q4) a) Attempt any one of the following : **[5]**

- i) Explain scientific method and its characteristics.
- ii) Explain the concept of impact factor of Journals

b) Attempt any one of the following : **[4]**

- i) Write a short note on Research ethics.
- ii) Explain plagiarism in Research.



Total No. of Questions : 4]

SEAT No. :

PD-3225

[Total No. of Pages : 2

[6474]-201
F.Y. M.A./M.Sc.
MATHEMATICS
MTS - 551 - MJ : Topology
(2023 Pattern) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following :

[6]

- a) Show that Q_+ of positive rational numbers is countably infinite.
- b) Is $\overline{A-B} = \overline{A}-\overline{B}$? Justify your answer.
- c) Show that $[0,1]$ and $(0,1)$ are not homeomorphic.
- d) State 'The Lebesgue Number Lemma'.

Q2) Attempt any two of the following :

[10]

- a) Let X be an ordered set in the order topology. Let Y be a subset of X that is convex in X . Then prove that the order topology on Y is same as the topology on Y inherits as a subspace of X .
- b) Prove that every closed subspace of a compact space is compact.
- c) Show that IR_l is normal.

Q3) Attempt any two of the following :

[10]

- a) Prove that every finite point set in a hausdorff space X is closed.
- b) Show that if $f : X \rightarrow Y$ is continuous, where X is compact and Y is hausdorff then f is a closed mop.
- c) Show that closed subspace of a normal space is normal.

P.T.O.

Q4) a) Attempt any one of the following : [4]

- i) Show that $[a,b]$ is homeomorphic to $[0,1]$.
- ii) Let $f : X \rightarrow Y$ be a bijective continuous function. If X is compact and Y is hausdorff. Then prove that f is homeomorphism.

b) Attempt any one of the following : [5]

- i) Prove that a space X is locally connected iff for every open set U of X , each component of U is open in X .
- ii) Prove that every regular space with a countable basis is normal.



Total No. of Questions : 7]

SEAT No. :

PD-3226

[Total No. of Pages : 3

[6474]-202

M.A./M.Sc.

MATHEMATICS

MTS-553 MJ : Ring Theory

(2023 Pattern) (Credit System) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any five of the following :

[10]

- a) Let S_1 and S_2 be two subrings of ring R then prove that intersection of two subrings is also subrings.
- b) Define :
 - i) Integral Domain
 - ii) Characteristics of ring
- c) Let I and J be ideals of ring R then prove that $I + J$ is an ideal of ring R .
- d) If R is any non-trivial ring is not a commutative and $n \geq 2$ then prove that $M_n(R)$ is not commutative.
- e) Check whether the given polynomial $x^4 + 10x + 5$ in $\mathbb{Z}[x]$ is irreducible or not using Eisenstein's criterion applied for the prime 5.
- f) Find the greatest common divisor of the polynomials $x^3 - 2$ and $x + 1$ in $\mathbb{Q}[x]$.
- g) In an integral domain a prime element is always irreducible.

P.T.O.

Q2) Attempt any two of the following :

[10]

- a) Let R be a ring. Then prove the following statements.
 - i) $0.a = a.0 = 0$, for all $a, \in R$
 - ii) $(-a)b = a(-b) = -(ab)$, for all $a, b \in R$
 - iii) $(-a)(-b) = ab$, for all $a, b \in R$
 - iv) If R has identity 1 , then the identity is unique and $-a = (-1)a$.
- b) Let R be an integral domain and let $p(x)$ and $q(x)$ be non-zero elements of $R[x]$ then prove that $R[x]$ is an integral domain.
- c) Let R and S be rings and let $\phi : R \rightarrow S$ be a homomorphism, then prove that
 - i) The image of ϕ is a subring of S .
 - ii) The kernel of ϕ is a subring of R .

Q3) Attempt any two of the following :

[10]

- a) If $\phi : R \rightarrow S$ is a homomorphism of rings, then prove that kernel of ϕ is an ideal of R , the image of ϕ is a subring of S and $R/\ker\phi$ is isomorphic as a ring to $\phi(R)$.
- b) Prove that every Boolean ring is a commutative ring.
- c) Let R be a finite commutative ring with identity. Prove that every prime ideal of R is a maximal ideal.

Q4) Attempt any two of the following :

[10]

- a) Let R be a Unique Factorization Domain. Prove that a non-zero element is a prime if and only if it is irreducible.
- b) Prove that every Euclidean Domain is Unique Factorization Domain.
- c) Let the prime number $p \in \mathbb{Z}$, then prove that p divides an integer of the form $n^2 + 1$ if and only if p is either 2 or is an odd prime congruent to 1 (modulo 4).

Q5) Attempt any two of the following :

[10]

- a) If $N(\alpha)$ is a prime is irreducible in quadratic integer ring θ , where $\alpha \in \theta$.
- b) State and prove Gauss Lemma.
- c) Let F be a field and let $P(x) \in F(x)$. Then prove that $P(x)$ has a factor of degree one if and only if $P(x)$ has a root in F , there is an $\alpha \in F$ with $P(\alpha) = 0$.

Q6) Attempt any two of the following :

[10]

- a) Let I be an ideal of the ring R and let $(I) = I[x]$ be the ideal of $R[x]$ generated by I . Then prove that $R[x]/(I) \cong (R/I)[x]$.
- b) Let R be the quadratic integer ring $\mathbb{Z}[\sqrt{-5}]$ then prove that the quadratic integer ring $\mathbb{Z}[\sqrt{-5}]$ is not a principal ideal domain.
- c) Let $a = 2210$ and $b = 1131$, determine their greatest common divisor d and write d as a linear combination $ax + by$ of a and b .

Q7) Attempt any two of the following :

[10]

- a) Let $R[x]$ be the ring of polynomial then prove that R is commutative ring with unity if and only if $R[x]$ is commutative ring with unity.
- b) Let D be a rational number that is not a perfect square in $\mathbb{Q}$ and define $\mathbb{Q}(\sqrt{D}) = \{a + b\sqrt{D} \mid a, b \in \mathbb{Q}\}$ show that $\mathbb{Q}(\sqrt{D})$ is a field.
- c) Find all prime ideal and maximal ideal of ring $\mathbb{Z}_{12}$.



Total No. of Questions : 7]

SEAT No. :

PD-3227

[Total No. of Pages : 3

[6474]-203

F.Y. M.A./M.Sc.

MATHEMATICS

MTS - 554 MJ : Advanced Calculus

(2023 Pattern) (Semester -II) (NEP - 2020)

Time : 3 Hours]

[Max. Marks : 70

Instruction to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following : **[10]**

- a) Find the gradient vector of $f(x,y,z) = x^2+2y^2-3z^2$ at point (1,0,2).
- b) State Stoke's Theorem.
- c) Let f and g denote scalar fields that are differentiable on an open set S . Then show that $\text{grad} (fg) = f \cdot \text{grad} (g) + g \cdot \text{grad} (f)$.
- d) Compute magnitude of the fundamental vector product of $\vec{r}(u,v) = (u+v)\vec{i} + (u^2+v^2)\vec{j} + (u^3+v^3)\vec{k}$
- e) Define Jordan curve.
- f) Define area of parametric surface.
- g) Define line integral

Q2) Attempt any two of the following : **[10]**

- a) State and prove chain rule for the derivative of scalar fields.
- b) Let $Z = u(x,y) e^{ax+by}$ and $\frac{\partial^2 u}{\partial x \partial y} = 0$ find values of constant a and b such

that $\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} + z = 0$

P.T.O.

- c) Let $f(x, y) = x \sin \frac{1}{y}$ if $y \neq 0$ and $f(x, y) = 0$ if $y = 0$ show that, $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$ but $\lim_{y \rightarrow 0} \left[\lim_{x \rightarrow 0} f(x, y) \right] \neq \lim_{x \rightarrow 0} \left[\lim_{y \rightarrow 0} f(x, y) \right]$.

