Total No. of Questions : 8]

SEAT No. :

P250

[Total No. of Pages : 3

[5828]-101
M.A./M.Sc.
MATHEMATICS
MTUT 111 : LINEAR ALGEBRA
(2019 Pattern) (Credit System) (Semseter - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) Attempt any five questions.

2) Figures to the right side indicate full marks.

Q1) a) Prove that, a linear transformation $T \in L(V, V)$ is invertible if and only if T is one to one and onto. **[7]**

b) Let $S \in L(V, V)$ be given by **[4]**

$S(u_1) = u_1 + u_2$

$S(u_2) = -u_1 - u_2$

Where, $\{u_1, u_2\}$ is a basis for V.

i) Find the rank and nullity of s.

ii) Check S is invertible or not

c) Test the linear transformation $T : R_3 \to R_2$ defined by the system of equation $y_1 = x_1 - 2x_2 + x_3$

$y_2 = x_1 + x_3$

Determine whether the system T is one to one. **[3]**

Q2) a) If s is a subspace of V containing the vectors $a_1, \ldots\ldots a_m$, then every linear combination of $a_1, \ldots\ldots, a_m$ belongs to s. **[6]**

b) Let A, B, C be points in R_2. Then prove that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$. **[4]**

c) Determine whether the following set of points are vertices of parallelogram or not **[4]**

$\langle 0,0\rangle, \langle 1,1\rangle, \langle 4,2\rangle, \langle 3,1\rangle$

Q3) a) State and prove the cauchy-schwarz Inequality. **[4]**

b) For vectors $a = \langle \alpha_1, \alpha_2\rangle, b = \langle \beta_1, \beta_2\rangle$ Define their inner product **[3]**

$(a, b) = \alpha_1 \beta_1 + \alpha_2 \beta_2$

Show that, the inner product satiesfies the following

i) $(a, b) = (b, a)$

ii) $(a, b + c) = (a, b) + (a, c)$

P.T.O.

c) Show that, the functions fn (x) = sin nx, n = 1, 2, ... form an orthonormal set in the vector space c([–π, π]) of continuous real valued functions on the closed interval [–π, π] with respect to the inner product

$$(f \cdot g) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot g(x)\,dx$$

for continuous functions f, g $\in$ c ([–π, π]) **[7]**

Q4) a) Let V be the vector space over F and suppose there exist non-zero linear transformations $\{E_1, ..., E_s\}$ in L(V, V) such that the following conditions are satiesfied. **[7]**

i) $1 = E_1 + E_2 + ... + Es$

ii) $E_i E_j = E_j E_i = 0$, if $i \neq j$, $1 \leq i , j \leq s$

Then show that,

$E_i^2 = E_i$, $1 \leq i \leq s$ and

V is the direct sum

$V = E_1 V \oplus E_2 V \oplus ...... \oplus E_s V$ and each subspace $E_i V$ is different from zero. **[7]**

b) Let T be linear transformation on a vector space over the complex number such that

$T(v_1) = v_1 + 2v_2$

$T(v_2) = 4v_1 + 3v_2$

Where $\{v_1 = (1, 0), v_2 = (0, 1)\}$ is a basis for the vector space then find

i) Characteristic polynomial of T.

ii) Minimal polynomial of T.

iii) Characteristic roots of T.

iv) Characteristic vector of T.

Q5) a) Let U and V be finite dimensional vector spaces over F. Let $S_1, S_2 \in L(U,U)$ and Let $T_1, T_2, \in L(V_1 V)$ Then prove that, **[5]**

$(S_1 \otimes T_1)(S_2 \otimes T_2) = S_1 S_2 \otimes T_1 T_2$

b) Find the perpendicular distance from the point (1, 5) to the line passing through the points (1, 1) and (–2, 0) by using Gram schmidt process. **[6]**

c) Find the rational canonical form over the field of rational numbers of matrix A,

where $A = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix}$ **[3]**

Q6) a) Let V be a vector space over a field F and let Y be a subspace of V. Then prove that, the relation $\Re$ on the set V defined by $v \Re v'$ if $v - v' \in Y$ is an equivalence relation. **[6]**

b) Let $T \in L(V_1\ V)$, let $\{V_1, \ldots\ldots, V_n\}$ be a basis of V and $\{f_1, \ldots\ldots f_n\}$ the dual basis of V*, Let A be the matrix of T with respect to the basis $\{V_1, \ldots\ldots\ldots V_n\}$. Then prove that, the matrix of T* with respect to the basis $\{f_1, \ldots\ldots, f_n\}$ is the transpose matrix ${}^tA$. **[6]**

c) If $f(x_1, x_2) = x_1^2 - 6x_1x_2 - 5x_2^2$

Find symmetric matrix A whose quadratic equation is $f(x_1, x_2)$. **[2]**

Q7) a) If V be a vector space over an algebraically closed field F, then prove that every irreducible invariant subspace w relative to $T \in L(V, V)$ has dimension1. **[5]**

b) Compute $(A_1 \dot{\times} B_1)(A_2 \dot{\times} B_2)$ **[5]**

where, $A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$A_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$

c) Define an Unitary transformation **[4]**

Show that, $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

is unitary matrix

Q8) a) Let T be an invertible linear transformation on a vector space V over C with Hermition scalar product then prove that T can be expressed in the form T = US, where S is positive and U is unitary. **[7]**

b) Let T be a normal transformation on V. Then prove that, there exist common characteristic vectors for T and T'. For such a vector v, Tv = av and T'v = $\bar{a}$ v **[4]**

c) Test the matrix $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$ is similar to diagonal matrix in $M_2(R)$. If so, find the matrix D and S such that $D = S^{-1}AS$. **[3]**

♣ ♣ ♣

Total No. of Questions : 8]

SEAT No. :

P251

[Total No. of Pages : 3

[5828]-102
M.A./M.Sc.
MATHEMATICS
MTUT 112 : Real Analysis
(CBCS) (2019 Pattern) (Semseter - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) i) Let f and g are measurable functions on E. Prove that $f \cup g$ and $f \cap g$ are measurable. **[4]**

ii) Prove that countable set has outer measure zero. **[3]**

b) i) Let f be a Lipschitz function on $[a, b]$ show that f is of bounded variation of $[a, b]$ and $T_v(f) \leq c\,(b-a)$. Where $|f(u) - f(v)| \leq c|u-v|$, for all u, v in $[a, b]$. **[4]**

ii) Let f be a function defined on [0, 1] as

$$f(x) = \begin{cases} x\sin\left(\dfrac{\pi}{x}\right) & , \ if \ \ 0 < x \leq 1 \\ 0 & , \ if \ \ \ \ x = 0 \end{cases}$$

show that f is not of bounded variation **[3]**

Q2) a) Prove that any set E of real numbers with positive outer measure contains a subset that fails to be measurable. **[7]**

b) Let $\{E_k\}^{\infty}_{k=1}$ is any countable collection of sets which are disjoint or not, then prove that $m^*\left(\bigcup_{k=1}^{\infty} E_k\right) \leq \sum_{k=1}^{\infty} m^*(E_k)$ **[7]**

Q3) a) Let f be a simple function defined on E. Then prove that for each $\in > 0$, there is a continuous function g on $\mathbb{R}$ and a closed set F contained in E for which $f \equiv g$ on F and $m(E \sim F) < \in$. **[7]**

b) Let the function f have a measurable domain E, then the following statements are equivalent. **[7]**

i) For each real number c the set $\{x \in E / f(x) > c\}$ is measurable.

ii) For each real number c, the set $\{x \in E / f(x) \geq c\}$ is measurable.

iii) For each real number c, the set $\{x \in E / f(x) < c\}$ is measurable.

Q4) a) Let the function f be monotone on the closed, bounded interval [a,b]. Then prove that f is absolutely continuous on [a,b] if and only if

$$\int_a^b f' = f(b) - f(a)$$ **[7]**

b) Define, $f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x^2}\right), \text{if } x \neq 0, \\ 0, \text{if } x = 0, \quad x \in [-1,1] \end{cases}$

Is f of bounded variation on [-1,1]? Justify. **[7]**

Q5) a) Let $\{E_k\}_{k=1}^{\infty}$ is a countable disjoint collection of measurable sets then prove that $\bigcup_{k=1}^{\infty} E_k$ is also measurable and $m\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} m(E_k)$. **[7]**

b) Let $\{f_n\}_{n=1}^{\infty}$ be an increasing sequence of continuous functions on $[a, b]$ to function f on $[a, b]$. Show that $\{f_n\}_{n=1}^{\infty}$ converges uniformly on $[a, b]$. **[7]**

Q6) a) Let f be an extended real valued function on E. **[7]**

i) If f is measurable on E and $f = g$ almost every where on E. Prove that g is measurable.

ii) For a measurable subset D of E, prove that f is measurable on E if and only if the restrictions of f to D and $E \sim D$ are measurable.

b) Define an absolutely continuous function. With a suitable example prove that the function f is absolutely continuous but not Lipschitz's on closed and bounded interval. **[7]**

Q7) a) Let the function *f* be continuous on the closed, bounded interval [a, b]. The family of divided difference functions $\{\text{Diff}_h f\}_{0<h\leq 1}$ is uniformly integrable over [*a*, *b*] then prove that *f* is absolutely continuous on [*a*, *b*]. **[7]**

b) Let A and B are any two disjoint subsets of $\mathbb{R}$. Show that $m^*(A \cup B) = m^*(A) + m^*(B)$. **[7]**

Q8) a) Prove that the Cantor set C is closed, uncountable set of measure zero. **[7]**

b) Let E be a measurable set of finite outer measure. For each $\in > 0$ there is a finite disjoint collection of open intervals $\{I_k\}^{\infty}_{k=1}$ if $\theta = \bigcup_{k=1}^{n} I_k$ then prove that $m^*(E \sim \theta) + m^*(\theta \sim E) > \in$. **[7]**

♣ ♣ ♣

Total No. of Questions : 8]

SEAT No. :

P252

[Total No. of Pages : 3

[5828]-103
M.A./M.Sc. (Semester - I)
MATHEMATICS
MTUT 115 : Ordinary Differential Equations
(2019 Pattern) (CBCS)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Figures to the right indicate full marks.***
2) ***Attempt any five questions.***

Q1) Attempt the following :

a) i) Show that the function $\phi(x)=\frac{2}{3}+e^{-3x}$ is the solution of the equation $y' + 3y = 2$. **[2]**

ii) Show that every solution of the equation $x^2y' + 2xy = 1$ on $(0, \infty)$ tends to zero as $x \to \infty$. **[5]**

b) Explain the method of solving the equation $y' + ay = b(x)$, where a is constant and $b(x)$ is continuous function. **[7]**

Q2) a) i) Show that $\phi(x)=e^{-ax}\int_{x_0}^{x} e^{at}b(t)\,dt$ is a solution of the equation $y' + ay = b(x)$. **[3]**

ii) If $\phi(x)$ is the solution of the equation $y' + iy = x$ such that $\phi(0) = 2$ then find $\phi(\pi)$. **[4]**

b) Solve the equation $Ly' + Ry = E$ where L, R and E are constants. Also show that every solution of it tends to E/R as $x \to \infty$. **[7]**

P.T.O.

