Total No. of Questions : 8]

SEAT No. :

P2689

[Total No. of Pages : 2

[5528]-11
M.A./M.Sc.
MATHEMATICS
MT-501 : Real Analysis - I
(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:
1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $X=\mathbb{R}^2$ and $d:X\times X\to\mathbb{R}$ defined by $d(x,y)=|x_1-y_1|+|x_2+y_2|$, where $x=(x_1, x_2)$, $y=(y_1, y_2)$ then show that d is a metric on X. **[6]**

b) Define normed linear space and give examples of two different norms on $\mathbb{R}^n$. **[5]**

c) Define interior of a set in a metric space (X, d). If A, B $\subset$ X then it is true that $\text{Int}(A\cup B)=\text{Int}(A)\cup\text{Int}(B)$? Why? **[5]**

Q2) a) If M_f denote $\{A_k\subset\mathbb{R}^n / D(A_k, A)\to 0 \text{ as } k\to\infty\}$ for some sequence in A_k in $\in$ then prove that M_J is a ring. **[6]**

b) State and prove Heine - Borel theorem. **[5]**

c) With usual notations prove that m^* is translation Invariant. **[5]**

Q3) a) Let $f:\mathbb{R}^n\to\mathbb{R}\cup\{\pm\infty\}$ then show that the following statements are equivalent **[6]**

i) $\{x/f(x)>a\}$ is measurable.

ii) $\{x/f(x)\geq a\}$ is measurable for any $a\in\mathbb{R}$.

b) If f is measurable function then prove that $|f|$ is also measurable. **[5]**

c) Let A, B, C be subsets of $\mathbb{R}^n$ then with usual notations prove that

$S(A,C)\subseteq S(A,B)\cup S(B,C)$ and $D(A,C)\subseteq D(A,B)+D(B,C)$. **[5]**

P.T.O.

Q4) a) State and prove monotone convergence theorem. **[6]**

b) Prove that Cantor set is a Lebesque measurable set and it has measure zero. **[5]**

c) Find Interior of $A=\left\{\frac{1}{n}\Big/ n\in N\right\}$ and $Q\subset\mathbb{R}$. **[5]**

Q5) a) Show that a constant function and continuous function both are measurable. **[6]**

b) State and Fatou's Lemma. **[5]**

c) Give an example of non-measurable function. **[5]**

Q6) a) State and prove Holder's inequality. **[6]**

b) Define counting measure and probability measure. **[5]**

c) Prove that $f = 0$, a.e. on E. **[5]**

if $\int_E f\, dm = 0$, $\forall$ $E\subset M$.

Q7) a) Show that classical Fourier series for $f(x) = x$ is $2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin(nx)$. **[8]**

b) Apply Gram smidth process to functions $1, x, x^2$, ... to obtain formulas for first three Legender polynomials. **[8]**

Q8) a) State and prove Banach contraction principle. **[8]**

b) State and prove Riesz - Fischer theorem. **[8]**

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Total No. of Questions :8]

SEAT No. :

P2690

[Total No. of Pages :3

[5528]-12
M.A./ M.Sc.
MATHEMATICS
MT-502: Advanced Calculus
(2008 Pattern) (Semester - I)

Time : 3Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that the composition of continuous functions is always continuous. **[6]**

b) A scalar field f is defined on $\mathbb{R}^n$ by the equation $f(\overline{x}) = \overline{a}.\overline{x}$, where $\overline{a}$ is a constant vector. Compute $f'(x\ ;\ y)$ for arbitrary x and y. **[5]**

c) Define the gradient vector of a scalar field f at $\overline{a}$. Find the gradient vector for the scalar field $f(x, y) = (x^2+y^2)\sin(xy)$ at $\left(1, \frac{\pi}{2}\right)$. **[5]**

Q2) a) State and prove chain rule for the derivatives of vector fields. **[8]**

b) Let $u = \frac{x-y}{2}$ and $v = \frac{x+y}{2}$ changes $f(u,v)$ into $F(x, y)$. Use an appropriate from of the chain rule to express the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ in terms of the partial derivatives $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$. **[4]**

c) Evaluate the directional derivative of $f(x, y, z) = \left(x/y\right)^z$ at (1, 1, 1) in the direction of $2\overline{i} + \overline{j} - \overline{k}$. **[4]**

P.T.O.

Q3) a) Show that the work done by a constant force depends on the endpoints and not on the path joining them. **[6]**

b) Evaluate $\int_c \frac{(x+y)dx-(x-y)dy}{x^2+y^2}$,

Where c is the circle $x^2+y^2 = 4$, traversed once in counter clockwise direction. **[5]**

c) Determine whether or not the vector field $\overline{f}(x,y)=3x^2y\overline{i}+x^3y\overline{j}$ is a gradient on any open subset of $\mathbb{R}^2$. **[5]**

Q4) a) State and prove the second fundamental theorem of calculus for line integrals. **[6]**

b) Evaluate $\iint_Q \left(\sqrt{y}+x-3xy^2\right)dxdy$ where $Q = [0, 1]\times[1, 3]$. **[5]**

c) Compute the volume of the solid enclosed by the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **[5]**

Q5) a) State the formula for change of variable in double integrals. Prove the formula for a particular case when the region of integration is rectangle and the function with constant value 1. **[6]**

b) Let f be defined on the rectangle $Q=[0, 1]\times[0, 1]$ as $f(x,y)=\begin{cases}1-x-y & \text{if } x+y\le 1\\ 0 & \text{otherwise}\end{cases}$ make a sketch of the ordinate set of f over Q and compute the volume of this ordinate set by double integration. **[5]**

c) Evaluate the line integral $\int_C \left(5-xy-y^2\right)dx-\left(2xy-x^2\right)dy$, where C is the square with vertices (0, 0), (1, 0), (1, 1) (0, 1) traversed counterclockwise. **[5]**

Q6) a) Prove the transformation formula $\iiint\limits_S f(x,y,z)\,dxdydz = \iiint\limits_T F(\sigma,\theta\phi)\,\sigma^2 \sin\phi d\rho d\theta d\phi$

where $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$, $z = \rho\cos\phi$, and $\rho > 0$, $0 \le \theta < 2\pi$, and $0 \le \phi < \pi$. Justify your steps. **[6]**

b) Let S be a parametric surface whose vector representation is $\bar{r}(u,v) = (u+v)\bar{i} + (u-v)\bar{j} + (1-2u)\bar{k}$ Find the fundamental vector product and the unit normal to the surface. **[5]**

c) Use a suitable linear transformation to evaluate the double integral

$$\iint\limits_S (x - y^2)\sin^2(x+y)\,dxdy$$

Where S is the parallelogram with vertices (π, 0), (2π, π), (π, 2π), (0, π). **[5]**

Q7) a) State and prove stoke's theorem . **[8]**

b) Determine the Jacobian matrix and compute the curl and divergence of vector field $\bar{F}(x,y,z) = xy^2z^2\bar{i} + z^2\sin y\,\bar{j} + x^2e^y\bar{k}$. **[8]**

Q8) a) Calculate the curl and divergence of a gradient of scalar field. **[8]**

b) i) Let $\bar{r}$ and $\bar{R}$ be smoothly equivalent functions related by the equation $\bar{R}(s,t) = \bar{r}(\bar{G}(s,t))$, where $\bar{G}(s,t) = U(s,t)\bar{i} + V(s,t)\bar{j}$ is a one-one continuously differentiable mapping of a region B in the st-plane onto a region A in the uv-plane. Prove that **[6]**

$$\frac{\partial \bar{R}}{\partial s} \times \frac{\partial \bar{R}}{\partial t} = \left(\frac{\partial \bar{r}}{\partial u} \times \frac{\partial \bar{r}}{\partial v}\right)\frac{\partial(u,v)}{\partial(s,t)}$$

Where the partial derivatives are evaluated at the point (*u(s, t)*, *v(s, t)*).

ii) State Gauss divergence theorem. **[2]**

Total No. of Questions : 8]

SEAT No. :

P2691

[Total No. of Pages : 4

[5528]-13

M.A./M.Sc.