Q3) Attempt any two of the following. **[10]**

- Find the Jacobian for the cylindrical transformation.
- State and prove Green's Theorem.
- Evaluate directional derivatives of the scalar field $F(x, y, z) = x^2 + 2y^2 + 3z^2$ at $(1, 1, 0)$ in the direction of $\vec{i} - \vec{j} + 2\vec{k}$.

Q4) Attempt any two of the following. **[10]**

- Evaluate $\iint_Q \sqrt{y - x^2} \, dx \, dy$ where $Q = [-1, 1] \times [0, 2]$
- Prove that If a scalar field F is differentiable at $\vec{a}$ then f is continuous at $\vec{a}$.
- Show that, f is not continuous at $(0, 0)$ where f be the scalar field defined on $\mathbb{R}^2$ as $f(x, y) = \frac{xy^2}{x^2 + y^4}$ if $x \neq 0$ and $f(0, y) = 0$ if $x = 0$.

Q5) Attempt any two of the following. **[10]**

- Explain line integral with respect to arc length.
- Use Green's theorem, compute work done by the force field $F(x, y) = (y + 3x)\vec{i} + (2y - x)\vec{j}$ in moving particle once around the ellipse $4x^2 + y^2 = 4$ in the clockwise direction.
- Evaluate $\iiint_S \sqrt{x^2 + y^2} \, dx \, dy \, dz$ where S is the solid formed by the upper half of the cone $z^2 = x^2 + y^2$ and the plane $z = 1$.

Q6) Attempt any two of the following.

[10]

- a) State Gauss-divergence theorem and explain each terms.
- b) Let $\vec{F} = (F_1, F_2, \dots, F_n)$ be continuously differentiable vector field on an open connected set S in $\mathbb{R}^n$. If $\vec{F}$ is gradient on S then show that the partial derivatives of the components of $\vec{F}$ are related by the equations $D_i f_j(x) = D_j f_i(x)$ for $i, j = 1, 2, \dots, n$ and every x in S .
- c) Let F be a two dimensional vector field given by $f(x, y) = Jy\vec{i} + (x^3 + y)\vec{j}$.

Calculate

- i) line integral of F from $(0,0)$ to $(1,1)$ along the line with parametric equations. $x = t, y = t \ 0 \leq t \leq 1$.
- ii) line integral along path with parametric equation $x = t^2, y = t^3 \ 0 \leq t \leq 1$.

Q7) Attempt any two of the following.

[10]

- a) Let f is differentiable at $\vec{a}$ with total derivative T_a . Then show that the derivative $f'(\vec{a}, \vec{y})$ exists for every $\vec{y}$ in $\mathbb{R}^n$ such that $T_a(\vec{y}) = f'(\vec{a}, \vec{y})$.
- b) Give any two properties of double integral.
- c) Show that a vector field $f(x, y) = \frac{-y}{x^2 + y^2} \vec{i} + x^3 y \vec{j}$ is not gradient.



Total No. of Questions : 4]

SEAT No. :

PD-3228

[Total No. of Pages : 3

[6474]-204

M.A./M.Sc.

MATHEMATICS

MTS-556 (A) MJ : Graph Theory
(2023 Pattern) (Semester - II) (NEP - 2020)

Time : 2 Hours]

[Max. Marks : 35

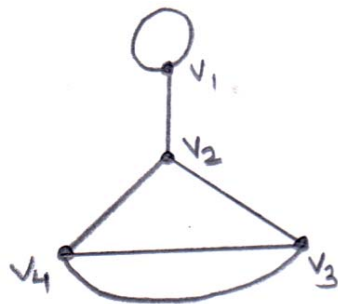
Instructions to the candidates :

- 1) Attempt all questions.
- 2) Figures to the right indicate full marks.

Q1) Attempt any Three of the following.

[6]

- a) Define perfect matching with suitable example.
- b) Draw a 4 - critical graph.
- c) Draw a graph G which is strongly connected digraph.
- d) List the degrees of each of the vertices of the graph G and find adjacency matrix of the following graph.



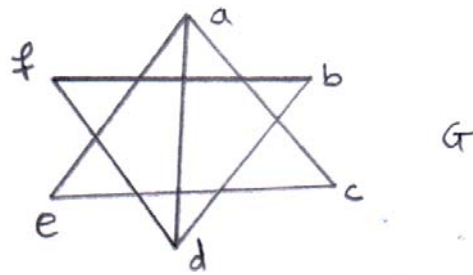
Q2) Attempt any two of the following .

[10]

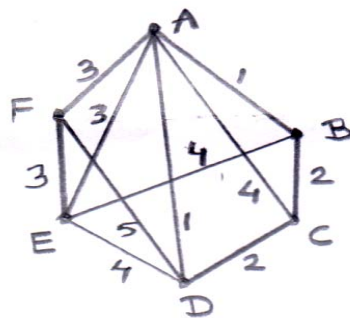
- a) Prove that λ be a feasible vertex labelling for the weighted complete bipartite graph G . If the equality subgraph G_λ has a perfect matching M^* then M^* is an optimal matching for G .

P.T.O.

- b) Consider the graph given below and answer the following questions.



- i) Draw a graph of it's complement.
 - ii) How many even vertices does G have?
 - iii) Find $G - U$ where, $U = \{f, e\}$
 - iv) How many odd vertices does G have.
 - iv) Find $G(F)$ where $F = \{a, b, d\}$.
- c) Find a minimal spanning tree for the connected weighted graph of the following by using Kruskal's algorithm.



Q3) Attempt any two of the following :

[10]

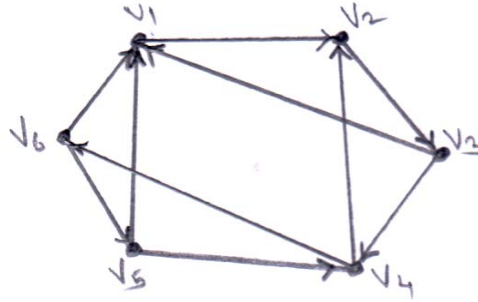
- a) Prove that the following statements about a graph G with n - vertices are equivalent.
 - i) G is a tree
 - ii) G is an acyclic graph with n-1 edges.
 - iii) G is a connected graph with n-1 edges.
- b) Define Hamiltonian path and Hamiltonian cycle and Draw a graph G_1 has no Hamiltonian path, G_2 has a Hamiltonian path but no. Hamiltonian cycle, G_3 has Hamiltonian cycle.
- c) Prove that a simple graph G is Hamiltonian if and only if it's closure $c(G)$ is Hamiltonian.

Q4) a) Attempt any one of the following : **[5]**

- i) Prove that a K -regular bipartite graph G with $K > 0$ has a perfect matching.
- ii) Let G be a K -critical graph then prove that G is connected and the degree of every vertex of G is at least $K-1$.

b) Attempt any one of the following : **[4]**

- i) Consider the graph given below and answer the following.



- I) Find $od(v_i)$, $id(v_i)$ for each vertex of diagram.
 - II) Find a directed walk of length 4.
 - III) Find a directed cycle of length 5.
 - IV) Find a directed path of longest possible length.
- ii) Define Graph with suitable example and Draw the graph having the following matrix as their adjacency matrix.

$$A(G) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$



Total No. of Questions : 4]

SEAT No. :

PD-3229

[Total No. of Pages : 2

[6474]-205
F.Y. M.A./M.Sc.
MATHEMATICS
MTS - 556(B) - MJ : Dynamical Systems
(2023 Pattern) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following :

[3 × 2 = 6]

- a) Prove that a linear combination of solutions to $\dot{X} = Ax$ is also a solution.
- b) Solve the system $\dot{X} = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix} X$.
- c) Find e^{At} of $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$.
- d) State Liouville Formula for Wroanskian.