Q3) a) Show that by Formal substitution $Z = y^{1-k}$ transforms the equation $y' + \alpha(x)y = \beta(x)y^k$ into $Z' + (1-k)\alpha(x)Z = (1-k)\beta(x)$. Hence Find all the solutions of $y' - 2xy = xy^2$. **[7]**

b) Show that every solution of the constant coefficient equation $y'' + a_1y' + a_2y = 0$ tends to 0 as $x \to \infty$ if and only if the real parts of the roots of the characteristics polynomial are negative. **[7]**

Q4) a) Compute $W(\phi_1, \phi_2, \phi_3)(x)$ at a point $x = 0$ for the function $\phi_1 = e^x$, $\phi_2 = xe^x$ and $\phi_3 = x^2e^x$. **[7]**

b) If $\phi(x)$ is a function having continuous derivative on $[0, \infty)$ such that $\phi'(x) + 2\phi(x) \leq 1,\ \forall x\ \varepsilon\ [0,\infty)$ and $\phi(0) = 0$. Then show that $\phi(x) < \frac{1}{2}$ for $x \geq 0$. **[7]**

Q5) a) Find solution of the equation $y'' - y' - 2y = e^{-x}$. **[7]**

b) i) Compute three linearly independent solutions of the equation $y''' - 4y' = 0$. **[3]**

ii) Find the solution of the initial value problem

$y'' + (1 + 4i)y' + y = 0$ with $y(0) = 0 = y'(0)$ **[4]**

Q6) a) Explain the method for solving non-homogeneous equation with constant co-efficient of order n. **[7]**

b) Define Wronskian of ϕ_1, ϕ_2. Hence, show that two solutions ϕ_1 and ϕ_2 of $y'' + a_1y' + a_2y = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2) \neq 0,\ \forall x \varepsilon I$. **[7]**

Q7) a) Explain the method of reduction of order for solving n^{th} order homogeneous equation. **[7]**

b) Show that $\phi_1(x) = |x|^i$ and $\phi_2(x) = |x|^{-i}$ are linearly independent solutions of the equation $x^2y'' + xy' + y = 0$. **[7]**

Q8) a) Explain the variable separable method for first order differential equation $y' = F(x, y)$. **[7]**

b) Show that the function $\phi(x)=\frac{y_0}{1-y_0(x-x_0)}$ which passes through the point (x_0, y_0) is a solution of the equation $y' = y^2$. **[7]**

Total No. of Questions : 8]

SEAT No. :

P253

[Total No. of Pages : 3

[5828]-104
M.A./M.Sc. (Semester - I)
MATHEMATICS
MTUT 114 : Advance Calculus
(2019 Pattern)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Figures to the right indicate full marks.***
2) ***Attempt any five questions.***

Q1) A) If the derivative $f'(a + ty{:}y)$ exist, for each t in the interval $0 \le t \le 1$. Then show that for some real θ in the open interval $0 < \theta < 1$, **[6]**

we have,

$F(a + y) - F(a) = F'(z : y)$, where $z = a + \theta y$

B) Evaluate the directional derivative of the scaler field $F(x, y, z) = x^2 + 2y^2 + 3z^2$ at point (1, 1, 0) in the direction of $\bar{i} - \bar{j} + 2\bar{k}$. **[4]**

C) Determine the set of point (x, y) at which function f is continuous, where

$F(x,y) = \tan\left(\dfrac{x^2}{y}\right)$. **[4]**

Q2) A) A particle of Mass m moves along a curve under the action of a force field $\bar{F}$. If the speed of the particle at time t is $\bar{v}(t)$, it's kinetic energy is defined to be $\frac{1}{2}m\bar{v}^2(t)$. Prove that, the change in kinetic energy in any time interval is equal to the work-done by $\bar{F}$ during this time interval. **[6]**

B) Find the gradient vector of the function $F(x, y, z) = \log(x^2 + 2y^2 - 3z^2)$. **[4]**

C) Make a sketch to describe the level set corresponding to given values of C for $f(x, y) = x^2 + y^2$, $C = 0, 1, 4, 9$ **[4]**

P.T.O.

Q3) A) If A_1, A_2 are open subset of $\mathbb{R}$, then prove that $A_1 \times A_2$ is open subset of $\mathbb{R}^2$. **[6]**

B) Calculate the line integral of the vector field $\overline{F}$ along the path described.

$\overline{F}(x,y,z)=(y^2-z^2)\overline{i}+2yz\,\overline{j}-x^2\overline{k}$, along the

path $\overline{\alpha}(t)=t\overline{i}+t^2\,\overline{j}+t^3\overline{k}$, $0 \leq t \leq 1$ **[4]**

C) Give any two basic properties of line integral. **[4]**

Q4) A) Prove that, if $\overline{T}$ denotes unit tangent vector, then $\int \overline{f}\cdot d\overline{\alpha} = \int \phi\, ds$. **[6]**

B) Find the amount of work done by the force, $f(x,y)=(x^2-y^2)\overline{i}+2xy\,\overline{j}$ in moving a particle (in counterclockwise direction) once around these square bounded by the coordinate axes and the line $x = a$ and $y = a$, $a > 0$. **[8]**

Q5) A) Prove that, if ϕ be real-valued function that is continuous on an interval $[a, b]$. Then the graph of ϕ has content zero. **[6]**

B) Evaluate $\iiint\limits_S xyz\, dx\, dy\, dz$, **[8]**

where S = $\{(x, y, z) / x^2 + y^2 + z^2 \leq 1, x \geq 0\ y \geq 0, z \geq 0\}$

Q6) A) Determine the region S and interchange the order of integration **[6]**

$$\int_0^1 \left[\int_{x^2}^{x} F(x,y)\,dy\right] dx$$

B) Use Green's theorem to evaluate the line integral $\oint\limits_C y^2 dx + x dy$, when C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2) **[4]**

C) Define simply connected plane set. **[4]**

Q7) A) If r and R be smoothly equivalent function related by equation R (s, t) = r[G(s, t)], where $G = u\bar{i} + v\bar{j}$ is a one to one continuously differentiable mapping of a region B in the st plane onto a region A in the uv-plane. Then show that **[6]**

$$\frac{\partial R}{\partial s} \times \frac{\partial R}{\partial t} = \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v}\right) \frac{\partial(u, v)}{\partial(s, t)}$$

Where, the partial derivatives $\frac{\partial r}{\partial u}$ and $\frac{\partial r}{\partial v}$ are to be evaluated at point (U(s, t), V(s.t)).

B) Define : **[4]**

i) Curl $\overline{F}$

ii) Div $\overline{F}$, for vector field $\overline{F}$

C) Write any two methods of representation of surface and explain it. **[4]**

Q8) A) Prove that, Fundamental vector product is normal to the surface. **[6]**

B) Determine whether or not a vector field $\overline{F}(x, y) = 3x^2 y\bar{i} + x^3 y\bar{j}$ is gradient on any open subset of $\mathbb{R}^2$. **[4]**

C) Let T : $\mathbb{R}^n \to \mathbb{R}^n$ be a given linear transformation compute the derivative F′(*x* ; *y*) for the scaler field defined on $\mathbb{R}^n$ by the equation F(*x*) = *x*.T(*x*). **[4]**

Total No. of Questions : 8]

SEAT No. :

P254

[Total No. of Pages : 3

[5828]-105
M.A./M.Sc.
MATHEMATICS
MTUT113 : Group Theory
(2019 Pattern) (Semester - I) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define a subgroup of a group G. Let H be a non-empty subset of G. Prove that H is a subgroup of G if and only if for $a, b \in$ H, $ab^{-1} \in$ H. **[5]**

b) Let G be a group and 'a' be an element of G of order n in G. Prove that

$\langle a^k \rangle = \langle a^{g.c.d(n,k)} \rangle$ and $|a^k| = \dfrac{n}{g.c.d(n,k)}$ where k is a positive integer. **[5]**

c) Prove that a group G is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all a, b in G. **[4]**

Q2) a) Let G be a finite group. Prove that the number of elements of order d is a multiple of $\varphi(d)$. **[5]**

b) Let S be a finite set and σ denote a permutation of S. Prove that $\sigma = \alpha_1\alpha_2....\alpha_n$. Where $\alpha_1, \alpha_2....\alpha_n$ are disjoint cycles. **[5]**

c) Let $\alpha = (1\ 2)\ (4\ 5)$ and $\beta = (1\ 6\ 5\ 3\ 2)$ be permutations in S_6. Compute each of the following : **[4]**

i) α^{-1}

ii) $\beta\alpha$

iii) $\alpha\beta$

Q3) a) Prove that every group is isomorphic to a group of permutations. **[5]**

b) Let G be a group and Aut (G) denote the automorphisms of G then prove that Aut(G) is a group. **[5]**

c) Let $\mathbb{R}^+$ be the group of positive real numbers under multiplication. Show that the mapping $\phi(x) = \sqrt{x}$ is an automorphism of $\mathbb{R}^+$. **[4]**

Q4) a) State and prove Lagrange's theorem. **[5]**

b) Let H and K be two subgroups of a group G define HK = {hk|h∈H, k∈K}, prove that $|HK| = \frac{|H||K|}{|H \cap K|}$. **[5]**

c) Let G = {(1), (132) (465) (78), (132) (465), (123) (456), (123) (456) (78), (78)}. Find the following : **[4]**

i) $orb_G(1)$, $stab_G(1)$

ii) $orb_G(2)$, $orb_G(2)$

Q5) a) Let G and H be finite cyclic group, prove that G⊕H is cyclic if and only if |G| and |H| are relatively prime. **[5]**

b) Prove that a group of order 4 is isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$. **[5]**

c) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$. **[4]**

Q6) a) Let G be a group and Z(G) denote the center of G, prove that G/Z(G) ≈ Inn(G) where Inn(G) – inner automorphism of G. **[5]**

b) Prove that if H is a subgroup of G having index 2 in G then H is normal in G. **[5]**

c) Prove that an Abelian group of order 33 is cyclic. **[4]**

Q7) a) State and prove first isomorphism theorem. **[5]**

b) Let φ be a group homomorphism from G to $\overline{G}$ then prove that Ker ϕ - kernel of ϕ is a normal subgroup of G. **[5]**

c) Determine all group homomorphism from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{30}$. **[4]**

Q8) a) If G is a group of order pq, where p,q are primes p<q, and p does not divide q–1, then prove that G is cyclic and $G \approx \mathbb{Z}_{pq}$. **[5]**

b) Let G be a group of order 99. Then prove that $G \approx \mathbb{Z}_{99}$ or $G \approx \mathbb{Z}_3 \oplus \mathbb{Z}_{33}$ **[5]**

c) Show that $cl(a) = \{a\}$ if and only if $a \in Z(G)$, where Z(G) is center of group G and $cl(a)$ denote conjugacy class of $a \in G$. **[4]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

P255

[Total No. of Pages : 3

[5828]-201
M.A./M.Sc.
MATHEMATICS
MTUT-121 : Complex Analysis
(2019 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $f : A \to \mathbb{C}$, $g : B \to \mathbb{C}$, $f(A) \subset B$ and $Z_0 \in A$. Suppose that $f'(Z_0)$ and $g'(f(Z_0))$ exist then show that $(gof)'(Z_0)$ exists and $(gof)'(Z_0) = g'(f(Z_0)).f'(Z_0)$. **[5]**

b) Let Z_1, Z_2 be the complex numbers. Then prove the following results. **[5]**

i) $|z_1 + z_2| = |z_1|^2 + |z_2|^2 + 2\text{Re}\,(z_1 \overline{z}_2)$

ii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

iii) $|z_1 + z_2| \leq |z_1| + |z_2|$

c) Identify the real and imaginary parts of the complex number $z = (1 + i)^4 + (1 - i)^4$. **[4]**

Q2) a) Let U be an open set in $\mathbb{R}^3$ and $f : U \to \mathbb{R}$ be a function having partial derivatives which are continuous at (x_0, y_0). Then show that f is frechet differentiable at (x_0, y_0). **[5]**

b) Let $f(x,y) = \dfrac{x-y}{x+y}$ for $(x, y) \neq (0, 0)$. Show that two iterated limits exists but are not equal. Does $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists? **[4]**

c) Use Cauchy Riemann equations to determine whether the function $f(z) = \text{Re}(z)$ is complex differentiable for $z \in \mathbb{C}$ or not. **[3]**

d) Compute the value of the integral $\int_\omega x\,dz$ where ω is the line segment from 0 to $1 + i$. **[2]**

P.T.O.