MATHEMATICS

MT-503 : Linear Algebra

(2008 Pattern) (Semester-I)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*
3) *V denotes a finite dimensional vector space over the field* **K.**

Q1) a) Let X and Y be finite subsets of a vector space V. If Y is a linearly independent and $V = \langle X \rangle$, then prove that $|Y| \leq |X|$. **[6]**

b) Find dimensions of the following subspaces of the vector space $\mathbb{R}^{n \times n}$ of all $n \times n$ matrices over $\mathbb{R}$: **[6]**

$$W_1 = \left\{ A \in \mathbb{R}^{n \times n} \,/\, A = A^t \right\}$$
$$W_2 = \left\{ A \in \mathbb{R}^{n \times n} \,/\, A = -A^t \right\}$$
$$W_3 = \left\{ A \in \mathbb{R}^{n \times n} \,/\, \text{trace } A = 0 \right\}$$

c) Let $T : \mathbb{R}[x] \to \mathbb{R}[x]$ be defined by $T\big(p(x)\big) = x\, p(x)$. Show that T is a linear operator on $\mathbb{R}[x]$. Also if D is the differential operator on $\mathbb{R}[x]$, then show that DT–TD=I. **[4]**

P.T.O.

Q2) a) State and prove first and second isomorphism theorem for vector spaces. **[6]**

b) Let $W = \langle [1,2,1,0,1]^t, [1,0,1,1,1]^t, [1,2,1,3,1]^t \rangle$ a subspace of $\mathbb{R}^5$. Find a basis of $\mathbb{R}^5/W$. **[5]**

c) Give a one-one linear map $T : \mathbb{R}^2 \to \mathbb{R}^3$. Can this map be onto? Justify. **[5]**

Q3) a) Let W_1, ..., W_m be subspaces of a vector space V. If **[6]**

i) $V = W_1 + ...+ W_m$, and

ii) For each k, $W_k \cap \sum_{i \neq k} W_i = \{0\}$ then prove that $V \simeq W_1 \oplus ...... \oplus W_m$.

b) Let D be the differential operator on the vector space $\mathbb{R}_4[x]$. Find the matrix of D with respect to the basis $\{1, x, x^2, x^3, x^5\}$. **[5]**

c) Let A and B be similar matrices over $\mathbb{R}$. Show that A and B have same characteristic polynomials. **[5]**

Q4) a) State and prove primary decomposition theorem. **[6]**

b) Let A be a 5×5 matrix with minimal polynomial $x^3(x-1)$. What can be its characteristic polynomial? Is A diagonalizable? Is A invertible? **[5]**

c) Show that if λ is an eigen value of T and $p(x) \in k[x]$, then $p(\lambda)$ is an eigen value of $p(T)$. **[5]**

Q5) a) Define geometric multiplicity and algebraic multiplicity of an eigen value of an operator. Prove that the geometric multiplicity of an eigen value cannot exceed its algebraic multiplicity. **[6]**

b) Write all possible Jordan Canonical form of the matrix whose characteristic polynomial is $(x^3 - 1)^2$. **[5]**

c) What do you mean by a diagonalizable matrix. Give two non-diagonal 3×3 matrices A and B such that A is diagonalizable but B is not diagonalizable. **[5]**

Q6) a) Explain the rational canonical form of a matrix. Prove that two matrices are similar if and only if they have same rational canonical forms. **[6]**

b) Define an inner product on the vector space of $n \times n$ matrices over $\mathbb{C}$.

Show that $\left|tr\left(AB^*\right)\right| \leq \sqrt{tr\left(AA^*\right)tr\left(BB^*\right)} \leq \dfrac{tr\left(AA^*\right)+tr\left(BB^*\right)}{2}$ for

$A, B \in \mathbb{C}^{n \times n}$. **[5]**

c) Prove the polarization identities for the inner product space. **[5]**

Q7) a) State and prove Riesz representation theorem for finite dimensional inner product space. **[6]**

b) Let T be a self adjoint operator on an inner product space V. Prove that **[5]**

i) For all $v \in V$, $\langle Tv, v \rangle$ is real,

ii) If $\langle Tv, v \rangle = 0$, for all $v \in V$, then $T \equiv 0$.

c) Let T be a self adjoint operator on a finite dimensional inner product space V. Then prove that T is positive definite if and only if all eigen values of T are positive. **[5]**

Q8) a) Let V be a finite dimensional inner product space and let $T \in L(V)$. Then prove that the following statements are equivalent. **[6]**

i) T is unitary.

ii) $\langle Tu, Tv \rangle = \langle u, v \rangle$, for all $u, v \in V$.

iii) $\|Tu\| = \|u\|$, for all $u \in V$.

b) Find a polar decomposition of the following matrix. **[5]**

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

c) Prove that a bilinear form is reflexive if and only if it is either symmetric or alternating. **[5]**

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Total No. of Questions :8]

SEAT No. :

P2692

[Total No. of Pages :3

[5528]-14
M.A./ M.Sc.
MATHEMATICS
MT-504: Number Theory
(2008 Pattern) (Semester - I)

Time : 3Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and prove wilson's Theorem. **[6]**

b) Solve the congruence $x^2 + x + 7 \equiv 0$ (mod15). **[5]**

c) What are the last two digits of 3^{545}. **[5]**

Q2) a) Let p be a prime. Prove that $x^2 \equiv -1(\bmod p)$ has a solution if and only if p=2 or $p \equiv 1(\bmod 4)$. **[6]**

b) Solve the set of congruences $x \equiv 1(\bmod 4), x \equiv 0(\bmod 3)$, $x \equiv 5(\bmod 7)$ **[5]**

c) Exhibit reduced residue system modulo7 composed entirely of powers of 3. **[5]**

Q3) a) Let a be an integer and p be an odd prime and (a,p)=1. Consider the integers a, 2a, 3a,_____$\left(\frac{p-1}{2}\right)$a and their least positive residues modulo p. If n denotes the number of these residues that exceed $\frac{p}{2}$ then prove that $\left(\frac{a}{p}\right) = (-1)^n$. **[6]**

P.T.O.

b) Let a and b be integers and p be a prime, P>2, such that (a,p) = (b,p)=1. Prove that if $x^2 \equiv a(\text{mod } p)$ and $x^2 \equiv b(\text{mod } p)$ are not solvable then $x^2 \equiv ab(\text{mod } p)$ is solvable. **[5]**

c) Prove that 3 is quadratic residue of 13 but a quadratic non-residue of 7. **[5]**

Q4) a) Let Q be an odd positive integer then prove that **[6]**

i) $\left(\frac{-1}{Q}\right)=(-1)^{\frac{Q-1}{2}}$

ii) $\left(\frac{2}{Q}\right)=(-1)^{\frac{Q^2-1}{8}}$

b) Verify that $x^2 \equiv 10(\text{mod } 89)$ is solvable. **[5]**

c) Find all primes p such that $\left(\frac{-2}{p}\right)=1$. **[5]**

Q5) a) Let f(n) be a multiplicative function and let $F(n)=\sum_{d/n} f(d)$ then prove that F(n) is multiplicative function. **[6]**

b) Prove that $\prod_{d/n} d = n^{d(n)/2}$ for any positive integer n. **[5]**

c) For what real number x. **[5]**

i) $[x+3]=[x]+3$

ii) $[9x]=9$

iii) $[x+3]=x+3$

Q6) a) Let p be a prime, prove that the largest exponent e such that $P^e / n!$ is

$$e = \sum_{i=1}^{\infty}\left[\frac{n}{p^i}\right].$$ **[6]**

b) Evaluate $\sum_{j=1}^{\infty}\mu(j!)$ **[5]**

c) Find all integers x and y such that $147x+258y=369$. **[5]**

Q7) a) Let f(x) ba a monic polynomial with integral coefficients, f(x) = g(x) h(x) where g(x) and h(x) are monic polynomials with rational coefficients then prove that g(x) and h(x) has integral coefficients. **[6]**

b) Let α be an integer in $Q\left(\sqrt{m}\right)$ such that N (α) = ± p then prove that α is a prime in $Q\left(\sqrt{m}\right)$ **[5]**

c) If α and $\beta \neq 0$ are integers in $Q\left(\sqrt{m}\right)$ and α/β then prove that α/β and N(α)|N(β). **[5]**