Q2) Attempt any two of the following :

[2 × 5 = 10]

- a) Let $A(t)$ be $n \times n$ matrix whose entries depend continuously on t , Given x_0 in $\mathbb{R}^n$ prove that There is at most one solution $x(t, x_0)$ of $\dot{X} = A(t)x$ with $X(0, x_0) = x_0$.
- b) Find general solution of the non homogeneous system
$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
- c) Consider the differential equations $\dot{x} = x^2 - 9$.
Find stability type of each fixed point and sketch phase portrait on the line.

P.T.O.

Q3) Attempt any two of the following :

[2 × 5 = 10]

- a) Assume that $A(t)$ is a real $n \times n$ matrix with bounded entries then prove that solutions of $\dot{X} = AX$ exists for all time.

- b) Consider the system of differential equations

$$\dot{x} = xy$$

$$\dot{y} = 1 - y - (1 + y)x$$

Determine the fixed points and linear type of each fixed point also find nullclines.

- c) Sketch phase portrait of $\dot{X} = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} X$.

Q4) A) Attempt any one of the following :

[1 × 4 = 4]

- a) Consider system $\dot{x} = x(a - by - fx)$ $\dot{y} = y(-c + ex - hy)$ where all the parameters a, b, c, e, f & h are positive. Using Dulac criterion show that this system does not have periodic orbit. Also find expression of poicare map p and using 'p' show that again has no periodic orbit.

- b) Find the general solutions of the system $\dot{X} = \begin{pmatrix} 0 & 4 \\ -1 & 0 \end{pmatrix} X$. Sketch the phase portrait for the system.

B) Attempt any one of the following :

[1 × 5 = 5]

- a) Let A and B be two $n \times n$ matrices that commute prove that $e^{A+B} = e^A \cdot e^B$.
- b) Show that for a gradient system $\dot{x} = -\nabla G(x)$, any α or ω limit point is a fixed point.



[6474]-206

F.Y. M.A./M.Sc.

MATHEMATICS

MTS 556(C) MJ : Coding Theory
(2023 Credit Pattern) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any Three of the following : **[6]**

- a) Define Hamming distance with suitable example.
- b) Define error detection and corrections.
- c) Find length of cyclic run of 0 in the $e = (0, 0, 1, 2, 0, 0, 0, 1, 0, 0)$
- d) Write generator polynomial for a (7, 4) cyclic code.

Q2) Attempt any two of the following : **[10]**

- a) Let C be the linear [7, 4] code over F_2 write generator matrix and find parity check matrix.
- b) Find the weight distribution of the binary code generated by $C_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ find the weight enumerator polynomial of the code.
- c) The narrow sense BCH code of length 63 with distance $\delta = 5$. Find its dimension.

Q3) Attempt any two of the following : **[10]**

- a) Show that every cyclic code is an ideal in $F_q[x]/(x^n-1)$
- b) Describe perfect code and its examples.

c) Let $H = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array} \right]$

Assume that $V = (1001011)$ is transmitted and $r = (1001001)$ is received then find syndrome of r .

P.T.O.

Q4) A) Attempt any One of the following : [4]

- i) Prove that MDS code attains the singleton bound.
- ii) Find the parity check matrix for $(7, 4)$ Hamming code.

B) Attempt any One of the following : [5]

- i) Construct (15, 11) BCH code over $C_1 F(2)$
 - ii) By using neighbour decoding rule to decode
 - a) 01111
 - b) 11011for the binary code
- $c = \{01101, 00011, 10110, 11000\}$



Total No. of Questions : 4]

SEAT No. :

PD-3231

[Total No. of Pages : 3

[6474]-207

F.Y. M.A./M.Sc.

MATHEMATICS

MTS - 556 (D) MJ : Operations Research

(2023 Pattern) (Semester -II)

Time : 2 Hours]

[Max. Marks : 35

Instruction to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any Three of the following :

[6]

- a) Define Beale's Method in Quadratic programming.
- b) Write the dual of the following LPP

$$\text{Max } z = x + 2y$$

$$\text{Subject to constraints } 2x + 3y \geq 4$$

$$3x + 4y = 5$$

$$x \geq 0 \text{ and } y \text{ is unrestricted.}$$

- c) What is the role of artificial variables in the Big-M method?
- d) Write quadratic form of the following function

$$f(x_1, x_2, x_3) = x_1^2 + 3x_2^2 - x_3^2 + 4x_1x_2 + 5x_2x_3 - 6x_3x_1 \text{ in the matrix notation.}$$

Q2) Attempt any two of the following :

[10]

- a) Use two-phase simplex method to solve

$$\text{Max } Z = 5x_1 + 3x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 1$$

$$x_1 + 4x_2 \geq 6$$

$$x_1, x_2 \geq 0$$

P.T.O.

- b) What is sensitivity Analysis? How does it help in decision - making in linear programming?
- c) A packaged food manufacture produces two kinds of products chips and soda. The unit profit from a packet of chips is ₹ 80, and of a bottle of soda is ₹40. The goal of the plant manager is to earn a total profit of exactly ₹640 in the next week.

Q3) Attempt any two of the following. **[10]**

- a) Solve the following Linear Programming Problem Gomory's cutting plane method

$$\text{Max } Z = x_1 + 2x_2$$

Subject to the constraints $2x_1 \leq 7$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

$x_1, x_2 \geq 0$ and x_1, x_2 are integers

- b) Use penalty method to solve

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to the constraints $2x_1 + x_2 \leq 2$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0$$

- c) Discuss the Branch and Bound method with detailed example.

Q4) a) Attempt any one of the following. **[4]**

- i) Solve the following Non-linear Programming Problem using Kuhn-Tucker conditions

$$\text{Maximize } Z = x_1^2 - x_1x_2 - 2x_2^2$$

$$\text{Subject to } 4x_1 + 2x_2 \leq 24$$

$$5x_1 + 10x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

- ii) Explain Lagrange in multipliers method for Non-linear programming problem.

b) Attempt any one of the following:

[5]

i) Use Simplex method to solve

$$\text{Max. } Z = 3x_1 + 2x_2 + 5x_3$$

$$\text{Subject to constraints } x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_2 \leq 200$$

$$x_1 + 4x_2 \leq 420$$

$$x_1, x_2, x_3 \geq 0$$

ii) Explain Wolfe's Modified Simplex method.



Total No. of Questions : 4]

SEAT No. :

PD-3232

[Total No. of Pages : 2

[6474]-301

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-601MJ : Complex Analysis

(2023 Pattern) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any Three of the following :

[6]

- a) Show that z is purely real number if and only if $z = \bar{z}$
- b) Find the radius of convergence of $\sum_{n=0}^{\infty} a^n z^n, a \in \mathbb{C}$
- c) Evaluate $\int_{\gamma} \frac{e^{iz}}{z^2} dz$ Where $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$
- d) Evaluate cross ratios $(7+i, 1, 0, \infty)$

Q2) Attempt any two of the following :

[10]

- a) Let f and g be analytic on G and Ω respectively and suppose $f(G) \subset \Omega$ then show that $g \circ f$ is analytic on G and $(g \circ f)'(z) = g'(f(z)) f'(z)$ for all z in G .
- b) Calculate the square roots of i , i is Imaginary number ($i = \sqrt{-1}$)
- c) If f is bounded entire function then show that f is constant.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Let G be a region and let $f:G \rightarrow \mathbb{C}$ be a continuous function such that

$$\int_T f = 0 \text{ for every triangular path } T \text{ in } G \text{ then show that } f \text{ is analytic in } G.$$

b) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

- c) Prove that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i \sin n\theta$, $n \in \mathbb{Z}$

Q4) a) Attempt any one of the following :

[5]

- i) Let $f:G \rightarrow \mathbb{C}$ be analytic and suppose $\bar{B}(a,r) \subset G$ ($r > 0$). if $\gamma(t) = a + re^{it}$,

$$0 \leq t \leq 2\pi \text{ then show that } f(z) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(w)}{w-z} dw$$

- ii) Let G be an open set and let $f:G \rightarrow \mathbb{C}$ be a differentiable function then show that f is analytic on G .

b) Attempt any one of the following :

[4]

- i) Let f be a function defined on $(0,\infty)$ such that $f(x) > 0$ for all $x > 0$ suppose that f has the following properties

I) $\log f(x)$ is a convex function.