Q3) a) Let f be a continuous function in a region Ω and complex differentiable in $\Omega\backslash A$, where A is discrete subset of Ω. Let T be a triangle completely contained in Ω. Then show that $\int_{\partial T} f(z)dz = 0$. **[5]**

b) Let f be a complex differentiable in a region Ω. Then prove that f has complex derivatives of all order in Ω. More over if D is a disc whose closure is contained in Ω and z belongs to interior of D then for all integers $n \geq 0$ prove that $f^{(n)}(z) = \frac{n!}{2\pi i}\int_{\partial D} \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi$ where ∂D denotes the boundary of the disc. **[5]**

c) Evaluate $\int_C \frac{\sin z}{z+3i} dz$ where C : $|z - 2 + 3i| = 1$ traversed in counter clockwise direction. **[4]**

Q4) a) Let $f : \Omega \to \mathbb{C}$ be a non constant complex differentiable function on a domain Ω. Then prove that there does not exists any point $w \in \Omega$ such that $|f(z)| \leq |f(w)|\ \forall\ z \in \Omega$. **[5]**

b) Let C be a circle $|z| = 3$ traced in the counterclockwise sense for any z with $z \neq 3$ let $g(z) = \int_C \frac{2w^2 - w - 2}{w - z} dw$. Prove that $g(2) = 8\pi i$. Find $g(4)$. **[4]**

c) Is the function $f(z) = \frac{z^2+1}{z(z-1)}$ meromorphic? Why? **[3]**

d) Find the value of the integration $\int_C \frac{e^{az}}{z} dz$. Where C is the unit circle traversed in counterclockwise direction. **[2]**

Q5) a) Let f be a nonzero holomorphic function in a domain Ω and $a \in \Omega$ be a zero of f of order k. Then prove that there is a unique holomorphic function ϕ in a neighbourhood of a such that $\phi(a) \neq 0$ and $f(z) = (z-a)^k.\phi(z) \forall z \in \Omega$. **[5]**

b) Prove that a nonconstant holomorphic function on an open set is an open mapping. **[5]**

c) Compute the residues at all singular points of the function $f(z) = \frac{5}{(z^2-1)^2}$. **[4]**

Q6) a) Let Ω be a holomorphic function on $A(r_1,r_2)$ Let $r_1 < \rho_1 < \rho_2 < r_2$. Then for $\rho_1 < |z| < \rho_2$, show that $f(z)=\frac{1}{2\pi i}\int_{|w|=\rho_2}\frac{f(w)}{w-z}dw-\frac{1}{2\pi i}\int_{|w|=\rho_1}\frac{f(w)}{w-z}dw$. **[5]**

b) Obtain the Laurent series expansion for the function $f(z)=\frac{1}{1-z}$ for the region $A = \{z / |z-2| > 1\}$. **[5]**

c) Find the function $f(z)$ to evaluate the improper integral $\int_{-\infty}^{\infty}\frac{\cos 3x}{(x^2+1)^2}dx$ by using the complex method. **[4]**

Q7) a) Find the value of the integral $\int_0^{\infty}\frac{2x^3-1}{x^4+5x^2+4}dx$. **[7]**

b) Show that $\int_0^{2\pi}\frac{d\theta}{1+a\sin\theta}=\frac{2\pi}{\sqrt{1-a^2}}, -1<a<1$. **[7]**

Q8) a) Let $f : \mathbb{D} \to \bar{\mathbb{D}}$ be a holomorphic function such that $f(0) = 0$ then prove that $|f(z)| \le |z|$ and $|f'(0) \le 1$. Further prove that the following conditions are equivalent. **[6]**

i) there exists $Z_0 \neq 0$ with $|z_o| < 1$ and $|f(z_o)| = |z_o|$

ii) $|f'(0)| = 1$.

iii) $f(z) = cz$ for some $|c| = 1$.

b) Let f, g be holomorphic in an open set containing the closure $\bar{\mathbb{D}}$ of a disc $\mathbb{D}$ and satisfy the inequality $|f(z) - g(z)| < |g(z)|$ $\forall z \in \partial\mathbb{D}$ then show that f and g have same number of zeros inside c. **[5]**

c) Find the Cauchy's principal value of $\int_{-\infty}^{\infty}\frac{e^{iax}}{1-x}dx$ for $H : |z| < 1$. **[3]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

P256

[Total No. of Pages : 3

[5828] - 202
M.A./M.Sc. (Mathematics)
MTUT - 122 : GENERAL TOPOLOGY
(2019 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any Five questions.*
2) *Figures to the right, indicate full marks.*

Q1) a) Let A be a set. prove that there is no injective map $f : p(A) \to A$ and there is no surjective map $g : A \to p(A)$. **[6]**

b) Let B be a nonempty set. Then show that the following are equivalent :**[4]**

i) B is countable.

ii) There is surjective function

$f : \mathbb{Z}_+ \to B$.

iii) There is an injective function

$g : B \to \mathbb{Z}_+$

c) Show that there is bijective correspondence of $A \times B$ with $B \times A$. **[4]**

Q2) a) Show that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another. **[6]**

b) Consider the following topologies on $\mathbb{R}$:

τ_1 = the standard topology,

τ_2 = the topology of $\mathbb{R}_k$,

τ_3 = the finite complement topology,

τ_4 = the upper limit topology having all sets (a, b] as basis,

τ_5 = the topology having all sets $(-\infty, a) = \{x \mid x < a\}$ as basis.

Determine, for each of these topologies, which of the others it contains.**[4]**

c) Show that the order topology on the set $X = \{1,2\} \times \mathbb{Z}_+$ in the dictionary order is not the discrete topology. **[4]**

P.T.O.

Q3) a) Let x be an ordered set in the order topology; let y be a subset of x that is convex in y. Then show that the order topology on y is the same as the topology y inherits as subspace of x. **[6]**

b) If $\{\tau_\alpha\}$ is a family of topologies on x, show that $\bigcap_\alpha \tau_\alpha$ is a topology on x.

Is $\bigcap_\alpha \tau_\alpha$ is topology on x? Justify. **[4]**

c) Let A be a subset of the topological space x; let A′ be the set of all limit points of A. Then show that $\overline{A} = A \cup A'$. **[4]**

Q4) a) Show that every simply ordered set is a Hausdorff space in the order topology. The product of two Hausdorff spaces is a Hausdorff space. A subspace of a Hausdorff space is a Hausdorff space. **[7]**

b) Let $\rho : X \to Y$ be a quotient map. Let Z be a space and let $g : X \to Z$ be a map that is constant on each set $\rho^{-1}(\{y\})$, for $y \in Y$. Then show that g induces a map $f : YX \to Z$ such that $fog = g$. The induced map f is continuous if and only if g is continuous; f is quotient map if and only if g is a quotient map. **[7]**

Q5) a) Prove that a finite Cartesian product of connected spaces is connected. **[6]**

b) Let τ and τ^1 be two topologies on X. If $\tau \leq \tau^1$, what does connectedness of X in one topology imply about connectedness in the other? Justify? **[4]**

c) If the sets C and D forms a separation of a topological spaces X and if Y is a connected subspace of X, then prove that Y is entirely within either C or D. **[4]**

Q6) a) Prove that every compact subspace of Hausdorff space is closed. **[6]**

b) Define : **[4]**

i) First countable space,

ii) Second countable space,

iii) Lindelöf space,

iv) Separable space.

c) Let $f : X \to Y$ be a continuous map of the compact metric space (X, dx) to the metric space (Y, dy). Then prove that f is uniformly continuous. **[4]**

Q7) a) Prove that an arbitrary product of compact spaces is compact in the product topology. **[7]**

b) Let X be a normal space; let A be a closed subspace of X. Then prove that, **[7]**

i) Any continuous map of A into the closed interval $[a, b]$ of $\mathbb{R}$ may be extended to a continuous map of all of X into $[a, b]$.

ii) Any continuous map of A into $\mathbb{R}$ may be extended to a continuous map of all of X into $\mathbb{R}$.

Q8) a) Prove that a subspace of a completely regular space is completely regular. A product of completely regular spaces is completely regular. **[7]**

b) Show that the sorgenfrey plane $\mathbb{R}_l^2$ is not normal. **[7]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

P257

[Total No. of Pages : 2

[5828] - 203

M.A./M.Sc. (Mathematics)

MTUT - 123 : RING THEORY

(2019 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right, indicate full marks.***

Q1) a) Let R be a ring with 1 and non-units in R form a subgroup of (R, +), then prove that char (R) is either 0 or else a power of a prime. **[6]**

b) If R is the ring of all strictly upper triangular $n \times n$ matrices over the ring of integer Z, then show that R is non-commutative ring and each element of R is nilpotent ($n \geq 3$). **[5]**

c) Give examples of two zero-divisors in matrix ring $M_2(Z)$ whose sum is not a zero divisors in $M_2(Z)$. **[3]**

Q2) a) Let R = $\mathbb{Z}[i, j, k]$ be the ring of integral quaterntions. Then prove that the units in R is a group of order 8. **[6]**

b) Let R be a commutative ring with 1 and I be an ideal in R. Then prove that R/I is a field if and only if I is maximal ideal in R. **[6]**

c) Prove or dis prove $7-5\sqrt{2}$ is unit in $Z\left[\sqrt{2}\right]$. **[2]**

Q3) a) Let R be a ring with 1. Then prove that R is division ring if and only if (0) and R are the only left ideals in R. **[6]**

b) Prove that product of two ideals of the same kind is again an ideal of the same kind. **[5]**

c) Define local ring and give an example of non-local ring. **[3]**

P.T.O.

Q4) a) For $n \geq 2$, then prove that the ring $Z/_{nz}$ has no non-trivial nilpotent element if and only if n is square free. **[5]**

b) Let I be an ideal in a ring R. Then prove that I is a 2-sided ideal in R if and only if I is the kernel of some homomorphism $f : R \rightarrow S$ for a suitable rings. **[5]**

c) State Chinese Remainder theorem for a commutative ring R with 1. **[4]**

Q5) a) Prove that the ring $End_K(V)$ is a simple ring if and only if V is a finite dimensional vector space over the field K. **[7]**

b) If I is a 2-sided ideal of R, then prove that I[x] is a 2-sided ideal of R[x] and also show that ring $R[x]/_{I[x]}$ is naturally isomorphic to (R/I) [x]. **[7]**

Q6) a) Prove that every Euclidean domain is a principal ideal domain. What about converse? **[6]**

b) Prove that the ring Z[i] of Gaussian integers is Euclidean domain. **[5]**

c) With usual notations prove that $\sqrt{(9)} = \sqrt{(27)} = \sqrt{(3)}$. **[3]**

Q7) a) For a commutative integral domain R with unity prove that the following are equivalent. **[6]**

i) R is field.

ii) R[x] is Euclidean domain

iii) R[x] is PID

b) With usual notation show that $\dfrac{Q[x]}{<1+x^2>} \cong Q[i]$. **[5]**

c) Prove or disprove The polynomial $x^4 + 1$ is irreducible over R. **[3]**

Q8) a) Show that vector space is free module. **[7]**

b) State and prove schur's lemma for simple modules. **[7]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

P258

[Total No. of Pages : 4

[5828]-204

First Year. M. A./M. Sc.

MATHEMATICS

ADVANCED NUMERICAL ANALYSIS

(CBCS 2019 Pattern) (Semester - II) (MTUT 124)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Figures to the right indicate full marks.***
2) ***Attempt any five questions.***

Q1) a) Determine the corresponding rate of convergence for the function.

$$f(x) = \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$$ **[4]**

b) Let , f be a twice continuously differentiable function on the interval [a,b] with p∈(a, b) and f (p) = 0 further, suppose that $f'(p) \neq 0$ then, show that there exists a $\delta > 0$ such that for $P_0 \in I = [p - \delta, p + \delta]$ the sequence $\{P_n\}$ generated by Newton's method converges to P. **[5]**

c) Use the secant method to determine P_5, the fifth approximation to root of $f(x) = x^3 + 2x^2 - 3x - 1$ in (1, 2) with $P_0 = 2$, $P_1 = 1$. **[5]**

Q2) a) The sequence listed below was obtained from fixed point iteration applied to the function $g(x) = e^{-x}$, which has a unique fixed point. Applying Aitken's Δ^2 - method to the given sequence. **[6]**

Find $\hat{p}_3, \hat{p}_4$ and $\hat{p}_5$.