Q8) a) Prove that the fields $Q\left(\sqrt{m}\right)$ for m= –1,–2,–3,–7,2,3 are Euclidean so have unique factorisation property. **[8]**

b) Prove that the reciprocal of a unit is a unit and the set of units in an algebraic number field form a multiplicative group. **[5]**

c) Prove that $\sqrt{3}+1$ and $\sqrt{3}-1$ are associates in $Q\left(\sqrt{3}\right)$. **[3]**

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Total No. of Questions :8]

SEAT No. :

P2694

[Total No. of Pages :2

[5528]-21
M.A./M.Sc.
MATHEMATICS
MT-601: General Topology
(2008 Pattern) (Semester - II)

Time : 3Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks***

Q1) a) If $\{T_\alpha\}_{\alpha \in J}$ is a collection of topologies on X then show that $\bigcap \tau_\alpha$ is again topology on X. Give an example to show that $\bigcup \tau_\alpha$ may not be a topology on X. **[8]**

b) Prove that countable union of countable sets is again countable. **[5]**

c) Define:Basis for a topology τ on X with an example. **[3]**

Q2) a) Prove that every compact Hausdorff space is regular. **[8]**

b) Define order topology on set X having simple order relation with more than one element.. **[5]**

c) State intermediate value theorem **[3]**

Q3) a) State and prove Pasting lemma. **[8]**

b) Show that every retraction map is a quotient map. **[5]**

c) State Tietz extension theorem. **[3]**

Q4) a) State and prove tube lemma. **[8]**

b) Prove that continuous image of connected set is again connected. **[5]**

c) Define: **[3]**

1) Lower limit Topology on $\mathbb{R}$
2) K-topology on $\mathbb{R}$
3) Upper limit Topology on $\mathbb{R}$

P.T.O.

Q5) a) State and prove Urysohns lemma. **[8]**

b) State and prove Lebesgue number lemma. **[8]**

Q6) a) State and prove Tychnoff's theorem. **[8]**

b) Let $\Delta = \{x \times x \mid x \in X\}$ be the set in X×X where X is a topological space. Then show that X is Hausdorff space if and only if Δ is closed in ×××. **[5]**

c) Prove that [0,1] and [1,2] are homeomorphic. . **[3]**

Q7) a) Prove that compactness implies limit point compactness. but not conversally. **[8]**

b) Let f:A $\rightarrow$ X×Y is a map given by f(a)= ($f_1(a), f_2(a)$) $\forall\, a \in$ A then prove that f is continuous if and only if f_1: A $\rightarrow$ X and f_2:A $\rightarrow$ Y are continuous **[8]**

Q8) a) Let $\{A\alpha\}\ \alpha \in J$ be the arbitrary collection of subsets of X, then show that $\bigcup_{\alpha\in J} \overline{A}_\alpha \subseteq \overline{\bigcup_{\alpha\in J} A_\alpha}$ but not conversally. **[8]**

b) Prove that continuous image of a compact set is gain compact. **[5]**

c) Show that every retraction map is quotient map. **[3]**

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Total No. of Questions : 8]

SEAT No. :

P2696

[Total No. of Pages : 2

[5528]-23
M.A./M.Sc.
MATHEMATICS
MT-603 : Groups and Rings
(2008 Pattern) (Semester-II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any Five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define normal subgroup. Give an example of a group which is non abelian but has a proper normal subgroup. Justify the answer. **[5]**

b) Find the inverse of the element $A=\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}$ in $GL(2,\mathbb{Z}_{11})$. **[5]**

c) Prove that a cyclic group is isomorphic to $\mathbb{Z}$ or $\mathbb{Z}_n$, for some $n\in\mathbb{N}$. **[6]**

Q2) a) Let H be finite subset of a group G. Then prove that H is a subgroup of G if and only if H is closed under the operation of G. Can $(\mathbb{Z},+)$ contain a finite subgroup other than (0)? Justify. **[5]**

b) Give examples of two non isomorphic groups of order 4 with justification. **[5]**

c) Prove that every subgroup of a cyclic group is cyclic. Moreover prove that, if $|\langle a\rangle| = n$, then the order of any subgroup of $\langle a\rangle$ is a divisor of n; and for each positive divisor k of n, the group $\langle a\rangle$ has exactly one subgroup of order k. **[6]**

Q3) a) Suppose that H is a proper subgroup of $\mathbb{Z}$ under addition and H contains 10 and 14. Determine H. **[5]**

b) Let G be a finite abelian group and let p be a prime that divides the order of G. Then prove that G has an element of order p. **[5]**

c) Let G be a finite group and p be a prime. If p^k divides |G|, then prove that G has at least one subgroup of order p^k. **[6]**

P.T.O.

Q4) a) Find the inverse and the order of each of the following permutations in S_{14}. **[5]**

i) (11 3 9) (2 4 10) (7 12 6 5) ii) (12 10 3 13 4) (7 1 5) (2 8 9).

b) If N is a normal subgroup of a group G and |G/N| = m, show that $x^m \in N$ for all x in G. **[5]**

c) State and prove the Lagrange's theorem for finite groups. Is the converse of the theorem true? Justify. **[6]**

Q5) a) State and prove the orbit stabilizer theorem. **[5]**

b) Is $(\mathbb{R},+)$ isomorphic to $(\mathbb{R}^*,\times)$? Justify your answer. **[5]**

c) If $\tau = (6\ 10\ 4)\ (5\ 11\ 9)$, $\rho = (5\ 6\ 8\ 3\ 1)\ (9\ 2\ 10) \in S_{19}$. Then find $\tau^{-1}\rho\tau$ and $\rho^{-1}\tau\rho$. **[6]**

Q6) a) Prove that if G is a group, then set of automorphisms of G, Aut(G) is a group. **[5]**

b) Determine all the homomorphisms from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{24}$. **[5]**

c) Find all the non isomorphic abelian groups of order 4900. **[6]**

Q7) a) Let H be an index 2 subgroup at group G. Prove that $a^2 \in H, \forall a \in G$. **[5]**

b) Find the group of inner automorphisms of dihedral group D_4 i.e. find $Inn(D_4)$. **[5]**

c) let G be a finite group and let p be a prime. Then prove that the number of Sylow p-subgroups of G is equal to 1 modulo p and divides |G|. Also prove that, any two Sylow p subgroups of G are conjugate. **[6]**

Q8) a) Suppose that G is an abelian group with an odd number of elements. Show that the product of all of the elements of G is the identity. **[5]**

b) Prove that the group of order 56 is not simple. **[5]**

c) Let p be a prime integer and let G be a group such $|G|=p^2$. Prove that G is abelian. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2697

[Total No. of Pages : 3

[5528]-24
M.A./M.Sc.
MATHEMATICS
MT-604 : Complex Analysis
(2008 Pattern) (Semester-II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Solve any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and prove Liouville's theorem. Using it, prove that sin z is an unbounded function on $\mathbb{C}$. **[6]**

b) Let R be the radius of convergence of the power series $\sum a_n z^n$. Prove that $R = \lim\left|\frac{a_n}{a_{n+1}}\right|$. **[5]**

c) Let $\gamma(t) = e^{it},\ 0 \le t \le 2\pi$. Then evaluate the integrals: **[5]**

i) $\int_\gamma \frac{e^{iz}}{z^2}\,dz$

ii) $\int_\gamma \frac{dz}{(z-2)^5}$

Q2) a) Let $u : \mathbb{C} \to \mathbb{C}$ be a harmonic function. Prove that u has a harmonic conjugate. **[6]**

b) Prove that if both f and $\overline{f}$ are analytic on $\mathbb{C}$, then f is constant. **[5]**

c) Prove that a Mobius transformation takes circles onto circles. **[5]**

Q3) a) Find a Mobius transformation which takes right half plane onto the unit disc. **[6]**

b) Find the radius of convergence for the following series: **[5]**

i) $\sum 3^n z^n$

ii) $\sum \frac{z^n}{n!}$

c) Show that if $f = u + iv$ is analytic on $\mathbb{C}$, then u, v and uv are harmonic functions. **[5]**