II) $f(x+1) = x f(x)$ for all x

III) $f(1) = 1$

Then show that $f(x) = \Gamma(x)$ for all x .

- ii) Is $u(x,y) = \log(x^2+y^2)^{1/2}$ harmonic on $G = \mathbb{C} - \{0\}$? Justify.



Total No. of Questions : 7]

SEAT No. :

PD-3233

[Total No. of Pages : 2

[6474]-302

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-603MJ : Field Theory

(2023 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right side indicates full marks.

Q1) Attempt any Five of the following :

[10]

- a) Define the characteristics of a field. Write the characteristics of $\mathbb{R}$ and $\mathbb{Q}$.
- b) Is x^2-2 separable over $\mathbb{Q}$? Justify.
- c) Show that any automorphism of a field K fixes it's prime subfield.
- d) Find the minimal polynomial for $\alpha=1+\sqrt{5}$ over $\mathbb{Q}$.
- e) Is x^2+x+1 irreducible over $\mathbb{Z}_2$? Justify.
- f) Find the smallest extension of $\mathbb{Q}$ having a root of $x^2-2 \in \mathbb{Q}[x]$.
- g) Define
 - i) Frobenius automorphism.
 - ii) Algebraic closure of a field F .

Q2) Attempt any two of the following :

[10]

- a) Prove that $F(\alpha) = F(\alpha^2)$ if $[F(\alpha) : F]$ is odd.
- b) If $p(x) \in F[x]$ is irreducible polynomial of degree n and $\theta = x \pmod{p(x)}$ then prove that $K = F[x]/\langle p(x) \rangle = \{a_0 + a_1\theta + \dots + a_{n-1}\theta^{n-1} \mid a_i \in F\}$ and hence $[K : F] = n$.
- c) Show that $[Q(\sqrt[6]{2}) : Q] = 6$ and hence prove that $x^3 - \sqrt{2}$ is irreducible polynomial over $Q(\sqrt{2})$.

P.T.O.

Q3) Attempt any two of the following : [10]

- a) If F is any field then prove that $F = \bar{F}$ if and only if F is algebraically closed.
- b) Show that the degree of a splitting field of n^{th} degree polynomial over a field F is at most $n!$ over F .
- c) Prove that the extension K/F is finite if and only if K is generated by a finite number of algebraic element over F .

Q4) Attempt any two of the following : [10]

- a) If K is algebraic over F and L is algebraic over K then prove that L is algebraic over F .
- b) Show that every irreducible polynomial over a finite field $\mathbb{F}^p$ is separable
- c) Show that the cyclotomic polynomial $\Phi_n(x)$ is a monic polynomial in $\mathbb{Z}(x)$ of degree $\phi(n)$

Q5) Attempt any two of the following : [10]

- a) Show that a polynomial $f(x)$ has multiple root α if and only if α is also root of $D_x(f(x))$.
- b) If L is an extension of F and α is an algebraic over both F and L then prove that, $M_{\alpha L}(x) | M_{\alpha F}(x)$ in $L[x]$.
- c) Prove that $(f(x))^p = (f(x^p))$ for $f(x) \in \mathbb{F}_p[x]$.

Q6) Attempt any two of the following : [10]

- a) State fundamental theorem of Galois theory.
- b) Let G be a finite subgroup of automorphism of a field K and F be a fixed then prove that $\text{Aut}(K/F) = G$ and the extension K/F is Galois with G as Galois group.
- c) If $G_1 \neq G_2$ are distinct finite subgroups of $\text{Aut}(K)$ then prove that their fixed field are distinct.

Q7) Attempt any two of the following : [10]

- a) Find $\text{Aut}(\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q})$
- b) Show that Galois group of $x^3 - 2 \in \mathbb{Q}[x]$ is a group of symmetric of triangle.
- c) Show that the irreducible polynomial $x^4 + 1 \in \mathbb{Z}[x]$ is reducible over $\mathbb{Z}_p$



Total No. of Questions : 7]

SEAT No. :

PD-3234

[Total No. of Pages : 3

[6474]-303

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 604 MJ : Differential Geometry

(2023 Credit Pattern) (Semester -III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following :

[10]

- a) Define maximal integral curve with an example.
- b) What is the height of the level set.
- c) Explain the difference between plane and a hyperplane.
- d) Give 2 examples of oriented 2-surface in $\mathbb{R}^3$.
- e) If $\alpha(t) = (t, 1+t)$. Find $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$.
- f) What is the range of the Gauss map?
- g) Define Weingarten map.

Q2) Attempt any two of the following :

[10]

- a) Define complete vector field and check wheather the vector field $X(x_1, x_2) = (x_1 x_2, 1, 0)$ is complete or not.
- b) Explain why an integral curve cannot cross itself as does the parametrized curve.
- c) If $f(x_1, x_2) = x_1^2 + x_2^2$ then sketch the level sets for $c = 0, 1, 4$.

P.T.O.

Q3) Attempt any two of the following. [10]

- a) Show that the velocity vector field along a parametrized curve $\alpha(t)$ in an n -surface S is parallel if and only if $\alpha(t)$ is a geodesic.
- b) Let $g : I \rightarrow \mathbb{R}$ is a smooth function and C be the graph of $g(t)$. Then show that the Curvature of C at a point $(t, g(t))$ is $g''(t)/(1+(g'(t)^2)^{\frac{3}{2}}}$.
- c) Show that the set S of all unit vectors at all points of $\mathbb{R}^2$ forms a 3-surface in $\mathbb{R}^4$

Q4) Attempt any two of the following. [10]

- a) If S is connected n -surface in $\mathbb{R}^{n+1}$. Then show that there exist on S exactly two smooth unit normal vector field N_1 and N_2 with $N_2(P) = -N_1(P)$ for all $P \in S$.
- b) Find the Global parametrization of the curve $(x_1 - a)^2 + (x_2 - b)^2 = r^2$
- c) State and prove Lagrange's Multiplier Theorem.

Q5) Attempt any two of the following. [10]

- a) If C is connected oriented plane curve and $\beta : I \rightarrow C$ is unit speed global parametrization of C . Then show that β is either one-to-one or periodic.
- b) Define geodesics. Hence, show that geodesics have constant speed.
- c) If S is an oriented 2-surface in $\mathbb{R}^3$ and $P \in S$. Then show that for each $u, v \in T_p S$, $L_p(u) \times L_p(v) = k(p)u \times v$.

Q6) Attempt any two of the following. [10]

- a) Explain the difference between local and global parametrization of the curve C .
- b) Explain how level sets is used in geography.
- c) State and prove any two properties of levi-civita parallel.

Q7) Attempt any two of the following.

[10]

- a) Show that the Weingarten map is self -adjoint.
- b) Show that the unit n -sphere $x_1^2 + x_2^2 + \dots + x_{n+1}^2 = 1$ is connected for $n > 1$.
- c) Show that Möbius band is an unorientable 2-surface in $\mathbb{R}^3$.



Total No. of Questions : 4]

SEAT No. :

PD-3235

[Total No. of Pages : 2

[6474]-304

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-611 (A) MJ : Mathematical Statistics

(2023 Pattern) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any Three of the following :

[6]

- a) Prove that, the probability of an impossible event is zero.
- b) If x is uniformly distributed over the interval $[0,10]$. Compute the probability that, $2 < x < 9$
- c) Define simple hypothesis and statistical hypothesis.
- d) Define correlation coefficient.