1	1. 000000
2	0. 3678794412
3	0. 6922006276
4.	0. 5004735006
5	0. 6062435351

P.T.O.

b) Show that when Newton's method is applied to the equation $x^2 - a = 0$, the resulting iteration function is $g(x) = \frac{1}{2}(x + \frac{a}{x})$ **[4]**

c) Define : **[4]**

i) Orthogonal Matrix

ii) Round off Error

Q3) a) Solve the following system of equations by using Gaussian elimination with scaled partial pivoting. **[5]**

$2x_1 + 3x_2 + x_3 = -4$

$4x_1 + x_2 + 4x_3 = 9$

$3x_1 + 4x_2 + 6x_3 = 0$

b) Show that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$ has no LU decomposition. **[4]**

c) Solve the following system by Jacobi method starting with vector $x^{(0)} = [0\ 0\ 0]^T$ perform two iteration. **[5]**

$3x_1 + x_2 = -1$

$-x_1 + 2x_2 + x_3 = 3$

$x_2 + 3x_3 = 4$

Q4) a) Solve the following system of linear equations by SOR method, start with $x^{(0)} = [0\ 0\ 0]^T$ and $w = 0.9$ (perform 3 iteration) **[5]**

$5x_1 + x_2 + 2x_3 = 10$

$-3x_1 + 9x_2 + 4x_3 = -14$

$x_1 + 2x_2 - 7x_3 = -33$

b) Solve the following system of non-linear algebric equations by using Broyden's method start with $x^{(0)} = [1\ 1\ 1]^T$ and (Perform 3 iteration) **[5]**

$x_1^3 - 2x_2 - 2 = 0$

$x_1^3 - 5x_3^2 + 7 = 0$

$x_2 x_3^2 - 1 = 0$

c) Let A be an n×n matrix with eigenvalues λ_1, λ_2, ________, λ_n and associated eigenvectors v_1, v_2, ____________, v_n. then prove that,

If B = $a_0 I + a_1 A + a_2 A^2 +$ __________ $+ a_m A^m = p(A)$ where p is the polynomial $p(x) = a_0 + a_1 x + a_2 x^2 +$ __________ $+ a_m x^m$, then the eigen values of B are $p(\lambda_1), p(\lambda_2)$, _________, $p(\lambda_n)$. With associated eigenvector v_1, v_2, ___________, v_n. **[4]**

Q5) a) Use the QR factorization of a symmetric Tridiagonal Matrix. **[7]**

$$A = \begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Find the product $R^{(0)} Q^{(0)}$.

b) Define Householder matrix and show that it is symmetric and orthogonal. **[4]**

c) For the following differential equation, identify the function $f(t, x)$ and calculate $\frac{df}{dt}, \frac{d^2 f}{dt^2}$. $x^1 = t^2 - 1 - 2x^2$. **[3]**

Q6) a) Derive the open Newton - cotes formula with $n = 3$: **[5]**

$I(f) \approx I_3$, open $(f) = \frac{b-a}{24}\left[11 f(a + \Delta x) + f(a + 2\Delta x) + f(a + 3\Delta x) + 11 f(a + 4\Delta x)\right]$

b) Derive the following forward difference approximation for the second derivative : $f^{11}(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}$ **[4]**

c) Show that : If f is continuous on [a, b], g is integrable on [a, b,] and $g(x)$ does not change sign on [a, b], then there exist a number $\xi \in$ [a, b] such that, $\int_a^b f(x)\, g(x)\, dx = f(\xi) \int_a^b g(x)\, dx$. **[5]**

Q7) a) Use Euler's method to solve initial value problem. **[7]**

$$\frac{dx}{dt} = t^2 + x,\ 0 \le t \le 0.03,\ x(0) = 1$$

b) i) Solve the initial value problem.

$$\frac{dx}{dt} = 1 + \frac{x}{t},\ 1 \le t \le 1.5,\ x(1) = 1,\ h = 0.25$$

by using Taylor method of order N = 2.

ii) Solve the initial value problem. **[7]**

$$\frac{dx}{dt} = 1 + \frac{x}{t},\ 1 \le t \le 2,\ x(1) = 1,\ h = 0.5$$

by using Taylor method of order N = 4.

Q8) a) Evaluate $\int_{-2}^{2} \frac{x}{5+2x}\,dx$ by using Trapezoidal rule by dividing the interval [–2, 2] into five equal subintervals. **[3]**

b) Define : **[4]**

i) Relative error

ii) Triangular Matrix

c) Derive the difference equation for the four-step Adams -Bash forth method :

$$\frac{w_{i+1} - w_i}{h} = \frac{55}{24} f(t_i, w_i) - \frac{59}{24} f(t_{i-1}, w_{i-1}) + \frac{37}{24} f(t_{i-2}, w_{i-2}) - \frac{9}{24} f(t_{i-3}, w_{i-3})$$

Also derive the associated truncation error : $\tilde{L}_i = \frac{251\,h^4}{720}\ y^{(5)}(\xi)$ **[7]**

❄ ❄ ❄

Total No. of Questions : 8]

SEAT No. :

P259

[Total No. of Pages : 3

[5828]-205
M.A./M.Sc.
Mathematics
PARTIAL DIFFERENTIAL EQUATIONS (MTUT125)
(CBCS 2019 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Figures to the right side indicate full marks.***
2) ***Attempt any 05 questions.***

Q1) a) Show that the PDE's $f(x,y,z,p,q)=0$ and $g(x,y,z,p,q)=0$ are compatible . **[5]**

b) Attempt the following. **[4]**

i) Find the complete integral of the partial differential Equation $z = px + qy + p^2 + q^2$ by charpit's method.

ii) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible hence find its solution. **[5]**

Q2) a) Explain charpits method for separable first order partial differential equation $f(x,p) = g(y,q)$ **[4]**

b) Attempt the following.

i) Obtain the PDE by eliminating the arbitrary constants 'a' and 'b' from $\log(az-1) = x + ay + b$ **[4]**

ii) Solve $p^2 x + q^2 y = z$ by Jacobi's Method. **[6]**

Q3) a) Prove that if $\alpha_\gamma D + \beta_\gamma D' + \gamma_\gamma$ is a factor of $F(D,D')$ and $\phi_r(\xi)$ is an arbitrary function of the single variable ξ, then $u_\gamma = \exp\left(\frac{-\gamma_\gamma x}{\alpha_\gamma}\right)$ $\phi_\gamma(\beta_\gamma x - \alpha_\gamma y)$ for $\alpha_\gamma \neq 0$. **[6]**

P.T.O.

b) Attempt the following **[4]**

i) Find the complementary function of the partial differential equation.

$$\frac{\partial^3 z}{\partial x^3} - \frac{2\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + \frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

ii) Solve $(D + D' - 1)(D + 2D' - 3)z = 0$ **[4]**

Q4) a) Explain the method of second order partial differential Rr+Ss+Tt+f(x,y,z,p,q) = 0 to a cannonical form if $S^2 - 4RT > 0$. **[5]**

b) Attempt the following.

i) Reduce the equation

$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to a cannonical form and hence solve it. **[5]**

ii) Classify the PDE

1) $u_{xx} + 2u_{xy} + u_{yy} = 0$ **[2]**

2) $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$ **[2]**

Q5) a) Derive the Laplace equation of second order partial differential equation. **[4]**

b) Attempt the following **[6]**

i) Solve

PDE : $\nabla^2 u = 0$, $0 \le x \le$ a, $0 \le$ y $\le$ b

BCs : $u(x,b) = u(a,y) = 0, u(0,y) = 0, u(x,0) = f(x)$

ii) Find the complete integral of $u_t - 3u^2 u_x = 0$ with $u(x,0) = \text{Cos}\, x$. **[4]**

Q6) a) Derive the diffusion equation of second order differential equation. **[5]**

b) Attempt the following.

i) Find by method of separation of variables the solution $u(x,t)$ of the boundary value problem. $u_t = 3u_{xx}, t > 0, 0 < x < 2, u(0,t) = 0$,

$u(2,t) = 0, t > 0, u(x,0) = x, 0 < x < 2$ **[6]**

ii) Find the characteristics of the partial differential equation **[3]**

$(\sin^2 x)r + (2\cos x)s - t = 0$

Q7) a) Find the solution of one dimensional wave equation by cannonical reduction method. **[5]**

b) Attempt the following

i) Solve the wave equation $u_{tt} = C^2 u_{xx}$ where $u = P_0 \cos pt$ (P_0 is constant) when $x = l$ and $u = 0$ when $x = 0$. **[5]**

ii) Find the complete integral of $z^2\left(1 + p^2 + q^2\right) = 1$. **[4]**

Q8) a) Find the steady state temperature distribution in the thin rectangular plate bounded by lines $x = 0$, $x = a$, $y = 0$, $y = b$. The edges $x = 0$, $x = a$, $y = a$, are kept at temperature zero while the edge $y = b$ is kept at 100°C. **[7]**

b) A uniform rod 20cm in length is instead over its sides. Its ends are kept at 0°C its initial temperature is $\sin\left(\frac{\pi x}{20}\right)$ at a distance '*x*' from an end, find temperature $u(x,t)$ at time 't', Given that $\frac{\partial u}{\partial t} = a^2\left(\frac{\partial^2 u}{\partial x^2}\right)$. **[7]**

❍❍❍❍

Total No. of Questions : 8]

SEAT No. :

P260

[Total No. of Pages : 3

[5828]-301
M.A./M.Sc.
MATHEMATICS
MTUT - 131 : Functional Analysis
(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five of the following questions.***
2) ***Figures to the right side indicate full marks.***
3) ***Symbols have their usual meanings.***

Q1) a) Give a definition of normed linear space. Define one norm on vector space $\mathbb{R}^2$ Justify. **[4]**

b) Let M be any closed linear subspace of normed linear space N. Define norm on N/M by $||x + M|| = \inf\{||x + m|| : m \in M\}$. Show that N/M is a normed linear space also, prove that if N is complete then N/M is a Banach space. **[6]**

c) Consider a norm on $\mathbb{R}$ given by $||x|| = |x|$ for all $x \in \mathbb{R}$. Show that $\mathbb{R}$ is a Banach space by showing $\mathbb{R}$ is complete. **[4]**

Q2) a) Let M be a linear subspace of normed linear space N, and Let f be a functional on M if $x_0 \notin M$ and $M_0 = M + [x_0]$ then prove that f can be extended to a linear functional f_0 such that $||f_0|| = ||f||$. **[7]**

b) Let B and B' be two Banach spaces and T is a continious linear transformation of B onto B'. Then prove that image of each open sphere centred at origine in B contains an open sphere centred at origine in B'.**[7]**

P.T.O.

Q3) a) Let $\mathcal{B}(N, N')$ be the set of all continious linear transformation of N into N'. Prove that $\mathcal{B}(N, N')$ is a normed linear space. Also, prove that if N' is a complete then $\mathcal{B}(N, N')$ is a Banach space. **[7]**

b) State and prove the open mapping theorem. **[5]**

c) Let $T : N \to N$ be continious linear transformation.