Q4) a) Let f be analytic in the disc B(a, R) and let γ be a closed rectifiable curve in B(a, R). Prove that $\int_{\gamma} f = 0$. **[6]**

b) Let f be analytic in B(a, R) and suppose $f(a) = 0$. Show that a is a zero of multiplicity m if and only if $f^{(m-1)}(a) = ... = f(a) = 0$ and $f^{(m)}(a) \neq 0$. **[5]**

c) Let G be simply connected and $f : \mathrm{G} \to \mathbb{C}$ be analytic in G. Prove that f has a primitive in G. **[5]**

Q5) a) State and prove the Casorati-Weierstrass theorem. **[6]**

b) Find a Mobius transformation which map the points $0, 1, \infty$ onto the points 0, 1/2, 1, respectively. **[5]**

c) Identify the analytic function f on the unit disc such that **[5]**

$$f(1/n) = \frac{n}{2n+1} \text{ for } n = 2, 3, \; .....$$

Q6) a) Suppose f has an isolated singularity at $z = a$. Prove that $z = a$ is a removable singularity if and only if $\lim_{z \to a}(z-a)f(z) = 0$. **[6]**

b) Classify the singularities of the following functions: **[5]**

i) $\dfrac{\sin z}{z(z-2)}$

ii) $\sin(1/z)$

c) Let f and g be entire functions such that $fg \equiv 0$. Prove that $f \equiv 0$ or $g \equiv 0$. **[5]**

Q7) a) State: **[6]**

i) Morera's theorem

ii) Open mapping theorem

iii) Goursat's theorem

b) Let f be analytic and non-vanishing in a region G. Prove that there is analytic g such that $f(z) = e^{g(z)}$. **[5]**

c) State and prove Schwarz's lemma. **[5]**

Q8) a) State Rouche's theorem. Using it, prove the fundamental theorem of algebra. **[6]**

b) Let f be a non-constant analytic function on a bounded open set G and is continuous on $\overline{G}$. Prove that either f has a zero in G or $|f|$ assumes its minimum value on the boundary of G. **[5]**

c) Using residue theorem, show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2698

[Total No. of Pages : 2

[5528]-25
M.A./M.Sc.
MATHEMATICS
MT - 605 : Partial Differential Equations
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Eliminate the arbitrary function from the function F $\left(x+y, x^2+y^2+z^2\right)$ and find the corresponding partial differential Equations. **[6]**

b) Find the general solution of $z(z-y)dx+z(z+x)dy+x(x+y)dz=0$. **[6]**

c) Define the following terms and example of each **[4]**

i) Linear equation

ii) Non-linear equation

Q2) a) Find general solution of $y^2p-xyq=x(z-2y)$. **[6]**

b) State the conditions for equation $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ to be compatible on the Domain D. **[4]**

c) Find the complete integral of $p^2+q^2=x+y$. **[6]**

Q3) a) If $h_1=0$ and $h_2=0$ are compatible with $f=0$, then, prove that h_1 and h_2 satisfy $\frac{\partial(f,h)}{\partial(x,u_x)}+\frac{\partial(f,h)}{\partial(y,u_y)}+\frac{\partial(f,h)}{\partial(z,u_z)}=0$ where $h_i, i=1,2$. **[6]**

b) Verify that the equation is integrable

$yz(y+z)dx+zx(x+z)\,dy+xy\,(x+y)dz=0$. **[6]**

c) State auxiliary equations of Jacobi's method of Non-linear partial differential equations. **[4]**

Q4) a) Reduce the equation $U_{xx}+2U_{xy}+17U_{yy}$ to canonical form and solve it. **[6]**

b) State and prove Kelvin's inversion theorem. **[6]**

c) Find the two initial strips of equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ which passes through X-axis. **[4]**

Q5) a) State and prove Harnack's theorem. **[6]**

b) Verify that the pfaffian differential equation is integrable and find its primitive of $(1+yz)dx+z(z-x)dy-(1+xy)dz=0$. **[6]**

c) Find auxiliary of equations $z^2=pqxy$ by charpit's method. **[4]**

Q6) a) Find the solution of the Heat-equation in an infinite rod which is defined as $\begin{matrix} U_t=kU_{xx}, & -\infty<x<\infty, t>0 \\ U(x,0)=f(x), & -\infty<x<\infty \end{matrix}$. **[6]**

b) State Dirichlet problem for rectangle and its solution. **[6]**

c) Classify the following equation into hyperbolic, elliptic, parabolic type $U_{xx}+2(1+\alpha y)U_{yz}=0$. **[4]**

Q7) a) Prove that the pfaffian differential equation $X\cdot dr=P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=0$ is integrable iff $X\cdot(nr)X=0$. **[8]**

b) Use Duhamel's principle and solve the non-homogeneous wave equation $U_{tt}-C^2U_{xx}=F(x,t), t>0$ with conditions $u(x,0)=f(x), 0<x<l$, $u_t(x,0)=g(x)$, $0<x<l$, $u(0,t)=u(1,t)=0, t>0$. **[8]**

Q8) a) Find the solution by method of characteristic, the integral surface of $pq=z$ which pass through curve $xz=a^2, y=0$. **[8]**

b) State Dirichlet's problem for rectangle and find it's solution. **[8]**

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Total No. of Questions : 4]

SEAT No. :

P2699

[Total No. of Pages : 2

[5528]-26
M. A./M. Sc.
MATHEMATICS
MT - 606 : Object Oriented Programming Using C++
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Question 1 is compulsory.***
2) ***Attempt any two from questions 2, 3, 4.***
3) ***Figures to the right indicate full marks.***

Q1) Attempt the following questions. **[20]**

a) Write a short note on function prototype.

b) What is data encapsulation?

c) What is use of scope resolution operator?

d) Write a function to read a matrix of size $m \times n$ from the keyboard using 'for' loop.

e) Write a function to find LCM of two numbers.

f) Give an example of structure in C++.

g) Write a note on operator 'new'.

h) Write a note on function overloading.

i) Which operator we cannot overload?

j) What is difference between private and protected members?

Q2) a) Define a class 'complex' having two data members 'real' and 'imaginary'. Overload necessary constructors, and overload operators + and –. Find addition and substraction of two complex numbers. **[10]**

b) Illustrate by example the use of virtual functions. **[5]**

Q3) a) Illustrate by example use of static member functions. **[10]**

b) Write a note on inline functions. **[5]**

Q4) a) Write a note on compile time polymorphism and run time polymorphism. **[10]**

b) Write a note on const member function. **[5]**

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Total No. of Questions :8]

SEAT No. :

P2700

[Total No. of Pages :3

[5528]-31
M.A./ M.Sc.
MATHEMATICS
MT-701: Functional Analysis
(2008 Pattern) (Semester - III)

Time : 3Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*
3) *Figures to the right indicate full marks.*

Q1) a) State and prove the uniform boundedness principle. **[8]**

b) Show that if the conjugate space of a normed linear space X is separable then X is separable. **[6]**

c) Show that the orthogonal subset of inner product space is linearly independent. **[2]**

Q2) a) Show that any two norms on a finite dimensional normed linear space are equivalent. **[4]**

b) Let X be a normed linear space and let E be a convex subset of X. Prove that interior E° and closure $\overline{E}$ of E is convex and also prove that If $E^\circ \neq \phi$, then $\overline{E} = \overline{E^\circ}$. **[6]**

c) Let k(s,t) be a square integrable function on the unit square [0,1]×[0,1]. For $x \in L^2[0,1]$, let $(Ax)(s) = \int_0^1 k(s,t)x(t)\,dt, 0 \le s \le 1$. Show that A is a bounded operator on $L^2[0,1]$. Show also that A is self adjoint if $k(s,t) = \overline{k(s,t)}$ for all (s,t) **[6]**

P.T.O.