Q2) Attempt any two of the following :

[10]

- a) Define independent event. Show that, if A and B are independent events then A and $\bar{B}$ are also independent events.
- b) Show that, the moment generating function of a gamma random variable

$$x \text{ is } \left(\frac{\lambda}{\lambda - t} \right)^{\alpha}$$

- c) Calculate the correlation coefficient for the following heights (in inches) of father (X) and their sons (Y).

X	65	66	67	67	68	69	69	70	72
Y	67	68	65	68	72	72	72	69	71

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) A random variable x has the following probability distribution.

x	0	1	2	3	4	5	6	7
$\phi(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

- i) Find k
- ii) Evaluate $p(x \leq 6)$, $p(x \geq 6)$
If $p(x \leq c) > \frac{1}{2}$, Find the minimum value of c .
- b) State and prove Bayes Theorem.
- c) Prove that, coefficient is independent of change of origin and scale.

Q4) a) Attempt any one of the following :

[5]

- i) Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M that is, they arrive at 7:00, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30 Find the probability that, he walks.
- I) Less than 5 minutes for a buses.
- II) At least 12 minutes for a bus.
- ii) Derive the expression for the least square estimation for simple linear regression (B_0 and B_1)

b) Attempt any one of the following :

[4]

- i) If two dice are thrown, what is the probability that, the sum is
- I) Greater than 8 and
- II) Neither 7 nor 11?
- ii) Let $x_1, x_2, \dots, x_n$ be a random sample from a normal population with mean μ and variance σ^2 where μ and σ^2 are unknown We wish to test $H_0 : \mu = \mu_0$ (specified) against $H_1 : \mu \neq \mu_0$, $0 < \sigma^2 < \infty$, show that, the likelihood ratio test is same as two tailed t-test.



Total No. of Questions : 4]

SEAT No. :

PD-3236

[Total No. of Pages : 2

[6474]-305

S.Y. M.A./M.Sc.

MATHEMATICS

MTS 611(B) MJ : Algebraic Topology
(2023 Credits Pattern) (CBCS) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any Three of the following : [6]

- a) Define simply connected space.
- b) State the fundamental theorem of algebra.
- c) Define star convex set.
- d) State Borsuk-Ulam Theorem for S^2 .

Q2) Attempt any two of the following : [10]

- a) Prove that path homotopy relation is an equivalence relation.
- b) Let α be a path in X from x_0 to x_1 and β be a path in X from x_1 to x_2 . Show that if $\gamma = \alpha * \beta$, then $\hat{\gamma} = \hat{\beta} \circ \hat{\alpha}$
- c) Let $f : X \rightarrow Y$ be continuous and let $f(x_0) = y_0$. If f is a homotopy equivalence, prove that $f_* : \pi_1(X, x_0) \rightarrow \pi_1(Y, y_0)$ is an isomorphism.

Q3) Attempt any two of the following : [10]

- a) Let $p : E \rightarrow B$ be a covering map, with E simply connected. Show that, given any covering map $r : Y \rightarrow B$, there is a covering map $q : E \rightarrow Y$ such that $r \circ q = p$.
- b) Let X be the wedge of the circles S_α for $\alpha \in J$. Let p be the common point of these circles, then prove that $\pi_1(X, p)$ is a free group.
- c) Show that if $G = G_1 \oplus G_2$, where G_1 and G_2 are cyclic of orders m and n respectively, then m and n are not uniquely determined by G in general.

P.T.O.

Q4) a) Attempt any One of the following : [4]

- i) Define retraction. Prove that there is no retraction of B^2 onto S^1 .
- ii) Show that if $n > 1$, every continuous map $f : S^n \rightarrow S^1$ is nulhomotopic.

b) Attempt any One of the following : [5]

- i) Prove that the fundamental group of the torus has a presentation consisting of two generators α, β and a single relation $\alpha \beta \alpha^{-1} \beta^{-1}$.
- ii) Show that in a simply connected space X , any two paths having the same initial and final points are path homotopic.



Total No. of Questions : 4]

SEAT No. :

PD-3237

[Total No. of Pages : 2

[6474]-306

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 611(C) - MJ : Integral Transforms and Special Functions
(2023 Credit Pattern) (CBCS) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Obtain the Laplace transform of $f(t) = t^2 \sin 3t$.
- b) With usual notations, find $S_n\{t\}$.
- c) Evaluate $\Gamma\left(\frac{-1}{2}\right)$.
- d) Prove that $B(x, y+1) = \frac{y}{x+y} B(x, y)$.

Q2) Attempt any two of the following.

[10]

- a) Using partial fractions, find $L^{-1}\left\{\frac{s^2 - 8s + 15}{(s-1)(s-2)(s-3)}\right\}$.
- b) Solve the following initial value problem :
 $y''(t) + y'(t) - 6y(t) = e^t, y(0) = 1, y'(0) = 0$
- c) When n is integral, prove that $Y_{-n}(x) = (-1)^n Y_n(x)$

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Solve the integral equation and verify the solution if

$$y(t) = 1 + \int_0^t e^{\tau} y(t - \tau) d\tau$$

- b) Prove that, the general solution of $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (\lambda^2 x^2 - n^2) y = 0$ is

$$A J_n(\lambda x) + B y_n(\lambda x)$$

- c) Prove that $\Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma\left(x + \frac{1}{2}\right)$.

Q4) a) Attempt any one of the following.

[4]

- i) Prove that :

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

- ii) If $F_d(\omega)$ is the DFT of $f(t)$, then prove that :

$$\text{I) } F_d(\omega + 2\pi) = F_d(\omega)$$

$$\text{II) } F_d(2\pi - \omega) = F_d(-\omega)$$

- b) Attempt any one of the following.

[5]

- i) Prove that :

$$\text{I) } \frac{d}{dx} \{x^n K_n(x)\} = -x^n K_{n-1}(x)$$

$$\text{II) } \frac{d}{dx} \{x^{-n} K_n(x)\} = -x^{-n} K_{n+1}(x)$$

- ii) State and prove convolution theorem for Fourier transforms



Total No. of Questions : 4]

SEAT No. :

PD-3238

[Total No. of Pages : 2

[6474]-307

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 611 (D) MJ : Mechanics

(2023 Credit Pattern) (Semester -III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Find the extremal of the functional $I = \int_1^2 \left(\frac{x^3}{y^{1/2}} \right) dx$ subject to the conditions that $y(1) = 0$ $y(2) = 3$
- b) Explain Holonomic and Non holonomic constraints.
- c) What are Generalised co-ordinates?
- d) Explain the D'Alembert's principle.

Q2) Attempt any two of the following :

[10]

- a) Show that Lagranges equation of motion can also be written as
$$\frac{\partial L}{\partial t} - \frac{d}{dt} \left(L - \sum \dot{q}_j \frac{\partial L}{\partial \dot{q}_j} \right) = 0$$
- b) Prove that the extremal of the isoperimetric problem $I = \int_1^4 y^{1/2} dx$ subject to the condition $\int_1^4 y dx = 36$ and $y(1) = 3$, $y(4) = 24$ is a parabola.
- c) The length of simple pendulum changes with time such that $l = a + bt$, where a and b are constants. Find Lagrangian equation of motion.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Show that the Hamilton's principle $\delta \int_{t_0}^{t_1} L dt = 0$ also hold for non conservative system.
- b) Find the value of α and β so that equation $Q = q^\alpha \cos(\beta p)$, $p = q^\alpha \sin(\beta p)$ represent a Canonical transformation.
- c) State Hamilton's principle for non conservative system and Hence derive from it the Lagranges equation of motion for non conservative holonomic system.