Define $T^* : N^* \to N^*$ by $T^*(f) = f \circ T$

Prove that $\|T^*\| \leq \|T\|$. **[2]**

Q4) a) Prove that a closed convex subset of Hilbert space H contains a unique vector of smallest norm. **[5]**

b) Prove that for any two vector's x and y. In a Hilbert space H, $|<x, y>| \leq \|x\| \cdot \|y\|$. **[4]**

c) Let M be a proper closed linear subspace of a Hilbert space H. Prove that there exist's a non zero vector Z_0 in H such that $Z_0 \perp$ H. **[5]**

Q5) a) Prove that if M is a closed linear subspace of a Hilbert space H then $H = M \oplus M^{\perp}$. **[5]**

b) Let y be any fixed vector in H. define $f_y : H \to \mathbb{F}$ by $f_y(x) = (x, y)$. Show that f_y is linear and continious. **[4]**

c) Let T be any operator on H for which $(T(x), x) = 0$ for all $x \in$ H. Prove that T is identically zero function, that is $T(x) = 0, \forall x \in$ H. **[5]**

Q6) a) Let H be a Hilbert space and let $f \in H^*$ be an arbitrary functional. Prove that there exist's a unique vector y in H such that $f(x) = (x, y)$. **[6]**

b) Prove that T is self adjoint operator on H if and only if $(T(x), x)$ is real for all x. **[4]**

c) If N_1 and N_2 are normal operator on H such that $N_1 \circ N_2^* = N_2^* \circ N_1$ and $N_2 \circ N_1^* = N_1^* \circ N_2$ then prove that $N_1 + N_2$ is a normal operator. **[4]**

Q7) a) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be given by T(x, y, z) = (x, y + z, y). Find the matrix of T with respect to basis. B{(1, 1, 1), (1, 1, 0), (1, 0, 0)}. **[4]**

b) Let T be a normal operator. Prove that λ is an eigenvalue of T with eigenvector x if f $\overline{\lambda}$ is a eigenvalue of T* with eigenvector x. **[4]**

c) Let T be any arbitrary operator on H and N be a normal operator. Prove that if T commutes with N then T commutes with N*. **[4]**

d) Give a statement of open mapping theorem. **[2]**

Q8) a) Let B = {e_i} be an ordered basis for Hilbert space H and [T] is a matrix of operator T on H relative to basis B. Prove that the mapping T $\to$ [T] is a one-to-one homomorphism. **[6]**

b) Let T be an operator on H. Prove that T is singular if and only if 0 is a eigenvalue of T. **[4]**

c) Let M be closed linear subspace of H prove that M is invarient under T if and only if $M^{\perp}$ is invarient under T*. **[4]**

☯☯☯☯

Total No. of Questions : 8]

SEAT No. :

P261

[Total No. of Pages : 2

[5828]-302

M.A./M.Sc.

MATHEMATICS

MTUT - 132 : Field Theory

(2019 CBCS Pattern) (Semester - III)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions of the following.***
2) ***Figures to the right side indicate full marks.***

Q1) a) If P(x) is an irreducible polynomial of degree 'n' in F(x) and u is a root of P(x) in an extension E of F then prove that $\{1, u, u^2, ..., u^{n-1}\}$ forms a basis of F(u) over F. **[7]**

b) Show that every finite extension of a field F is algebraic. **[5]**

c) Find the smallest extension of φ having a root $x^4 - 2 \in Q[x]$. **[2]**

Q2) a) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals. **[7]**

b) Show that doubling of a circle is not possible by using ruler and compass only. **[5]**

c) Define radical extension of a field with example. **[2]**

Q3) a) Let $E = Q\left(\sqrt[3]{2}, \omega\right)$ where $\omega^3 = 1$, $\omega \neq 1$, Let G = {1, σ}

Where 1 : $\begin{cases} \sqrt[3]{2} \to \sqrt[3]{2} \\ \omega \to \omega \end{cases}$ and σ:$\begin{cases} \sqrt[3]{2} \to \omega\sqrt[3]{2} \\ \omega \to \omega 2 \end{cases}$ are automorphism of E then find E_G. **[7]**

b) Let F be field and E be a finite normal separable extension of F then prove that F is a fixed field of G (E/F). **[5]**

c) Find the degree of $Q\left(\sqrt{2}\right)$ over Q. **[2]**

P.T.O.

Q4) a) Show that the Galois Group of $x^3-2 \in Q[x]$ is the group of symmetrices of the triangle. **[7]**

b) State fundamental theorem of Galois theory. **[5]**

c) Is C a normal extension of R?. Justify. **[2]**

Q5) a) Let F be a field containing n^{th} root of unity and E be a finite cyclic extension of degree n over F then prove that E is the splitting field of an irreducible polynomial $x^n-a \in F[x]$. **[7]**

b) Prove that every polynomial $f(x) \in Q[x]$. factors into linear factor in $C[x]$. **[7]**

Q6) a) Let E be an extension of a field F and $\alpha \in F$ be algebraic element over F then prove that α is separable over F if and only if $F(\alpha)$ is separable extension of F. **[7]**

b) Let $f(x) \in F[x]$ be a polynomial of degree ≥ 1 and α as a root. Then prove that α is a multiple root of $f(x)$ if and only if $f^1(\alpha) = 0$. **[7]**

Q7) a) Show that the degree of extension of the spliting field of $x^3-2 \in Q[x]$ is 6. **[6]**

b) Prove that a finite extension of a finite field is separable. **[5]**

c) Show that $x^p - x - 1$ is irreducible over $\mathbb{Z}p$. **[3]**

Q8) a) Show that every polynomial in $K[x]$ is of degree $\perp$ if K is algebraically closed. **[6]**

b) If $f(x) = a_0 + a_1x + ... + a_{n-1} x^{n-1} + x^n \in \mathbb{Z}[x]$ is a monic polynomial having α as a root in Q then prove that $\alpha \in \mathbb{Z}$ & $\alpha | a_0$. **[5]**

c) If E is an extension of a field F and [E : F] is prime then prove that there is no field properly between E and F. **[3]**

☯☯☯☯

Total No. of Questions : 4]

SEAT No. :

P262

[Total No. of Pages : 1

[5828]-303

M.A./M.Sc. - II

MATHEMATICS

MTUT - 133 : Programming with Python

(2019 CBCS Pattern) (Semester - III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***Figures to the right side indicate full marks.***
2) ***Question 1 is compulsory.***
3) ***Attempt any 2 questions from Q.2, Q.3 and Q.4.***

Q1) Attempt the following :

a) Explain any 3 features of Python. **[3]**

b) Explain the chronology of the development of Python. **[3]**

c) Does Python have data types? **[1]**

Q2) Attempt the following :

a) i) Write a Python program which accept positive integer and display wheather it is odd or not. **[5]**

ii) Write a Python program to swap two numbers. **[2]**

b) Write a note on conditional statements in Python with an example. **[7]**

Q3) Attempt the following :

a) i) Write a note on string operators in Python. **[5]**

ii) Explain the meaning of "Tuples are immutable". **[2]**

b) Write a note on For loop in Python with an example. **[7]**

Q4) Attempt the following :

a) i) Write a note on functions in Python. **[5]**

ii) Explain minimum one difference between For loop and While loop in Python. **[2]**

b) Write a note on operator overloading in Python. **[7]**

☯☯☯☯

Total No. of Questions : 8]

SEAT No. :

P263

[Total No. of Pages : 3

[5828]-304

M.A./M.Sc.

MATHEMATICS

MTUTO 134 : Discrete Mathematics

(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) How many ways are there to pick two different cards from a standard 52-card deck such that **[7]**

i) the first card is an Ace and the second card is not a Queen?

ii) the first card is a spade and the second card is not a Queen?

b) Prove that the isomorphism relation defined on set of simple graphs is an equivalence relation. **[5]**

c) Prove or disprove the following statement. **[2]**

'If every vertex of a simple graph G has degree Z, then G is cycle.

Q2) a) What is the probability that an arrangement of a, b, c, d, e, f has a and b side by side? **[5]**

b) If two vertices are nonadjacent in the petersen graph, then prove that they have exactly one common neighbour. **[5]**

c) If every vertex of a graph G has degree at least Z, then prove that G contains a cycle. **[4]**

Q3) a) Prove that a graph G is Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **[7]**

P.T.O.

b) Show that number of partitions of an integer r as a sum of m positive integers is equal to the number of partitions of r as a sum of positive integers in which largest is m. **[5]**

c) Find the coefficient of $\frac{x^r}{r!}$ in $\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots\right)^n$. **[2]**

Q4) a) Find a recurrence relation for a_n, where a_n is the number of n-digit ternary sequences without any occurence of the sequence "0 1 2". **[5]**

b) Solve the following recurrence relation. **[5]**

$a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = 1$, $a_1 = 1$

c) Prove that every graph has an even number of vertices of odd degree. **[4]**

Q5) a) State and prove Inclusion - Exclusion formula. **[7]**

b) Let G be an n-vertex graph with $n \geq 1$. Prove that following statements are equivalent. **[7]**

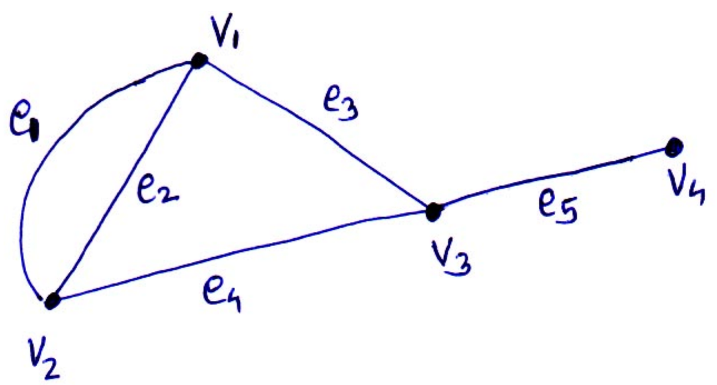
i) G is connected and has no cycles.

ii) G is connected and has $n - 1$ edges.

Q6) a) How many 8-letter words using the 26-letter alphabet (letters can be repeated) either begin or end with vowel? **[6]**

b) Prove that an X, Y - bigraph G has a matching that saturates X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. **[6]**

c) Find the adjacency matrix for the following graph. **[2]**



Q7) a) Let T be a tree with average vertex degree a. Determine n (T) in terms of a. **[5]**

b) Show by a combinatorial argument that **[5]**

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+.....+\binom{n}{n}=2^n$$

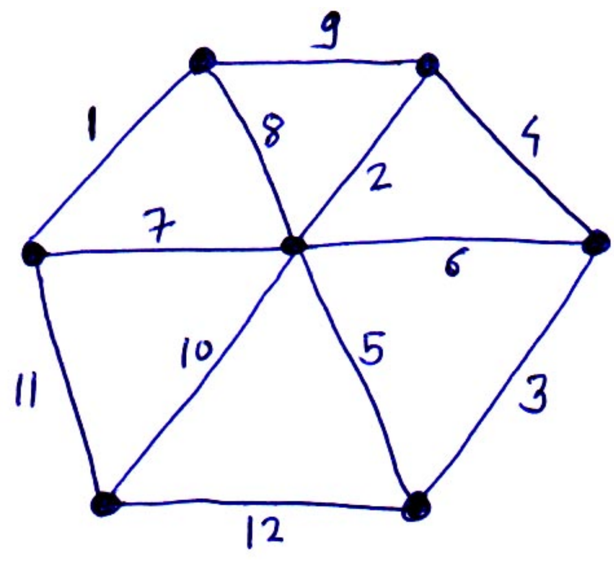
c) If the recurrence relation $a_n = C_1 a_{n-1} + C_2 a_{n-2}$ has a general solution $a_n = A_1 3^n + A_2 6^n$, where A_1, A_2 are constants, then find C_1 and C_2. **[4]**

Q8) a) Find an exponenitial generating function with **[8]**

i) $a_r = \dfrac{1}{r+1}$

ii) $ar = r!$

b) Use Kruskal's algorithm to find the minimum spanning tree for the following weighted graph. **[6]**



❦❦❦

Total No. of Questions : 8]

SEAT No. :

P264

[Total No. of Pages : 2

[5828]-305

M.A./M.Sc. (Mathematics)

MTUTO 135 : MECHANICS

(2019 Pattern) (Semester - III)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the principle of virtual work and derive D'Alembert equation of motion. **[5]**

b) Determine the equation of motion of simple pendulum by using D'Alembert principle. **[5]**

c) A particle is constrained to move on the plane curve xy = C, where C is constant under gravity obtain Lagrangian and hence equation of motion.**[4]**

Q2) a) Show that the Lagrang equation of motion can also be written as **[5]**

$$\frac{\partial L}{\partial t}-\frac{d}{dt}\left(L-\sum\dot{q}_j\frac{\partial L}{\partial q_j}\right)=0$$

b) A particle of mass m is projected with initial velocity u at an angle α with the horizontal. Use Lagranges equation to describe the motion of the projectile. **[5]**

c) Explain the following terms : **[4]**

i) Linear Momentum

ii) Angular Momentum

P.T.O.