Q3) a) Let X be a normed linear space, Y be a subspace of X and $g \in Y'$. Prove that there is some $f \in X'$ such that $f/Y = g$ and $||f|| = ||g||$. **[6]**

b) Prove that a normed space X is Banach if and only if every absolutely summable series of elements in X is summable in X. **[6]**

c) Does there exists a discontinuous linear function from an infinite dimensional normed linear space X to a normed linear space Y? Justify your answer. **[4]**

Q4) a) Let X be a normed space and $P : X \to X$ be a projection. Prove that P is closed map if and only if the subspaces $R(P)$ and $Z(P)$ are closed. **[6]**

b) Let $B(X, Y)$ be the set of all continuous linear maps from a normed linear space X to a normed linear space Y. Prove that if Y is Banach then $B(X, Y)$ is Banach. **[6]**

c) Let X and Y be normed linear spaces and $F : X \to Y$ be a linear map. Show that F is continuous at origin if and only if it is continuous on X. **[4]**

Q5) a) Let X be an inner product space. For all $x, y \in X$, prove that $|\langle x, y\rangle| \leq \langle x, x\rangle\langle y, y\rangle$ and equality holds if and only if the set $\{x, y\}$ is linearly dependent. **[6]**

b) Show that a nonzero normed space X is Banach if and only if the set $S = \{x \in X : \|x\| = 1\}$ is complete. **[6]**

c) Consider $X = (\ell^1, \|.\|_1)$, a normed linear space. Define $f : \ell^1 \to \mathbb{R}$ by $f(x) = \sum_{i=1}^{\infty} x_i$. Shows that f is a bounded linear functional with $||f|| = 1$. **[4]**

Q6) a) Let H be a Hilbert space and $A \in BL(H)$. Prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y\rangle = \langle x, B(y)\rangle$. **[6]**

b) Show that even a discontinuous linear map can have a closed graph. Does this contradict the closed graph theorem? Explain. **[6]**

c) Let F be a subspace of an inner product space X and $x \in X$. Show that $y \in F$ is a best approximation from F to x if and only if $x - y \perp F$. **[4]**

Q7) a) Let H be a Hilbert space and $A \in BL(H)$. Prove that the closure of $R(A)$ equals $Z(A^*)^{\perp}$ and the closure of $R(A^*)$ equals $Z(A)^{\perp}$. **[6]**

b) Let H be a Hilbert space and $A \in BL(H)$. Prove that $\|A^*\| = \|A\|$ and $\|A^*A\| = \|A\|^2$. **[6]**

c) Let T be an operator on a finite dimensional Hilbert space H and $M_1, M_2, ...M_n$ be all the eigenspaces of H. If T is normal, then prove that the $M_i's$ spans H. **[4]**

Q8) a) Let H be a Hilbert space. Prove the $A \in BL(H)$ can be expressed as $A = B + iC$, where B and C are self adjoint operators on H. **[6]**

b) If M is a linear subspace of a Hilbert space H, then show that M is closed if and only if $M = M^{\perp\perp}$. **[6]**

c) If T is an operator on a Hilbert space H with $<Tx, x> = 0$ for all $x \in H$ then prove that $T = 0$. **[4]**

Total No. of Questions :8]

SEAT No. :

P2701

[Total No. of Pages :2

[5528]-32
M.A./ M.Sc.
MATHEMATICS
MT-702: Ring Theory
(2008 Pattern) (Semester - III)

Time : 3Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***All the symbols have their usual meanings.***

Q1) a) State and prove first isomorphism theorem for rings. **[6]**

b) If F is a field, then show that $F[x]$ is an Euclidean domain. **[6]**

c) Show that a prime number p divides an integer of the form n^2+1 if and only if p is either 2 or $p\equiv 1 (\text{mod } 4)$. **[4]**

Q2) a) State and prove Chinese Remainder theorem for rings. **[6]**

b) Prove that every non-zero prime ideal in a PID is maximal ideal. **[6]**

c) Let F be a field and let $p(x) \in F[x]$. Then show that $p(x)$ has a factor of degree one if and only if $p(x)$ has a root in F. **[4]**

Q3) a) Let R be commutative ring with unity. Then show that an ideal P in ring R is prime ideal if and only if R/P is an integral domain. **[8]**

b) Prove that in an integral domain, a prime element is always irreducible. **[4]**

c) Give any two examples of module M over ring R. **[4]**

Q4) a) Let M be an R-module. Let A and B be submodules of M with $A \subset B$, then show that $(M/A)/ (B/A) \cong M/B$. **[10]**

b) Let R be an integral domain and let Q be the field of fractions of R. If a field F contains a subring R' isomorphic to R, then show that subfield of F generated by R' is isomorphic to Q. **[6]**

P.T.O.

Q5) a) Prove that in a ring with identity every proper ideal is contained in a maximal ideal. **[8]**

b) If R is any commutative ring such that $R[x]$ is PID, then prove that R is necessarily a field. **[6]**

c) Define free module along with suitable example. **[2]**

Q6) a) Is inverse image of maximal ideal under a ring homomorphism a maximal ideal? Is image of prime ideal under ring homomorphism a prime ideal? **[8]**

b) Show that ring $\mathbb{Z}[2i]$ is an integral domain but not UFD. **[4]**

c) If N is a submodule of M, then annihilator of N in R is defined as $Ann(N) = \{r \in R : rn = 0, \text{ for all } n \in N\}$. Then show that $Ann(N)$ is an ideal of R. **[4]**

Q7) a) Prove that every PID is UFD. **[8]**

b) Show that every ideal in a Euclidean domain is principal. **[6]**

c) Show that the polynomial $f(x) = x^4 + 10x + 5 \in \mathbb{Z}[x]$ is irreducible. (State all the results which will be used.) **[2]**

Q8) a) Is field (i) an integral domain (ii) PID? Justify your answer. **[4]**

b) Show that $\mathbb{R}[x]/ < x^2 + 1 >$ is field and $\mathbb{Z}[x]/ < x >$ is an integral domain. **[8]**

c) Let R be a ring and let M be an R-module. Show that a subset N of M is submodule of M if and only if $N \neq \phi$ and $x + \alpha y \in N$, for all $\alpha \in R$ and for all $x, y \in N$. **[4]**

Total No. of Questions : 8]

SEAT No. :

P2702

[Total No. of Pages : 3

[5528]-33
M.A./M.Sc.
MATHEMATICS
MT-703 : Mechanics
(2008 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Explain the terms: **[6]**

i) Canonical pendulum

ii) Generalized momentum

iii) Kepler's third law of planetary motion

b) A particle of mass m moves in one dimension such that it has the Langrangian $\mathrm{L} = \frac{m^2 x^4}{12} + mx^2 \mathrm{V}(x) - \mathrm{V}^2(x)$, where V is some differentiable function of x. Find the equation of motion for $x(t)$ and describe the physical nature of the system on the basis of this equation. **[4]**

c) Show that the Langrange's equation of Motion can also be written as

$$\frac{\partial \mathrm{L}}{\partial t} - \frac{d}{dt}\left(\mathrm{L} - \sum \dot{q}_j \frac{\partial \mathrm{L}}{\partial \dot{q}_j}\right) = 0.$$ **[6]**

Q2) a) Explain Atwood machine and discuss its motion. **[6]**

b) Find E-L differential equation satisfied by twice differentiable function $y(x)$ which extremizes the functional $\mathrm{I}(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\,dx$ where y is prescribed at the end points. **[8]**

c) Explain the Basic lemma. **[2]**

P.T.O.