Q4) a) Attempt any one of the following.

[4]

- i) Deduce Newton's second Law of motion from Hamilton's principle.
- ii) Derive the Hamilton's Canonical equations of motion from Hamiltonian function.

b) Attempt any one of the following.

[5]

- i) Use the Poission bracket, show that the transformation $q = \sqrt{2p} \sin Q$, $p = \sqrt{2p} \cos Q$, is canonical
- ii) Find the plane curve of fixed perimeter that encloses maximum area.



Total No. of Questions : 4]

SEAT No. :

PD-3239

[Total No. of Pages : 2

[6474]-401

M.A./M.Sc.

MATHEMATICS

MTS-651MJ : Functional Analysis
(2023 Credit Pattern) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Identify the sets $A = \{x \in \mathbb{R}^2 \mid \|x\|_1 = 1\}$
 $B = \{x \in \mathbb{R}^2 \mid \|x\|_\infty = 1\}$
- b) Prove that, the induced metric on a normed linear space X is translation invariant.
- c) Let $T : l^2 \rightarrow l^2$ be defined as $Tx = (a_1x_1, a_2x_2, \dots, a_nx_n, \dots)$ and $T \in BL(l^2)$. Then find T^* (adjoint of T)
- d) Define :
 - i) Normal operator on a Hilbert Space
 - ii) Hermitian operator on a Hilbert Space

Q2) Attempt any two of the following :

[10]

- a) Prove that, any Hilbert space has an Hilbert basis.
- b) Let $\langle H, \langle, \rangle \rangle$ be an inner product space. Show that $|\langle x, y \rangle| \leq \sqrt{\langle x, x \rangle} \cdot \sqrt{\langle y, y \rangle}$ and equality holds if and only if one of x and y is multiple of the other.
- c) Let H be a Hilbert space and $f : H \rightarrow \mathbb{K}$ be continuous linear functional. Show that there exists a unique $v \in H$ s.t $f(x) = \langle x, v \rangle$ for all $x \in H$.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Let X be a normed linear space over $\mathbb{R}$ and Y be any linear space. Let $F : Y \rightarrow \mathbb{R}$ be a continuous linear functional. Then Prove that there exists $g \in X^*$ such that $g(y) = f(y)$ for $y \in Y$ and $\|g\| = \|f\|$
- b) Let X and Y be Banach spaces. Let $T : X \rightarrow Y$ be a continuous linear operator such that T maps X onto Y . Prove that T is open.
- c) Show that dual of $(\mathbb{K}^n, \|\cdot\|_p)$ is $(\mathbb{K}^n, \|\cdot\|_q)$ where q is the conjugate index of p and $1 \leq p \leq \infty$

Q4) a) Attempt any one of the following :

[5]

- i) Let H be a complex Hilbert space and A and B are adjoint operators on $BL(H)$. Then prove that

$$\text{I) } (\alpha A + \beta B)^* = \bar{\alpha} A^* + \bar{\beta} B^* \text{ for, } \alpha, \beta \in \mathbb{C}, A, B \in BL(H)$$

$$\text{II) } (AB)^* = B^* A^* \forall A, B \in BL(H)$$

$$\text{III) } A^{**} = A \text{ for } A \in BL(H)$$

- ii) Let $\|\cdot\|_1$ and $\|\cdot\|_2$ be two equivalent norms on normed linear space X . Prove that the sequence (x_n) is Cauchy in $(X, \|\cdot\|_1)$ if and only if it is Cauchy in $(X, \|\cdot\|_2)$

b) Attempt any one of the following :

[4]

- i) Let $\langle H, \langle, \rangle \rangle$ be an inner product space. Show that the norm on $\langle H, \langle, \rangle \rangle$ satisfy parallelogram identity :

$$\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2), \forall x, y \in H$$

- ii) Let H be a finite dimensional space and $T : H \rightarrow H$ be compact and self-Adjoint operator show that one of the number $\pm\|T\|$ is an eigenvalue of T



Total No. of Questions : 7]

SEAT No. :

PD-3240

[Total No. of Pages : 3

[6474]-402

Second Year M.A./M.Sc.

MATHEMATICS

MTS - 653 - MJ : Partial Differential Equations and
Boundary Value Problems

(2023 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following :

[10]

- a) Explain the difference between linear and Non-linear Partial Differential Equation.
- b) When we say that, "System of linear PDE is Compatible"?
- c) Write a 2-dimensional Laplace equation in Polar form.
- d) Find the complete solution of $z^2(p^2 + q^2) = 1$.
- e) Explain the term : Canonical form of linear PDE.
- f) Write Lagrange's PDE of first order in 3-variables.
- g) Give an example of second order linear partial differential equation.

Q2) Attempt any two of the following :

[10]

- a) Form the PDE by eliminating the arbitrary function from

$$z = f(x + it) + g(x - it) \text{ where } i = \sqrt{-1}$$

- b) Explain the Charpit's method for finding the complete integral of a non linear PDE of first order of the form $f(x, y, z, p, q) = 0$.

- c) Classify and reduce the equation $y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$

to a Canonical form and solve it.

P.T.O.

Q3) Attempt any two of the following : **[10]**

- Derive the Diffusion equation of second order Partial Differential Equation.
- Find the Partial Differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.
- If $(b_i D' + c_i)$ is a factor of $F(D, D')u$ and $\phi_i(\xi)$ is an arbitrary function of a single variable ξ with $b_i \neq 0$. Then show that, $u_i = \exp\left(-\frac{c_i}{b_i} y\right) \phi_i(b_i x)$ is a solution of the equation $F(D, D') u = 0$.

Q4) Attempt any two of the following : **[10]**

- Find the solution of the equation $(2D - D' + 4) (D + 2D' + 1)^2 u = 0$.
- Find the Particular Integral (PI) of the PDE
 $(D^2 - 4DD' + 4D'^2) u = e^{2x+y}$
- Explain the method of solving second order Partial Differential Equation $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ under the condition that $B^2 - 4AC = 0$. Where coefficients are functions of x and y .

Q5) Attempt any two of the following : **[10]**

- Solve $u_{tt} - c^2 u_{xx} = f(x, t)$ subject to the initial condition $u(x, 0) = \eta(x)$ and $u_t(x, 0) = v(x)$
- Define Dirac delta function and hence show that it is an even function.
- Show that $u(x, t) = \phi(x - 3t) + \psi(x + 3t)$ is the solution of the equation $u_{tt} = 9u_{xx}$.

Q6) Attempt any two of the following : **[10]**

- Solve : $u_{xx} + u_{yy} = 0, 0 \leq x \leq a, 0 \leq y \leq b$
 BCS : $u(x, b) = u(a, y) = 0, u(0, y) = 0, u(x, 0) = f(x)$
 Using separation of variable method.
- Find the region in the x - y plane in which the following equation $[(x-y)^2 - 1]u_{xx} + 2u_{xy} + [(x-y)^2 - 1]u_{yy} = 0$ is hyperbolic.
- Show that the equations $p^2 + q^2 = 1$ and $(p^2 + q^2)x = pz$ are compatible and hence find its solution.

Q7) Attempt any two of the following :

[10]

- a) Show that a linear PDE of the type $\sum_i \sum_j a_{ij} x^i y^j \frac{\partial u^{i+j}}{\partial x^i \partial y^j} = f(x, y)$ can be reduced to a one with constant coefficients by the substitution $\xi = \log x, \eta = \log y$
- b) Show that $u(x, t) = Ae^{i(kx \pm wt)}$, where $i = \sqrt{-1}, k = \pm w/c$. is periodic solution of the wave equation.
- c) Show that $T(x, t) = \frac{1}{2\sqrt{\pi\alpha t}} \exp[-(x - \xi)^2 / 4\alpha t]$ is a solution of one dimensional diffusion equation $T_{xx} = \frac{1}{\alpha} T_t, -\infty < x < +\infty, t > 0$.