Q3) a) Show that the two Lagrangians $L_1 = (q + \dot{q})^2, L_2 = (q^2 + \dot{q}^2)$ are equivalent. **[7]**

b) Prove that if the force acting on a particle is conservative then the total energy is conserved. **[7]**

Q4) a) Find the curve, which extremizes the functional **[7]**

$$I(y(x)) = \int_0^{\pi/4} \left(y''^2 - y^2 + x^2\right) dx$$

with conditions that $y(0) = 0, y'(0) = 1,\ y\left(\pi/4\right) = y'\left(\pi/4\right) = \frac{1}{\sqrt{2}}$

b) State Hamiltons principle for non conservative system and hence derive from it the Lagranges equation of motion for non conservative holonomic system. **[7]**

Q5) a) Use Hamiltons principle to find the equation of motion of a simple pendulum. **[7]**

b) what is Hamiltonian function? Derive the Hamilton's Canonical equation of motion from Hamiltonian function. **[7]**

Q6) a) Write a note on Brachistochrone problem. **[7]**

b) Deduce Newton's Second Law from Hamilton's principle. **[7]**

Q7) a) Prove that field force motion is always motion in plane. **[5]**

b) Prove Keplars Second Law of planetary motion. **[5]**

c) Explain Principles of least action. **[4]**

Q8) a) Show that the Lagranges equations are necessary conditions for the action to have stationary value. **[7]**

b) Show that the geodesics on a right circular cylinder is a helix. **[7]**

Total No. of Questions : 8]

SEAT No. :

P265

[Total No. of Pages : 3

[5828]-306
M.A./M.Sc. (Mathematics)
MTUTO 136 : Advanced Complex Analysis
(2019 Pattern) (Semester - III) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and prove Morera's theorem. **[5]**

b) Prove that $\int_0^{\infty} \frac{1-\cos x}{x^2} dx = \frac{\pi}{2}$. **[5]**

c) Suppose that f is a holomorphic function in Ω^+ that extends continuously to I and such that f is real-valued on I. Prove that there exists a function F holomorphic in all of Ω such that $F = f$ on Ω^+. **[4]**

Q2) a) Prove that every non-constant polynomial $P(Z) = a_n z^n + \dots\dots\dots + a_0$ with complex coefficients has a root in $\mathbb{C}$. **[5]**

b) i) If f and g are holomorphic in a region Ω and $f(z) = g(z)$ for all z in some sequence of distinct point with limit point in Ω then prove that $f(z) = g(z)$ throughout Ω. **[3]**

ii) Show that the complex zeros of $\sin(\pi z)$ are exactly at the integers and each of order 1. **[2]**

c) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$. **[4]**

P.T.O.

Q3) a) If $f : U \to V$ is holomorphic and injective then prove that $f'(z) \neq 0$ for all $z \in U$. **[5]**

b) Prove that $f(z) = z^n$ is a conforal map from the sector $S = \left\{ z \in ₵ \mid 0 < \arg(z) < \frac{\pi}{n} \right\}$ to the upper half-plane. **[5]**

c) Let V and U be open sets in ₵ and $F : V \to U$ a holomorphic function. If $u : U \to ₵$ is a harmonic function, then prove that uoF is harmonic on V. **[4]**

Q4) a) Let $F : \mathbb{D} \to \mathbb{D}$ be holomorphic with $f(0) = 0$.

Then prove the following :

i) $|f(z)| \leq |z|$ for all $z \in \mathbb{D}$.

ii) If for some $z_0 \neq 0$ we have $|f(z_0)| = |z_0|$, then f is a rotation.

iii) $|f'(0)| \leq 1$, and if equality holds, then f is a rotation.

[7]

b) Let $F : H \to ₵$ be a holomorphic function that satisfies $|F(z)| \leq 1$ and $F(i) = 0$. Prove that $|F(z)| \leq \left| \frac{z-i}{z+i} \right|$ for all $z \in \mathbb{H}$. **[5]**

c) Define automorphism and give one example. **[2]**

Q5) a) State and prove Montel's theorem. **[7]**

b) Prove that any two proper simply connected open subsets in ₵ are conformally equivalent. **[5]**

c) State the Riemann mapping theorem. **[2]**

Q6) a) Let z_0 be a point on the unit circle and if $F : \mathbb{D} \to P$ is a conformal map, then prove that $F(z)$ tends to a limit as z approaches z_0 within the unit disc. **[7]**

b) Show that the function $\int_0^z \frac{d\xi}{\sqrt{\xi(\xi-1)(\xi-\lambda)}}$, with $\lambda \in \mathbb{R}$ and $\lambda \neq 1$ maps the upper half-plane conformally to a rectangle, one of whose vertices is the image of the point at infinity. **[5]**

c) Define the general Schwarz-Christoffel integral. **[2]**

Q7) a) Prove that there exist complex numbers c_1 and c_2 so that the conformal map F of $\mathbb{H}$ to p is given by $F(z) = c_1 S(z) + c_2$ where S is the Schwarz-Christoffel integral. **[7]**

b) If $F(z) = \int_1^z \frac{d\xi}{(1-\xi^n)^{2/n}}$, then show that F maps the unit disc conformally onto the interior of a regular polygon with n sides and perimeter $2^{\frac{n-2}{n}} \int_0^\pi (\sin\theta)^{-2/n} d\theta$. **[7]**

Q8) a) Prove that the total number of poles of an elliptic function in p_0 is always ≥ 2. **[7]**

b) Show that the two series $\sum_{(n,m)\neq(0,0)} \frac{1}{(|n|+|m|)^r}$ and $\sum_{n+mT\in A^*} \frac{1}{|n+mT|^r}$ where A* denote the lattice minus the origin, that is $A^* = A - \{(0,0)\}$, Converges if $r > 2$. **[7]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

P266

[Total No. of Pages : 3

[5828]-307
M.A./M.Sc.
MATHEMATICS
MTUTO-137 : Integral Equations
(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Convert the volterra integral equation $u(x)=1+x+\int_0^x (x-t)^2 u(t)dt$ to an equivalent initial value problem. **[4]**

b) Convert the initial value problem $y''' - 3y'' - 6y' + 5y = 0$ subject to the initial condition $y(0) = y'(0) = y''(0) = 1$ to an equivalent volterra integral equation. **[5]**

c) Derive an equivalent Fredholm integral equation to the boundary value problem $y''(x) + y(x) = x$, $0 < x < \pi$ subject to the boundary conditions $y(0) = 1$, $y(\pi) = \pi - 1$. **[5]**

Q2) a) Find the Taylor series generated by $f(x) = \cos x$ at $x = 0$. **[4]**

b) Solve the Fredholm integral equation

$u(x)=e^{3x}-\frac{1}{9}(2e^3+1)x+\int_0^1 x+u(t)dt$ using the modified decomposition scheme. **[5]**

c) Solve the fredholm integral equation $u(x)=x^2+\int_0^1 xt\,u(t)\,dt$ using the Adomian decomposition method. **[5]**

P.T.O.

Q3) a) Solve the Fredholm integral equation

$u(x) = \tan^{-1} x + \frac{1}{2}(ln2 - \frac{\pi}{2})x + \int_0^1 x u(t)\, dt$ using the modified decomposition method. **[4]**

b) Use the variational iteration method to solve the Fredholm integral equation $u(x) = x^2 - \frac{x}{3} + \int_0^1 x\, u(t)\, dt$. **[5]**

c) Solve the following Fredholm integral equation by using the direct computation method $u(x) = e^{2x} - \frac{1}{4}(e^2 + 1)x + \int_0^1 x\, t\, u(t)\, dt$. **[5]**

Q4) a) Solve the volterra integral equation $u(x) = 4x + 2x^2 - \int_0^x u(t)\, dt$ by using the Adomian decomposition method. **[4]**

b) Use the variational iteration method to solve volterra integral equation $u(x) = x + \int_0^x (x-t)u(t)\, dt$. **[5]**

c) Solve the following volterra integral equation by using the series solution method $u(x) = 1 + 2\sin x - \int_0^x u(t)\, dt$. **[5]**

Q5) a) Solve the following volterra integral equation

$u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t-x)u(t)\, dt$ by converting it to an equivalent initial value problem. **[7]**

b) Solve the following volterra integral equation by the successive substitution method $u(x) = \frac{x^3}{3!} + \int_0^x (x-t)u(t)\, dt$. **[7]**

OR

Q6) a) Find the solution of the volterra equation of the first kind

$$xe^x = \int_0^x e^{x-t} u(t)\, dt.$$ **[7]**

b) Solve the following Fredholm integro-differential equation

$$u'(x) = \cos x + \frac{x}{4} - \frac{1}{4}\int_0^{\pi/2} xt\, u(t)\ dt,\ u(0) = 0.$$ **[7]**

Q7) a) Solve the following Fredholm integro-differential equation by using the variational iteration method

$$u'(x) = 3 - 12x + \int_0^1 t\, u(t)\ dt,\ u(0) = 1.$$ **[5]**

b) Solve the following Fredholm integro-differential equation by converting it to a standard Fredholm integral equation

$$u'(x) = 1 - \frac{x}{3} + x\int_0^1 t\, u(t)\ dt,\ u(0) = 0.$$ **[5]**

c) Solve the following volterra integro-differential equation by using the series solution method $u'(x) = 1 - 2x\sin x + \int_0^x u(t)\, dt,\ u(0) = 0.$ **[4]**

OR

Q8) a) Solve the following volterra integro-differential equation by Adomian decomposition method

$$u'(x) = 2 + \int_0^x u(t)\ dt,\ u(0) = 2.$$ **[5]**

b) Solve the following Abel's integral equation

$$\frac{\pi}{2}(x^2 - x) = \int_0^x \frac{1}{\sqrt{x-t}} u(t)\ dt \cdot$$ **[4]**

c) Solve the volterra integro-differential equation of the first kind

$$\int_0^x (x - t + 1) u'(t)\, dt = 2e^x - x - 2,\ u(0) = 1.$$ **[5]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

P267

[Total No. of Pages : 4

[5828] - 308
M.A./M.Sc. (Semester - III)
MATHEMATICS
MTUTO 138 : Differential Manifolds
(2019 Pattern) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right, indicate full marks.***

Q1) a) Let W be a linear subspace of $\mathbb{R}^n$ of dimension *k*. Then prove that there is an orthonormal basis for $\mathbb{R}^n$ whose first k elements form a basis for W. **[5]**

b) Let $x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$ **[4]**

i) Find X^{tr}. X.

ii) Find V(x)

c) Let g : A $\rightarrow$ B be a diffeomorphism of open sets in $\mathbb{R}^k$. Let $\beta : B \rightarrow \mathbb{R}^n$ be a map of class C^r; Let $\gamma = \beta(B)$ - let $\alpha = \beta o g$; then $\alpha : A \rightarrow \mathbb{R}^n$ and $\gamma = \alpha(A)$. If $f : \gamma \rightarrow \mathbb{R}$ is a continuous function, then prove that f is integrable over γ_β if and only if it is integrable over γ_α; in this case **[5]**

$$\int_{\gamma_\alpha} f dv = \int_{\gamma_\beta} f dv$$

P.T.O.