Q3) a) State and prove the Principle of least action. **[8]**

b) Show that the kinetic energy of the system can always be written as the sum of three homogeneous functions of the generalized velocities. **[8]**

Q4) a) Describe the Routh's Procedure to solve the problem involving cyclic and non-cyclic co-ordinates. **[8]**

b) Prove that angular momentum of a particle in a central force field remains constant. **[4]**

c) Show that the Hamilton's Principle $\delta\int_{t_0}^{t} \mathrm{L}\,dt = 0$ also holds for the non-conservative system. **[4]**

Q5) a) Find the extremals for an isoperimetric problem $\mathrm{I}\left(y(x)\right) = \int_0^1 \left(y'^2 - y^2\right)dx$, subject to the conditions that $\int_0^1 y\,dx = 1,\ y(0) = 0,\ y(\pi) = 1$. **[7]**

b) For 2-D harmonic oscillator, the Hamiltonian is of the form $\mathrm{H}\left(x, y.\mathrm{P}_x, \mathrm{P}_y\right) = \frac{1}{2m}\left(\mathrm{P}_x^2 + \mathrm{P}_y^2\right) + \frac{1}{2}k\left(x^2 + y^2\right)$. Show that the quantity $\left(x\mathrm{P}_y - y\mathrm{P}_x\right)$ is conserved. **[7]**

c) Explain the Hamilton's Principle. **[2]**

Q6) a) Prove that central force motion is always motion in a plane. **[8]**

b) Prove the Kepler's first law of planetary motion. **[8]**

Q7) a) Show that the transformation $P = \frac{1}{2}\left(p^2 + q^2\right)$, $Q = \tan^{-1}\frac{q}{p}$ is canonical. **[8]**

b) Prove that Poisson brackets are invariant under canonical transformation. **[8]**

Q8) a) If the matrix of transformation form space set of axes to body set of axes is equivalent to a rotation through an angle χ about some axis through the origin then show that **[8]**

$$\cos\left(\frac{\chi}{2}\right) = \cos\left(\frac{\phi + \varphi}{2}\right).\cos\left(\frac{\theta}{2}\right).$$

b) Obtain the Lagrangian for the system of two bodies moving under central force field. **[8]**

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Total No. of Questions :8]

SEAT No. :

P2703

[Total No. of Pages :4

[5528]-34
M.A./ M.Sc.
MATHEMATICS
MT-704: Measure and Integration
(2008 Pattern) (Semester - III)

Time : 3Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *All symbols have their usual meanings.*

Q1) a) If E_i's are with $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that

$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$. **[6]**

b) Define a σ–algebra. Show that the class of measurable sets $\mathscr{M}$ is a σ–algebra. **[6]**

c) Show that every countable set has measure zero. **[4]**

Q2) a) Let $\{(X_\alpha, \mathscr{B}_\alpha, \mu_\alpha)\}$ be a collection of measurable spaces, and suppose that the sets $\{X_\alpha\}$ are disjoint and define

$$X = \bigcup X_\alpha,\ \mathscr{B} = \{B : (\alpha)[B \cap X_\alpha \in \mathscr{B}_\alpha]\} \text{ and } \mu(B) = \sum \mu_\alpha (B \cap X_\alpha).$$

i) Show that $\mathscr{B}$ is a σ-algebra. **[5]**

ii) Show that μ is a measure. **[5]**

b) Let c be any real number and let f and g be real valued measurable functions defined on the same measurable set E. Show that $f+c$, cf, $f+g$, $f-g$ and fg are measurable. **[6]**

P.T.O.

Q3) a) Let $\mathscr{R}$ be σ-ring and $\mathscr{B}$ be the smallest σ-algebra containing $\mathscr{R}$. if μ is measure on $\mathscr{R}$, define $\bar{\mu}$ on $\mathscr{B}$ by $\bar{\mu}E = \mu E$ if $E \in \mathscr{R}$ and $\bar{\mu}E = \infty$ if $E \in \mathscr{R}$.' Then show that $\bar{\mu}$ is a measure on $\mathscr{B}$. **[6]**

b) Show that for any set A and real number $\in > 0$, there is an open set O containing A such that $\mu^*(O) \le \mu^*(A) + \in$. **[5]**

c) Let E and F are measurable sets, $f \in L(E, \mu)$ and $\mu(E \Delta F) = 0$, then show that $f \in L(F, \mu)$ and $\int_F f\, d\mu = \int_F f\, d\mu$. **[5]**

Q4) a) Let $< f_n >$ be sequence of non-negative measurable functions that converges almost everywhere on a set E to a function f. Then show that $\int_E f \le \underline{\lim} \int_E f_n$. **[6]**

b) i) Show that, if f is integrable, then the set $\{x : f(x) \ne 0\}$ is a σ-finite measure. **[3]**

ii) Show that, if f is integrable, $f \ge 0$, then $f = \lim \phi_n$ for some increasing sequence of simple functions each of which vanishes outside a set of finite measure. **[3]**

c) If μ is a measure on a ring $\mathscr{R}$, then show that ρ defined by $\rho(A, B) = \mu(A \Delta B)$ is a pseudometric on $\mathscr{R}$. **[4]**

Q5) a) Let ν be a signed measure on the measurable space $(X, \mathscr{B})$ then prove that there is a positive set A and a negative set B such that $X = A \bigcup B \, and \, A \bigcap B = \phi$ **[6]**

b) Let $(X, \mathscr{B})$ be a measurable space and $< \mu_n >$ a sequance of measures on $\mathscr{B}$ such that $\mu_{n+1} E \geq \mu_n E$ and $\mu E = \lim_n \mu_n E$ for each E $\in \mathscr{B}$. Then show that μ is a measure on $\mathscr{B}$. **[6]**

c) Show that the function $x^{-1} \sin x$ is Riemann integrable on $(-\infty, \infty)$ but its Lebesgue integral does not exist. **[4]**

Q6) a) Let $(X, \mathscr{B}, \mu)$ be a σ-finite measure space, and let υ be a measure defined on $\mathscr{B}$ which is absolutely continuous with respect to μ. Then prove that there is a nonnegative measurable function f such that for each set E in $\mathscr{B}$ we have $\upsilon E = \int_E f \, d\mu.$

[6]

b) Let μ be a measure on an algebra $\mathscr{G}$ and μ^* the outer measure induced by μ. Then prove that the restriction $\overline{\mu}$ of μ^* to the μ^*– measurable sets is an extension of μ to σ –algebra containing $\mathscr{G}$. **[6]**

c) Define Hausdroff outer measure. Show that Hausdroff outer measure is invariant under translation. **[4]**

Q7) a) If μ is a measure on a ring $\mathscr{R}$ and the set function $\mu*$ is defined on $\mathscr{H}(\mathscr{R})$ by $\mu^*(E) = \inf\left[\sum_{n=1}^{\infty}\mu(E_n) : E_n \in \mathscr{R}, n = 1,2,\ldots, E \subset \bigcup_{n=1}^{\infty} E_n\right]$.

Then show that i) for E $\in \mathscr{R}$, $\mu^*(E) = \mu(E)$, ii) μ^* is an outer measure on $\mathscr{H}(\mathscr{R})$. **[5]**

b) If μ is a σ–finite measure on a aring $\mathscr{R}$ then show that it has unique extension to the σ–ring $\mathscr{S}(\mathscr{R})$. **[5]**

c) Let μ^* be a topologically regular outer measure on *X* then prove that each Borel set is μ^*-measurable. **[6]**

Q8) a) Define Borel set. Let E be a subset of X such that $E \bigcap K$ is a Borel set for each compact set *K*. Then show that *E* is a Borel set. **[6]**

b) Let μ be a finite measure defined on a σ–algebra $\mathscr{M}$ which contains all the Baire sets of locally compact space X.If μ is inner regular then show that μ is regular. **[4]**

c) Let μ^* be a topologically regular outer measure on *X*. Then each Borel set is μ^*–measurable. **[6]**

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Total No. of Questions :8]

SEAT No. :

P2704

[Total No. of Pages : 2

[5528]-35
M.A/M.Sc.
MATHEMATICS
MT 705 : Graph Theory
(2008 Pattern) (Semester -III)

Time : 3 Hours] *[Max. Marks : 80*

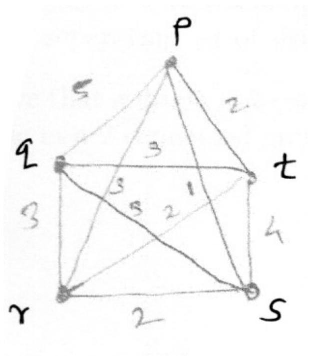
Instructions to the candidates:

1) ***Solve any five questions out of eight questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of non programmable scientific calculator is allowed.***