Total No. of Questions : 7]

SEAT No. :

PD-3241

[Total No. of Pages : 3

[6474]-403

M.A./M.Sc. (Part - II)

MATHEMATICS

MTS-654-MJ : Measure Theory and Integration
(2023 Credit Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any five of the following :

[10]

- a) Define lebesgue outer measure of an set.
- b) Show that, the constant function is measurable.
- c) Give an example where the strict inequality occurs in the Fatau's Lemma.
- d) Show that, $L^p(X, \mu)$ is a vector space over the real number.
- e) Show that, if $m^*(A) = 0$ then $m^*(A \cup B) = m^*(B)$ for any set B.
- f) Let $E_1 \supseteq E_2 \supseteq \dots \supseteq E_n$ show that, $\bigcup_{n=1}^{\infty} (E_1 - E_n) = E_1 - \bigcap_{n=1}^{\infty} E_n$
- g) Show that, if $F_n \rightarrow F$ in measure then $|F_n| \rightarrow |F|$ in measure.

Q2) Attempt any two of the following.

[10]

- a) State and prove Lindelöf's theorem.
- b) Prove that, every interval is measurable.
- c) Show that, for any set A and $\epsilon > 0$, there is an open set O containing A such that, $m^*(O) \leq m^*(A) + \epsilon$

P.T.O.

Q3) Attempt any two of the following. [10]

- a) Prove that, every non-empty open set G in $\mathbb{R}$ is the union of disjoint open intervals at most countable in number.
- b) Let $\{F_n\}$ be sequence of measurable function defined on the same measurable set then show that,
 - i) $\sup_{1 \leq i \leq n} F_i$ is measurable for each n .
 - ii) $\inf_{1 \leq i \leq n} F_i$ is measurable for each n .
 - iii) $\sup F_n$ is measurable
 - iv) $\limsup F_n$ is measurable
 - v) $\liminf F_n$ is measurable
- c) Show that, if $F \in \mathcal{M}$ and $m^*(F \Delta G) = 0$ then G is measurable.

Q4) Attempt any two of the following : [10]

- a) Let f and g be non-negative measurable function then, show that,
 $\int f dx + \int g dx = \int (f + g) dx$
- b) Let $\{F_n, n = 1, 2, \dots\}$ be a sequence of non-negative measurable function then show that $\liminf \int F_n dx \geq \int \liminf F_n dx$
- c) Show that, if F is non-negative measurable function then $F = 0$ almost everywhere if and only if $\int F dx = 0$

Q5) Attempt any two of the following : [10]

- a) Show that

$$\int_0^\infty \frac{\sin t}{e^t - x} dt = \sum_{n=1}^\infty \frac{x^{n-1}}{n^2 + 1}, -1 \leq x \leq 1$$
- b) Prove that, the function $F \in BV[a, b]$ if and only if F is difference of two finite valued monotone increasing function on $[a, b]$ where a and b are finite.
- c) If $F \in L[a, b]$ then show that,
 - i) $F(x) = \int_a^x f(t) dt$ is continuous on $[a, b]$
 - ii) $F \in BV[a, b]$

Q6) Attempt any two of the following.

[10]

- a) Let f and g be non negative measurable function. Prove that
- If $f \leq g$ then $\int f dx \leq \int g dx$
 - If A is measurable set and $f \leq g$ on A , then $\int_A f dx \leq \int_A g dx$
 - If $a \geq 0$ then $\int af dx = a \int f dx$
- b) Let $1 < p < \infty$, $1 < q < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$ and let $F \in L^p(\mu)$, $g \in L^q(\mu)$ then show that, $f \cdot g \in L^1(\mu)$ and
- $$\int |f \cdot g| d\mu \leq \left(\int |f|^p d\mu \right)^{1/p} \cdot \left(\int |g|^q d\mu \right)^{1/q}$$
- c) Let $[x, \delta, \mu]$ be a measure span with $\mu(x) = 1$. If ψ is convex on (a, b) where $-\infty < a < b < \infty$, and F is a measurable function such that, $a < F(x) < b$, for all x , then show that,
- $$\psi \left(\int F d\mu \right) \leq \int \psi_0 F d\mu$$

Q7) Attempt any two of the following.

[10]

- a) Show that, if $\mu(x) < \infty$ and $0 < p < q < \infty$ then show that $L^q(\mu) \leq L^p(\mu)$.
- b) If a sequence of measurable functions converges in measure, then show that the limit function is unique almost everywhere.
- c) Let $\{F_n\}$ be a sequence of measurable functions such that, $|F_n| < g$, an integrable function, and let $F_n \rightarrow F$ in measure, where F is measurable then show that, F is integrable,
- $$\lim \int F_n d\mu = \int F d\mu \text{ and } \lim \int |F_n - F| d\mu = 0$$



Total No. of Questions : 4]

SEAT No. :

PD-3242

[Total No. of Pages : 2

[6474]-404

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 661(A) - MJ : Commutative Algebra
(2023 Pattern) (Semester - IV) (CBCS)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Define the Jacobson radical of a ring A .
- b) Define integrally closed integral domain. Give an example.
- c) Let x be a nilpotent element of a ring A . Show that $1 + x$ is a unit in A .
- d) Give an example of a ring which satisfies neither A.C.C. nor D.C.C. on ideals.

Q2) Attempt any two of the following :

[10]

- a) Prove that every commutative ring $\neq 0$ with an identity element has at least one maximal ideal.
- b) Let A be a commutative ring $\neq 0$ with an identity element. Let M be an A - module. If M_1, M_2 are submodules of M , then prove that

$$(M_1 + M_2) / M_1 \cong M_2 / (M_1 \cap M_2)$$

- c) Show that

$$(\mathbb{Z} / m\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z} / n\mathbb{Z}) = 0$$

if m, n are coprime.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Let A be a ring, a an ideal and M an A -module. Show that

$$(A/a) \otimes_A M \cong M/aM.$$

- b) Let M be an A -module. Then prove that the following are equivalent:

- i) $M = 0$;
- ii) $M_P = 0$ for all prime ideals P of A ;
- iii) $M_m = 0$ for all maximal ideals m of A .

- c) Let a be a decomposable ideal and let $a = \bigcap_{i=1}^n q_i$ be a minimal primary decomposition of a . Let $p_i = r(q_i)$ ($1 \leq i \leq n$). Then prove that the P_i are precisely the prime ideals which occur in the set of ideals $r(a : x)$ ($x \in A$), and hence are independent of the particular decomposition of a .

Q4) A) Attempt any one of the following :

[4]

- a) Let A be a Artinian ring. Let M be finitely generated A module. Then prove that M is Artinian.
- b) If A is Noetherian and S is any multiplicatively closed subset of A , then prove that $S^{-1}A$ is Noetherian.

B) Attempt any one of the following :

[5]

- a) Let (B, g) be a maximal element of Σ . Then prove that B is a valuation ring of the field K .
- b) Prove that in an Artin ring A , every prime ideal is maximal.



Total No. of Questions : 4]

SEAT No. :

PD-3243

[Total No. of Pages : 2

[6474]-405
S.Y. M.A./M.Sc.
MATHEMATICS
MTS - 661(B) - MJ : Financial Mathematics
(2023 Pattern) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Symbols have their usual meanings.*

Q1) Attempt any three of the following : [6]

- a) What is the interest rate if a deposit subject to annual compounding is doubled after 10 years?
- b) Explain: Binomial Tree Model.
- c) Define :
 - i) Efficient portfolio
 - ii) Efficient frontier
- d) Define :
 - i) A forward contract
 - ii) A future contract.