Q2) a) Let A be open in $\mathbb{R}^k$; Let f : A $\to$ $\mathbb{R}$ be of class C^r; Let γ be the graph of f in $\mathbb{R}^{k+1}$, parametrized by the function $\alpha : A \to \mathbb{R}^{k+1}$, given by $\alpha(x) = (x, f_{(x)})$. Express $v(\gamma_\alpha)$ as an integral. **[5]**

b) Let M be a manifold in $\mathbb{R}^n$, and Let $\alpha : U \to v$ be a co-ordinate patch on M. If U_0 is a subset of U that is open in U, then prove that the restriction of α to U_0 is also a coordinate patch on M. **[5]**

c) Let $\beta : 1H^1 \to \mathbb{R}^2$ be the map $\beta(x) = (x, x^2)$; Let N be the image set of β. Show that N is a 1-manifold in $\mathbb{R}^2$. **[4]**

Q3) a) Let M be a k-manifold in $\mathbb{R}^n$, of class C^r. If ∂M is non empty, then prove that ∂M is a $k-1$ manifold without boundary in $\mathbb{R}^n$ of class C^r. **[5]**

b) Prove that if the support of f can be covered by a single coordinate patch, the integral $\int_M f \, dv$ is well-defined, independent of the choice of co-ordinate patch. **[5]**

c) Let M be a compact k-manifold in $\mathbb{R}^n$. Let $h : \mathbb{R}^n \to \mathbb{R}^h$ be an isometry; Let N = h (M). Let f : N $\to$ $\mathbb{R}$ be a continuous function. Show that N is a k-manifold in $\mathbb{R}^n$ and $\int_N f \, dv = \int_M (foh) dv.$ **[4]**

Q4) a) Let v be a vector space with basis a_1, ..., a_n. Let $I = (i_1, ... i_k)$ be a k-tuple of integers form the set {1, 2, ..., n}. Then prove that there is a unique k-tensor ϕ_I on V such that, for every k-tuple $J = (j_1, ..., j_k)$ from the set {1, 2, ..., n}, $\phi_I(aj_1,.........aj_k) = \begin{cases} 0 & \text{if} \quad I \neq J, \\ 1 & \text{if} \quad I = J. \end{cases}$ **[7]**

Also show that tensors ϕ_I form a basis for $L^k(V)$.

b) Let T : V $\to$ W be a linear transformation; Let T* : $L^k(W) \to L^k(V)$ be the dual transformation. Then prove that **[7]**

i) T* is linear.

ii) $T^* (f \otimes g) = T^* f \otimes T^* g$.

iii) If S : W $\to$ X is a linear transformation, then (SOT)* f = T*(S * f).

Q5) a) Let f be a k - tensor on V; Let 6, $T \in Sk$. Prove that the tensor f is alternating if and only if f^6 = (sgn 6) f for all 6. If f is alternating and if $v_p = v_q$ with $p \neq q$, then prove that $f(v_1, v_2, ... v_k) = 0$. **[5]**

b) Let T : V $\to$ W be a linear transformation. If f is an alternating tensor on W, then prove that T* f is an alternating tensor on v. **[5]**

c) Is $f(x, y) = x_1 y_2 - x_2 y_1 + x_1 y_1$ alternating tensors in $\mathbb{R}^4$? Why? **[4]**

Q6) a) Let $x, y, z \in \mathbb{R}^5$. Let

$G(x, y) = x_1 y_3 + x_3 y_1$

$F(x, y, z) = 2x_2 y_2 z_1 + x_1 y_5 z_4$.

Write AF and AG in terms of elementary alternating tensors. Express (AF) (x, y, z) as a function. **[5]**

b) Let w be a k-form on the open set A of $\mathbb{R}^n$. Then prove that w is of class C^r if and only if its component functions b_1 are of class C^r on A. **[5]**

c) Let $r : \mathbb{R} \to \mathbb{R}^n$ be of class C^r. Show that the velocity of γ corresponding to the parameter value t is the vector $\gamma_*(t, e_1)$. **[4]**

Q7) a) Let $A = \mathbb{R}^2 - 0$; consider the 1-form in A defined the equation

$w = (xdx + y\,dy)/(cx^2 + y^2)$. Show that w is closed, also show that w is exact on A. **[5]**

b) Let A be open in $\mathbb{R}^k$; Let $\alpha : A \to \mathbb{R}^n$ be of class C^∞. If w is an l - form defined in an open set of $\mathbb{R}^n$ containing $\alpha(A)$, then prove that $\alpha^*(dw) = d(\alpha^* w)$. **[7]**

c) Define parametrized - manifold of dimension k and volume. **[2]**

Q8) a) Let $k > 1$. Let M be a compact oriented k - manifold in $\mathbb{R}^n$; give ∂M the induced orientation if ∂M is not empty. Let w be a k–1 form defined in an open set of $\mathbb{R}^n$ containing M. Then prove that **[7]**

$$\int_{M} dw = \int_{\partial M} w$$

b) Let M be a compact oriented k-manifold in $\mathbb{R}^n$; Let w be a k-form defined in an open set of $\mathbb{R}^n$ containing M. Let λ be the scalar function on M defined by the equation $\lambda(p) = w(p)\ ((p\ ;\ a_1), \ldots\ldots.., (p\ ;\ q_k))$, where $((p\ ;\ q_1), \ldots\ldots.., (p\ ;\ q_k))$ is any orthonormal frame in the linear space $T_p(M)$ belonging to its natural orientation. Then prove that λ is continuous, and $\int_{M} w = \int_{M} \lambda\ dv$. **[7]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

P268

[Total No. of Pages : 3

[5828]-401
M.A./M.Sc.
MATHEMATICS
MTUT141 : Fourier Series and Boundary Value Problems
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let f denote a function that is piecewise continuous on the interval $-\pi < x < \pi$ and periodic with period 2π on entire x axis. Then prove that Fourier series converges to the mean value $\frac{f(x_1+)+f(x-)}{2}$ of the one sided limits of f at each point x $(-\infty < x < \infty)$. Where both of the one sided derivatives $f_R'(0)$ & $f_L'(0)$ exists. **[7]**

b) Find the Fourier cosine series for the function $f(x) = x^4$ $(0 < x < \pi)$. **[5]**

c) Prove or disprove all Fourier series are differentiable. **[2]**

Q2) a) Let f denote a function such that

i) f is continuous on the interval $-\pi \leq x \leq \pi$

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is piecewise continuous on the interval $-\pi < x < \pi$.

If a_n & b_n are the Fourier coefficients $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$ for f, then prove that the series

$\sum_{n=1}^{\infty}\sqrt{a_n^2 + b_n^2}$ converges. **[7]**

b) Find the Fourier sine series for the function $f(x) = x(\pi^2-x^2)$ $(0 < x < \pi)$. **[5]**

c) If $f(x) = \frac{e^x - 1}{x}$ $(x \neq 0)$, then find $f(0+)$ and $f_R'(0)$. **[2]**

P.T.O.

Q3) a) Let f be a function that is piecewise continuous on the interval $-\pi < x < \pi$. Then prove that

$$\int_{-\pi}^{x} f(s)\, ds = \frac{a_0}{2}(x+\pi) + \sum_{n=1}^{\infty} \frac{1}{n}[a_n \sin nx - b_n[\cos nx + (-1)]^{n+1}]$$

$-\pi \leq x \leq \pi$ **[6]**

b) Find the Fourier series on the interval $-\pi < x < \pi$ that corresponds to the function $f(x) = x + \frac{1}{4}x^2\ (-\pi < x < \pi)$ **[5]**

c) Obtain the Fourier cosine series on $0 < x < c$ from the following series on $0 < x < \pi$

$$x^2 \sim \frac{\pi^3}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx \quad 0 < x < \pi$$ **[3]**

Q4) a) Solve the following boundary value problem

$u_{xx}(x,y) + u_{yy}(x,y) = 0 \quad (0 < x < \pi,\ 0 < y < 2)$

$u_x(0,y) = u_x(\pi,y) = 0, \quad u(x,0) = 0$

$u(x,2) = f(x)$ **[6]**

b) Solve the following boundary value problem

$u_t(x, t) = k u_{xx}(x, t)\ (0 < x < \pi,\ t > 0)$

$u(0, t) = 0,\ u(\pi, t) = 0,\ u(x, 0) = f(x)$ **[6]**

c) If L is the linear operator $L = a^2 \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2}$ and $y_n = \sin\left(\frac{n\pi}{c}\right)x.\cos\left(\frac{n\pi a}{c}\right)t$, then show that $Ly_n = 0\ (n = 1, 2, ...)$ **[2]**

Q5) a) Solve the following boundary value problem

$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_\rho(\rho,\phi) + u_{\phi\phi}(\rho,\phi) = 0\ (1 < \rho < b,\ 0 < \phi < \pi)$

$u(\rho,0) = 0,\ u(\rho,\pi) = 0, \quad (1 < \rho < b)$

$u(1,\phi) = 0\ ,\ u(b,\phi) = u_0, \quad (0 < \phi < \pi)$ **[7]**

b) Solve the following boundary value problem

$u_{xx}(x,y) + u_{yy}(x,y) = 0\ (0 < x < a,\ 0 < y < b)$

$u(0,y) = 0,\ u(a,y) = 0\ (0 < y < b)$

$u(x, 0) = f(x),\ u(x, b) = 0\ (0 < x < a)$ **[7]**

Q6) a) Find the stedy temperatures $u(\rho, \phi)$ in a thin disk $\rho \leq 1$, with insulated surfaces when it's edge $\rho = 1$ is kept at temperatures $f(\phi)$. The variables ρ & ϕ are polar co-ordinates & 4 satisfies Laplaces equation $\nabla^2 u = 0$ i.e. $\rho^2 u_{\rho\rho}(\rho, \phi) + \rho u_{\rho}(\rho, \phi) + u_{\phi\phi}(\rho, \phi) = 0$ $(0 < \rho < 1$ $u(1, \phi) = f(\phi))$ u & it's first order partial derivatives $-\pi < \phi < \pi)$ are continuous on the ray $\phi = \pi$. **[7]**

b) If $X(x)$ & $Y(x)$ are eigen function's corresponding to the same eigenvalue of a regular Sturm-Liouville problem, then prove that $y(x) = cX(x)$ where c is non zero constant. **[5]**

c) Show that $\psi_1(x) = x$ and $\psi_2(x) = 1 - 3x^2$ are orthogonal on the interval $-1 < x < 1$. **[2]**

Q7) a) Prove that the orthonormal set $\{\phi_n(x)\}$ is complete in the space in which each function f has these properties : **[7]**

i) f is continuous on the interval $-\pi \leq x \leq \pi$.

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is piecewise continuous on the interval $-\pi < x < \pi$.

b) If $\phi_0(x) = \dfrac{1}{\sqrt{2\pi}}$, $\phi_{2n-1}(x) = \dfrac{1}{\sqrt{\pi}}\cos nx$,

$\phi_{2n}(x) = \dfrac{1}{\sqrt{\pi}}\sin nx$ $(n = 1, 2, \ldots\ldots\ldots)$ then show that the set $\{\phi_n(x)\}$ $(n = 0,1,2.....)$ is orthonormal on the interval $-\pi < x < \pi$. **[5]**

c) Show that each of the functions $y_1 = \dfrac{1}{x}$ and $y_2 = \dfrac{1}{1+x}$ satisfies the nonlinear differential equation $y' + y^2 = 0$ Then show that the sum $y_1 + y_2$ fails to satisfy that equation. **[2]**

Q8) a) If λ_m and λ_n are distinct eigenvalues of the Sturm - Liouville problem. $[r(x)X^1(x)]^1 + [g(x) + \lambda P(x)]\, X(x) = 0 \quad a < x < b$ under the condition $a_1\, X(a) + a_2\, X^1(a) = 0$, $b_1\, X(b) + b_2\, X^1(b) = 0$ then prove that corresponding eigen functions $X_m(x)$ and $X_n(x)$ are orthogonal with respect to weight function $p(x)$ on the interval $a < x < b$. **[6]**

b) Find eigenvalues & normalized eigen function of Sturm-Liouville problem. $X''(x) + \lambda X(x) = 0$, $X(0) = 0$, $hX(1) + X'(1) = 0$ $(h > 0)$. **[3]**

c) Solve the following boundary value problem.