Q1) a) Prove that every u, v-walk contains a u, v-path. **[8]**

b) Show that every closed odd walk contains an odd cycle. **[5]**

c) Write down the Adjacency and Incidence matrices of the following graph G. **[3]**



'G'

Q2) a) Prove that if every vertex of a graph G has degree at least 2, then G contains a cycle. **[8]**

b) Prove that a simple graph and its complement cannot both be disconnected. **[5]**

c) Write down the graphs of **[3]**

i) K_{10}

ii) C_7

Q3) a) If G is a graph then show that $\sum_{u \in v(G)} d(v) = 2e(G)$ **[8]**

b) Prove that the nonnegative integers $d_1, \ldots, d_n$ are the vertex degrees of some graph if and only if $\sum di$ is even. **[8]**

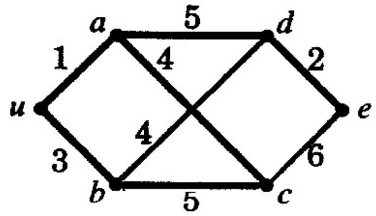
Q4) a) Prove that, there are n^{n-2} distinct labelled trees on n vertices. **[8]**

b) Show that every tree with at least two vertices has at least two leaves. Deleting a leaf from an n-vertex tree produces a tree with n-1 vertices. **[8]**

Q5) a) Show that Peterson Graph has diameter 2 **[8]**

b) Prove that in a connected weighted graph G, Kruskal's Algorithm constructs a minimum-weight spanning tree. **[8]**

Q6) a) By using Dijkstra's Algorithm find shortest route from vertex u to e. **[8]**



b) State and prove Hall's marriage theorem. **[8]**

Q7) a) Let G be an X,Y-bigraph having a matching that saturates X and Let S and T be subsets of X such that $|N(S)| = |(S)|$ and $|N(T)| = |(T)|$. Prove that $|N(S \cap T)| = (S \cap T)$ **[8]**

b) If G is a 3-regular graph, then prove that $K(G) = K'(G)$. **[8]**

Q8) a) Let G be an n-vertex simple graph other than Kn. Prove that if G is not k-connected, then G has a separating set of size k-1. **[8]**

b) Prove that a graph is 2-connected if and only if it has an ear decomposition. Furthermore, every cycle in a 2-connected graph is the initial cycle in some car decomposition. **[8]**

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Total No. of Questions :8]

SEAT No. :

P2705

[Total No. of Pages :2

[5528]-41
M.A./M.Sc.
MATHEMATICS
MT-801: Field Theory
(2008 Pattern) (Semester - IV)

Time : 3Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five question of the following.***
2) ***Figures to the right indicate full marks.***

Q1) a) If K is algebraically closed then prove that every irreducible polynomial in K[x] is of deqree L **[8]**

b) Let E be an extension of a field F. Define the group of F-automorphisms of E with an Example. **[5]**

c) Prove that $\sqrt{2}+\sqrt{3}$ is algebraic over $\mathbb{Q}$. **[3]**

Q2) a) Show that squaring the circle and doubling the cube are impossible by using rural and compass. **[8]**

b) If E is a finite extension of a field F then prove that $|G(E|F)| \leq [E:F]$.**[5]**

c) Show that IR is not a normal extension of $\mathbb{Q}$. **[3]**

Q3) a) If K is a splitting field of f(x) $\in$ F[x] over F then show that K is an algebraic extension of F **[8]**

b) Determine the minimal polynomial for the element 1+i over $\mathbb{Q}$. **[3]**

c) Examine whether the polynomial x^4+x+1 GQ[x] is a separable polynomial. **[5]**

Q4) a) Let E be a finite separable extension of a field F and H is a subgroup of G(E/F) then prove that $G(E/E_H)=H$ and $[E:E_H]=|G(E|E_H)|$. **[8]**

b) If [F(α):F] is odd then prove that $F(\alpha)=F(\alpha^2)$. **[5]**

c) Is $f(x)=x^2+x+1\in \mathbb{Z}_2[x]$ irreducible over $\mathbb{Z}_2$.? Justify. **[3]**

P.T.O.

Q5) a) Show that the polynomial $x^7-10x^5+15x+5$ is not solvable by radicals over $\mathbb{Q}$. **[8]**

b) Let $F=\mathbb{Q}\left(\sqrt{2}\right)$ and $E=\mathbb{Q}\left(\sqrt[4]{2}\right)$. Show that E is a normal extension of F and F is a normal extension of $\mathbb{Q}$ but E is not a normal extension of $\mathbb{Q}$ **[5]**

c) If f(x) is an irreducible polynomial over F then prove that f(x) has a multiple root iff $f^1(x)=0$ **[3]**

Q6) a) Show that the group G=G $\left(\mathbb{Q}\left(\sqrt[3]{2}\right)/\mathbb{Q}\right)$ of $\mathbb{Q}$-automorphisms of $\mathbb{Q}\left(\sqrt[3]{2}\right)$ is trivial. **[8]**

b) Let $\mathbb{Q}$ be a root of x^p-x-L over a field F of characteristic P then show that F(Q) is a separable extension of F. **[5]**

c) Construct a field with g-elements. . **[3]**

Q7) a) Show that Galois group of $x^4+1\in Q(x)$ is the klein four -group. **[8]**

b) If E is a Galois extension of F and K is any subfield of E containing F then prove that $K=E_{G(L\backslash K)}$ **[8]**

Q8) a) Show that $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right)=\mathbb{Q}\left(\sqrt{2}+\sqrt{3}\right)$ **[8]**

b) If L is algebraic over K and K is algebraic over a field F then prove that L is algebraic over F. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P2707

[Total No. of Pages : 2

[5528]-43
M.Sc./M.A.
MATHEMATICS
MT-803 : Differential Manifolds
(2008 Pattern) (Semester-IV)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define the differential operator d and for any k-form ω, show that $d(d\omega)=0$. **[8]**

b) Let A be open in $\mathbb{R}^k$ and let $f:\mathrm{A}\to\mathbb{R}$ be of class C^∞. Show that the graph of f is a k-manifold in $\mathbb{R}^{k+1}$. **[5]**

c) Define volume of parametrized surface in $\mathbb{R}^n$. **[3]**

Q2) a) Let U be an open set in $\mathbb{R}^n$ and $f:\mathrm{U}\to\mathbb{R}^n$ be of class C^r. Let $\mathrm{M}=\{x:f(x)=0\}$ and $\mathrm{N}=\{x:f(x)\geq 0\}$. If M is nonempty and D$f(x)$ has rank one at each point of M, then prove that N is an n-manifold in $\mathbb{R}^n$ and $\partial\mathrm{N}=\mathrm{M}$. **[8]**

b) Define an exact form and give an example. **[4]**

c) Give an example of a manifold without boundary. Justify. **[4]**

Q3) a) Define orientation of a manifold M and induced orientation on $\partial\mathrm{M}$. **[4]**

b) State Stoke's theorem. **[4]**

c) Suppose M is a 2-manifold in $\mathbb{R}^3$ whose intersection with the plane $z=t$ is the circle $(x-t)^2+(y-t)^2=1+t$; $z=t$ if $0\leq t\leq 1$ and is empty otherwise. Find the area of M. **[8]**

P.T.O.