Q2) Attempt any two of the following : [10]

- a) A sum of \$9, 000 paid into a bank account for two months (61 days) to attract simple interest will produce \$9, 020 at the end of the term. Find the interest rate r and the return on this investment.
- b) Given the following returns and assuming that $S(0)$ 45 dollars, find the possible stock prices in a three-step economy and sketch a tree of price movements :

Scenario	$K(1)$	$K(2)$	$K(3)$
ω_1	10%	5%	-10%
ω_2	5%	10%	10%
ω_3	5%	-10%	10%

- c) Given the initial wealth $V(0)$ and a predictable sequence $(x_1(n), \dots, x_m(n))$, $n = 1, 2, \dots$ of positions in risky assets, prove that it is always possible to find a sequence $y(n)$ of risk-free positions such that $(x_1(n), \dots, x_m(n), y(n))$ is a predictable self-financing investment strategy.

P.T.O.

Q3) Attempt any two of the following :

[10]

- a) Prove that the binomial tree model admits no arbitrage if and only if $d < r < u$.
- b) Let $u = 0.2$, $d = 0.1$ and $r = 0$. Find a risk-neutral probability. Draw a conclusion.
- c) Prove that the return K_V on a portfolio consisting of two securities is the weighted average $K_V = w_1 K_1 + w_2 K_2$, where w_1 and w_2 are the weights and K_1 and K_2 the returns on the two components.

Q4) a) Attempt any one of the following :

[4]

- i) Prove that the stock price at time $t = \tau n$ is given by $S(t) = S(0) \exp(mt + \sigma w(t))$.
- ii) Suppose that \$32, \$28 and x are the possible values of $S(2)$. Find x , assuming that stock prices follow a binomial tree. Can you complete the tree? Can this be done uniquely.

b) Attempt any one of the following :

[5]

- i) Compute the risk $\text{Var}(K_1)$, $\text{Var}(K_2)$ and $\text{Var}(K_3)$ in each of the following three investment projects, where the returns K_1 , K_2 and K_3 depend on the market scenario :

Scenario	Probability	Return K_1	Return K_2	Return K_3
ω_1	0.25	12%	11%	2%
ω_2	0.75	12%	13%	22%

Which of these is the most risky and the least risky project?

- ii) Find the optimal hedge ratio if the interest rates are constant.



Total No. of Questions : 4]

SEAT No. :

PD-3244

[Total No. of Pages : 2

[6474]-406
S.Y. M.A./M.Sc.
MATHEMATICS
MTS - 661(C) - MJ : Algebraic Curves
(2023 Credit Pattern) (CBCS) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following : **[6]**

- a) State Hilbert's Nullstellensatz.
- b) What is an affine variety? Provide an example.
- c) Define rational function on an affine variety.
- d) Define an ordinary multiple point of a projective plane curve.

Q2) Attempt any two of the following : **[10]**

- a) Show that there is only one conic passing through the five points $[0 : 0 : 1]$, $[0 : 1 : 0]$, $[1 : 0 : 0]$, $[1 : 1 : 1]$, $[1 : 2 : 3]$
- b) Describe the relationship between coordinate rings and affine varieties. Illustrate with an example.
- c) Show that $S = R[X]$, the ring of polynomials in one variable over a ring R , is ring-finite over R , but not module-finite over R ?

Q3) Attempt any two of the following : **[10]**

- a) Give an example of a countable collection of algebraic sets whose union is not algebraic.
- b) Let R be a domain that is not a field. Let R is Noetherian and local, and the maximal ideal is principal. Then prove that there is an irreducible element $t \in R$ such that every nonzero $z \in R$ may be written uniquely in the form $z = ut^n$, u a unit in R , n a nonnegative integer.
- c) Find the intersection number of the curves $A : Y - X^2$ and $B : Y^2 - X^3$ at the point $P = (0,0)$.

P.T.O.

Q4) a) Attempt any two of the following : [4]

- i) Find the multiple points and their multiplicities for the projective curve $F(X,Y,Z) = X^2 Y^3 + X^2 Z^3 + Y^2 Z^3$
- ii) Prove that $P = (0,0)$ is the only multiple point on the curve $C : (X^2 + Y^2)^2 + 3X^2 Y - Y^3 = 0$.

b) Attempt any one of the following : [5]

- i) Let $X \subset \mathbb{P}^n(k)$ be defined by a homogeneous ideal. Show that X is a projective algebraic set.
- ii) State and prove Bézout's theorem for curves in the projective plane.



Total No. of Questions : 4]

SEAT No. :

PD-3245

[Total No. of Pages : 4

[6474]-407

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-661(D) MJ : Optimization Techniques

(2023 Credit Pattern) (NEP-2020) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates :

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Explain the terms :
 - i) Carrying Cost
 - ii) Ordering Cost
- b) Write the Kuhn-Tucker necessary conditions for the following maximization problem

$$\text{Max. } Z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$$

Subject to the constraints :

$$x_1 + x_2 \leq 2$$

$$2x_1 + 3x_2 \leq 12$$

$$\text{and } x_1 \geq 0, x_2 \geq 0$$

- c) Draw the network diagram for the following relationship

Activity	(1-2)	(1-3)	(2-3)	(2-4)	(3-4)	(4-5)
Preceding Activity	-	-	(1-2)	(1-2)	(1-3),(2-3)	(2-4),(3-4)

P.T.O.

- d) Determine whether following two person zero sum game is strictly determinable or fair.

		Player B	
		I	II
Player A	I	1	1
	II	4	-3

Q2) Attempt any two of the following :

[10]

- a) Solve the following 2 x 4 game graphically.

Player A	Player B			
	B1	B2	B3	B4
A1	2	2	3	-2
A2	4	3	2	6

- b) Derive an EOQ formula with different rates of demand in different cycles.
- c) Solve the non-linear programming problem using the method of Lagrangian multiplier

$$\text{Min. } Z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$$

Subject to the constraints :

$$x_1 + x_2 + x_3 = 11$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

Q3) Attempt any two of the following :**[10]**

- a) A small project is composed of 9 activities whose time estimates are listed in the table below. Activities are identified by their beginning (i) and ending (j) node numbers.

Activity (i-j)	Optimistic (Weeks)	Most Likely (Weeks)	Pessimistic (Weeks)
(1-2)	3	6	15
(1-6)	2	5	14
(2-3)	6	12	30
(2-4)	2	5	8
(3-5)	5	11	17
(4-5)	3	6	15
(6-7)	3	9	27
(5-8)	1	4	7
(7-8)	4	19	28

- i) Draw the project network and identify all paths through it.
ii) Find expected duration and variance for each activity.
- b) Use Dominance principle to solve the following game:

Player A	Player B			
	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

- c) Prove that a necessary condition for a continuous function $f(X)$ with first and second order partial derivatives to have an extreme point (maximum or minimum) at $X = X_0$ is that each first order partial derivative of $f(X)$ exist at X_0 , vanish, that is $\nabla f(X_0) = 0$

where $\nabla = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n} \right)$ is the gradient vector.

Q4) a) Attempt any one of the following : **[4]**

- i) An aircraft company uses rivets at an approximate customer rate of 2500 kg. per year. Each unit costs Rs. 30 per kg. and the company personal estimate that it costs Rs. 130 to place an order and that the carrying cost of inventory is 10 % per year. How frequently should orders for revets be placed?
- ii) Determine the values of p and q that will make (A2, B2) a saddle point in the following game with pay-offs for player A.

	B1	B2	B3
A1	2	4	5
A2	10	7	q
A3	4	p	6

b) Attempt any one of the following : **[5]**

- i) Solve the non-linear programming problem using the Kuhn-Tucker conditions

$$\text{Max. } Z = 2x_1^2 + 12x_1x_2 - 7x_2^2$$

Subject to the constraint:

$$2x_1 + 5x_2 \leq 98$$

$$\text{and } x_1, x_2 \geq 0$$

- ii) Explain the four types of floats used in network Analysis.