Q4) a) What is the dimension of $A^k(V)$, the space of alternating k-tensors on an n dimensional vector space V? Justify. **[8]**

b) State Green's theorem for compact, oriented 2-manifold. **[4]**

c) Define a closed form and give an example. **[4]**

Q5) a) Let F be a k-tensor. With usual notation, if $AF = \sum_{\sigma \in S_k} (sign\ \sigma) F^{\sigma}$, then prove that AF is an alternating tensor. Find AF if F is already alternating. **[7]**

b) If $\omega = x^2 yzdx + xy^2 zdy + ze^y xdz$ and $\eta = yz\cos xdx + xyzdy + 4xyz^2 dz$, then find $(\omega \wedge \eta)$. **[5]**

c) Is [0, 1] × [0, 1] a manifold in $\mathbb{R}^2$? Justify. **[4]**

Q6) a) Let M be a k-manifold in $\mathbb{R}^n$. If ∂M is nonempty, then prove that ∂M is a k-1 manifold without boundary. **[7]**

b) If $\omega = x^2 y^2 z^2 dx + xz\sin ydy + e^x yzdz$ find $d\omega$. **[4]**

c) Let O(3) denote the set of all orthogonal 3 by 3 matrices. Show that it is a compact 3-manifold in $\mathbb{R}^9$ without boundary. **[5]**

Q7) a) If ω and η are k and l forms respectively, then prove that $d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$. **[8]**

b) Let $\omega = y^2 zdx + x^2 zdy + x^2 ydz$, and $\alpha(u,v) = (u - v, uv, u^2)$. Find $\alpha^*(d\omega)$. **[8]**

Q8) a) With usual notation, show that $\alpha^*(d\omega) = d(\alpha^* \omega)$. **[8]**

b) Let $A = \mathbb{R}^2 - \{0\}$. If $\omega = \dfrac{xdx + ydy}{x^2 + y^2}$, then show that ω is closed and exact on A. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P2708

[Total No. of Pages : 2

[5528]-44

M.A./M.Sc.

MATHEMATICS

MT-804 : Algebraic Topology

(2008 Pattern) (Semester-IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Solve any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that the relation of being of the same homotopy type is an equivalence relation. **[8]**

b) What is strong deformation retract? Show that S^n is a strong deformation retract of $\mathbb{R}^{n+1} \setminus 0$. **[4]**

c) Show that the unit closed solid n-sphere B^n is a contractible space. **[4]**

Q2) a) Let $x_0, x_1 \in X$. If there is a path in X from x_0 to x_1 then show that the groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **[8]**

b) Let $f : X \to Y$ be a continuous map of topological spaces. If $x_0 \in X$ then prove that there is a homomorphism $f^* : \pi_1(X, x_0) \to \pi_1(Y, f(x_0))$ induced by *f*. **[4]**

c) Show that a contractible space has trivial fundamental group. **[4]**

Q3) a) Show that fundamental group of circle is isomorphic to additive group of integers. **[9]**

b) The circle S^1 is not a retract of the disc B^2. **[4]**

c) What is the fundamental group of torus? Explain why? **[3]**

P.T.O.

Q4) a) Define the term covering space. Also give two examples of covering spaces with justification. **[6]**

b) Show that $f : S^1 \to S^1$ given by $f(z) = z^3$ is a covering projection. **[4]**

c) What is a local homeomorphism? Show that a covering map is local homeomorphism. **[6]**

Q5) a) Is every local homeomorphism a covering map? Justify your answer. **[4]**

b) Define the terms fibration and fiber and give two examples of each. **[4]**

c) Suppose that $p : E \to B$ is a fibration such that every fiber has no non-null path then show that p has unique path lifting. **[8]**

Q6) a) Show that two different complexes may have the same polyhedron. **[4]**

b) Show how to define simplicial mapping. Is simplicial mapping continuous? **[6]**

c) Show that $\mathbb{R}^n$ is homeomorphic to $\mathbb{R}^m$ if and only if $n = m$. **[6]**

Q7) a) Let $f : X \to Y$ be a homeomorphism and X has the fixed point property. Then show that Y has the fixed point property. **[8]**

b) Explain the terms with suitable examples: Geometrically independent set in $\mathbb{R}^n$; Simplex; Simplicial complex; oriented simplex. **[8]**

Q8) a) Define the term homotopy equivalent spaces. Also give examples of spaces that are homotopy equivalent but are not homeomorphic. **[4]**

b) Let X be a topological space and $x_0 \in X$. Define a group operation on elements of $\pi_1(X, x_0)$ and show that the operation is a well defined operation. **[6]**

c) Let X and Y be of the same homotopy type and $f : X \to Y$ be a homotopy equivalence. Then show that $f^* : \pi_1(X, x) \to \pi_1(Y, f(x))$ is an isomorphism for any $x \in X$. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2709

[Total No. of Pages : 3

[5528] - 45

M. A. / M. Sc.

MATHEMATICS

MT- 805: Lattice Theory

(2008 Course) (Old) (Semester - IV)

Time : 3.00 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that $N_5 \cong L \times K$ implies that L or K has only one element. **[5]**

b) Let the algebra (L, $\wedge, \vee$) be a lattice, set $a \leq b$ if and only if $a = a \wedge b$ then prove that (L, $\leq$) is a poset and as a poset it is lattice. **[5]**

c) Draw all non isomorphic lattices with 6 elements. **[6]**

Q2) a) Let Θ be a congruence relation on L, and $a \in L$ then prove that $[a]\,\Theta$ is a convex sublattice. **[5]**

b) Prove that sublattice of modular lattice is modular. **[4]**

c) Let L be a lattice. Prove that subset I of L is an ideal if and only if following two conditions hold. **[7]**

i) $a, b \in I$ implies that $a \vee b \in I$,

ii) $x \in L$ and $a \in I$ with $x \leq a$ implies that $x \in I$.

Q3) a) Define a congruence relation and show that a reflexive binary relation θ on a lattice L is a congruence relation if and only if the following three properties are satisfied, for $x, y, z \in$ L: **[9]**

i) $x \equiv y\ (\theta)$ if and only if $x \wedge y = x \vee y\ (\theta)$.

ii) $x \leq y \leq z$ $x \equiv y\ (\theta)$, and $y \equiv z\ (\theta)$ imply that $x \equiv z\ (\theta)$.

iii) $x \leq y$ $x \equiv y(\theta)$ imply that $x \wedge t \equiv y \wedge t(\theta)$ and $x \vee t \equiv y \vee t(\theta)$.

b) Prove that I is a prime ideal of L if and only if there is a homomorphism ϕ of L onto C_2 with $I = \phi^{-1}\ (0)$. **[7]**

P.T.O.

Q4) a) Let L, L_1, K, K_1 be lattices with $L \simeq L_1$ and $K \simeq K_1$, then prove that $L \times L_1 \simeq K_1 \times K$. **[6]**

b) Prove that collection of all complemented elements in a distributive lattice forms a sublattice. **[4]**

c) If in a poset P, $\wedge$ H exists for all H $\subset$ P then prove that P is complete lattice. **[6]**

Q5) a) Let L be a lattice and let $a \in L$ then prove that following conditions are equivalent. **[5]**

i) a is standard.

ii) Let α_a be the binary relation on L defined as : $x \equiv y\ (mod \alpha_a)$ if $(x \wedge y) \vee a_1 = x \vee y$ for some $a_1 \leq a$. Then α_a is a congruence relation.

iii) a is distributive and $a \vee x = a \vee y$ and $a \wedge x = a \wedge y$ imply that $x = y$.

b) Let L be distributive lattice, $a, b \in L$ and $a \neq b$, then prove that ther exist a prime ideal P of L containing exactly one of a and b. **[6]**

c) Show that the ideal lattice, $ID(L)$ is conditionally complemented for any lattice L. **[5]**

Q6) a) Prove that a lattice L is modular if and only if it does not contain a sublattice isomorphic to pentagon (N_5). **[10]**

b) Let L be a lattice of finite length. If L is semimodular then prove that any two maximal chains of L are of the same length. **[6]**

Q7) a) Prove that in a distributive lattice, $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ together implies that $x = y$. **[3]**

b) State and prove the Jordan-HÖlder theorem for semimodular lattices. **[7]**

c) State and prove the Stones separation theorem. **[6]**

Q8) a) Let L be a semimodular lattice. If p and q are atoms of L, $a \in L$, and $a < a \vee q \leq a \vee p$, then prove that $a \vee p = a \vee q$. **[6]**

b) Let $I = \{a_1, ..., a_n\}$ be a set of n atoms of a semimodular lattice. Then prove that the following conditions are equivalent. **[10]**

i) I is independent.

ii) $(a_1 \vee ... \vee a_i) \wedge a_{i+1} = 0$, for $i = 1, 2, ..., n - 1$.

iii) $h(a_1 \vee ... \vee a_n) = n$.

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