$u_{xx}(x, y) + u_{yy}(x, y) = 0 \quad (0 < x < \pi, y > 0)$

$u_x(0, y) = 0$, $u(\pi, y) = 0 \quad (y > 0)$

$-Ku_y(x, 0) = f(x) \quad (0 < x < \pi)$ where K is positive constant. **[5]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

P269

[Total No. of Pages : 3

[5828]-402
M.A/M.Sc.
MATHEMATICS
MTUT - 142 : Differential Geometry
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to right indicate full marks.***

Q1) a) Let S be an n-surface in R^{n+1}, let $\alpha : I \to S$ be a parametrized curve in S, let $t_0 \in I$ and let $\vec{v} \in S_{\alpha}(t_0)$. Prove that there exists a unique vector field $\vec{V}$ tangent to S along α, which is parallel and has $\vec{V}(t_0) = \vec{v}$. **[7]**

b) Compute Weingarten Map of n-sphere $x_1^2 + --- + x_{n+1}^2 = r^2, r > 0$ oriented by inward unit normal vector field $\vec{N}$. **[5]**

c) Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 + x_2$. **[2]**

Q2) a) Let $S \subseteq R^{n+1}$ be a connected n-surface in R^{n+1}. Prove that there exist on S exactly two smooth unit normal vector fields $\vec{N}_1$ and $\vec{N}_2$ with $\vec{N}_2(p) = -\vec{N}_1(p)$ for all $p \in S$. **[7]**

b) Let S be the hyperboloid $-x_1^2 + x_2^2 + x_3^2 = 1$ oriented by the unit normal vector field $N(p) = \left(p_1 \frac{-x_1}{\|p\|}, \frac{x_2}{\|p\|}, \frac{x_3}{\|p\|}\right), p = (x_1, x_2, x_3) \in S$ then find normal curvature of S at p = (0, 0, 1). **[5]**

c) Define term n-surface in R^{n+1} with an example. **[2]**

P.T.O.

Q3) a) State and Prove Lagranges Multiplier theorem. **[7]**

b) Show that gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p. **[5]**

c) Show by example that set of vectors tangent at a point p of a level set need not in general be a vector subspace of R_p^{n+1}. **[2]**

Q4) a) Let S be an oriented n-surface in R^{n+1} and let $\vec{v}$ be a unit vector in Sp, $p \in S$. Show that there exists an open set $V \subset R^{n+1}$ containing p such that $S \cap N(\vec{v}) \cap V$ is a plane curve. Also show that curvature at p of this curve is equal to normal curvature $k(\vec{v})$. **[7]**

b) Show that for each $a, b, c, d \in R$ the parametrized curve

$\alpha(t) = (\cos(at+b), \sin(at+b), ct+d)$ is a geodesic in the cylinder $x_1^2 + x_2^2 = 1$ in R^3. **[5]**

c) Let $\vec{X}$ be smooth vector field along the parametrized curve $\alpha : I \to R^{n+1}$ and f be smooth function along $\alpha(t)$. Prove that $(\dot{f\vec{X}}) = f'\vec{X} + f\dot{\vec{X}}$. **[2]**

Q5) a) Find global parametrization of circle $(x_1 - a)^2 + (x_2 - b)^2 = r^2$. **[5]**

b) Let S be an n-surface in R^{n+1}, oriented by unit normal vector field $\vec{N}$. Let $p \in S$ and $\vec{v} \in S_p$. Then show that for every parametrized curve $\alpha : I \to S$ with $\alpha(t_0) = \vec{v}$ for some $t_0 \in I$ $\ddot{\alpha}(t_0).\vec{N}(p) = L_p(\vec{v}).\vec{v}$ **[5]**

c) Let $f : U \to R$ be a smooth function, where $U \subset R^{n+1}$ is an open set and let $\alpha : I \to U$ be a parametrized curve. Show that $fo\alpha$ is constant if and only if α is everywhere orthogonal to gradient of f. **[4]**

Q6) a) Let S be a 2-surface in R^3 and let $\alpha : I \to S$ be a geodesic in S with $\dot{\alpha} \neq 0$. Show that vector field $\vec{X}$ tangent to S along α is parallel along α if and only if both $\|\vec{X}\|$ and angle between $\vec{X}$ and $\dot{\alpha}$ are constant along α. **[5]**

b) Let $\alpha(t) = (x(t), y(t))$ $(t \in I)$ be a local parametrization of oriented plane curve C. Show that $k o \alpha = \dfrac{x' y'' - y' x''}{\left(x'^2 + y'^2\right)^{3/2}}$. **[5]**

c) Show that $x_1^2 + --- + x_{n+1}^2 = 1$ is n-surface whenever $(x_1, ---, x_{n+1}) \neq (0, 0, --- 0)$. **[4]**

Q7) a) Let S be a compact oriented connected n-surface in R^{n+1} exhibited as a level set $f^{-1}(c)$ of a smooth function $f : R^{n+1} \to R$ with $\nabla f(p) \neq 0$ for all $p \in S$. Prove that Gauss Map Maps S onto unit sphere S^n. **[7]**

b) Let $\eta = \dfrac{-x_2}{x_1^2 + x_2^2} dx_1 + \dfrac{x_1}{x_1^2 + x_2^2} dx_2$ be the 1-form on $R^2 - \{0\}$ and C denote the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ oriented by its inward normal and $\alpha : [0, 2\pi] \to C$ defined by $\alpha(t) = (a\cos t, b\sin t)$ be parametric curve whose restriction to $[0, 2\pi]$ is one-one global parametrization of C, then find $\int_\alpha \eta$. Is η exact? **[7]**

Q8) a) Prove that on each compact oriented n-surface S in R^{n+1}, there exists a point p such that the second fundamental form at p is definite. **[7]**

b) Let $a, b, c \in R$ be such that $ac - b^2 > 0$ then show that the maximum and minimum values of the function $g(x_1, x_2) = ax_1^2 + 2bx_1x_2 + cx_2^2$ on unit circle $x_1^2 + x_2^2 = 1$ are of the form λ_1, λ_2 where λ_1, λ_2 are eigenvalues of Matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. **[7]**

Total No. of Questions : 4]

SEAT No. :

P270

[Total No. of Pages : 1

[5828]-403
M.A./M.Sc.
MATHEMATICS
MTUT 143 : Introduction to Data Science
(2019 Pattern) (Semester - IV)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***Figures to the right indicate full marks.***
2) ***Question 1 is compulsory.***
3) ***Attempt any two questions from Q.2, 3 and 4.***

Q1) Define the term big data and mention all steps involved in the process of data science. **[7]**

Q2) a) Give any five forms of data. **[5]**

b) Describe data transformation process. **[5]**

c) State details to be covered in a project charter. **[4]**

Q3) a) What is machine learning? Where it is used in data science process? Give any two Pythol tools used in machine learning. **[5]**

b) Write a short note on types of machine learning. **[5]**

c) State problems occurring in handling large data. **[4]**

Q4) a) Explain in detail concept of Hadoop and its Components. **[5]**

b) Write a short note on text mining and handling techniques to it. **[5]**

c) Give packages in Python which are used for text mining. **[4]**

Total No. of Questions : 8]

SEAT No. :

P271

[Total No. of Pages : 3

[5828]-404

M.A./M.Sc.

MATHEMATICS

MTUTO 144 : Number Theory

(2019 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any Five qustions.***
2) ***Figures to the right indicates full marks.***

Q1) a) Prove that every nonzero nonunit of an integral domain R is a product of irreducibles. **[7]**

b) If a, b and c are integers such that a/bc and a,b both are relatively prime then show that a/c. **[4]**

c) Show that $8/n^2-1$, for any odd integer n. **[3]**

Q2) a) Let $k[x]$ denotes ring of polynomials with coefficients in a field k and $f, g \in k[x]$. If $g \neq o$, then prove that there exist polynomials $h, r \in k[x]$ such that $f = hg + r$, where either $r = o$ or $r \neq o$ and deg r < deg g. **[6]**

b) If x and y are odd, then prove that $x^2 + y^2$ can not be a perfect square. **[4]**

c) Show that 2 is divisible by $(1+i)^2$ in $\mathbb{Z}[i]$. **[4]**

Q3) a) If $a, b, m \in \mathbb{Z}$ and $m \neq o$ then prove that

i) $\overline{a} = \overline{b}$ if and only if $a \equiv b (\text{mod}\, m)$, where $\overline{a}, \overline{b}$ are congruence class modulo m.

ii) $\overline{a} \neq \overline{b}$ if and only if $\overline{a} \cap \overline{b}$ is empty.

iii) There are precisely m distinct congruence classes modulo m. **[8]**

P.T.O.

b) Find all primes q such that $\left(\frac{5}{q}\right) = -1$. **[6]**

Q4) a) State and prove Eulers theorem. **[5]**

b) If p is an odd prime then prove that $\left(\frac{a}{p}\right) \equiv a^{\left(\frac{p-1}{2}\right)} (\bmod p)$. **[5]**

c) Find $\sigma(40), \phi(40)$. **[4]**

Q5) a) If p and q are district odd primes, then prove that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left(\frac{p-1}{2}\right)\left(\frac{q-1}{2}\right)}$

[7]

b) Find all integers that satisfy the following congruences simultaneously: **[5]**

$$x \equiv 1(\bmod 3)$$
$$x \equiv 2(\bmod 4)$$
$$x \equiv 3(\bmod 5)$$

c) Show that $(a, a+2) = 1 \text{ or } 2$ for every integer a. **[2]**

Q6) a) If *x* & *y* are any real numbers, then prove that.

i) $[x]+[y] \leq [x+y] \leq [x]+[y]+1$ and

ii) $\left[\frac{[x]}{m}\right] = \left[\frac{x}{m}\right]$ if m is a positive integer. **[8]**

b) State and prove de polignac's formula. **[6]**

Q7) a) Define a divisor function *d*. And if *n* is a positive integer then prove that $d(n) = \underset{p^{\alpha}\|n}{\pi}(\alpha+1)$ **[6]**

b) If α is any algebraic number then prove that there is a rational integer *b* such that bα is an algebraic integer. **[5]**

c) Find the smallest integer *x* for which $\phi(x) = 6$. **[3]**

Q8) a) If ξ is an algebraic number of degree n, then prove that every number in $Q(\xi)$ can be written uniquely in the form $a_0 + a_1\xi + ... + a_{n-1}\xi^{n-1}$ where the ai are rational numbers. **[7]**

b) If $F(n) = \sum_{d/n} f(d)$ for every positive integer *n*, then prove that $f(n) = \sum_{d/n} \mu(d) \, \mathrm{F}\left(\frac{n}{d}\right)$. **[5]**

c) Find the minimal polynomic of the algebraic number $\frac{1+\sqrt[3]{7}}{2}$. **[2]**

❍❍❍❍

Total No. of Questions : 8]

SEAT No. :

P272

[Total No. of Pages : 3

[5828]-405
M.A./M.Sc.
MATHEMATICS
MTUTO 145 : Algebraic Topology
(2019 Pattern) (CBCS) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any Five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that the homotopy relation is an equivalence relation. **[5]**

b) Show that if $h,h^1 : x \to y$ are homotopic and $k,k^1 : y \to z$ then koh and k′oh′ are homotopic. **[5]**

c) Given spaces *X* and *Y*. Let [*X*,*Y*] denote the set of homotopy classes of Maps of *X* into *Y*. Let I = [0, 1], show that for any X, the [X, I] has single element. **[4]**

Q2) a) A space X is said to be contractible if the identity map $\text{Ix} : x \to x$ is nulhomotopic then show that I and $\mathbb{R}$ contractible. **[5]**

b) Let α be a path in x from x_0 to x, define a map $\hat{\alpha} : \pi_1(\text{X}, x.) \to \pi_1(\text{X}, x_1)$ by $\hat{\alpha}([F]) = [\overline{\alpha}] * [F] * [\alpha]$ then show that $\hat{\alpha}$ is a group homomorphism. **[5]**

c) Define the following term **[4]**

i) Simply connected   ii) Star convex

Q3) a) Find the star convex set that is not convex. **[5]**

b) Let α be a path in X from x_0 to x_1 Let β be a path in X from x_1 to x_2 Show that if $\gamma = \alpha * \beta$

then $\hat{\gamma} = \hat{\beta} o \hat{\alpha}$. **[5]**

c) Define a covering map.Show that a covering map is a local homomorphism. **[4]**

P.T.O.

Q4) a) Give an example of a non identity covering map from S^1 on to S^1. **[5]**

b) Let $q : X \to Y$ and $r : Y \to Z$ be covering maps, Let p = roq. Show that $r^{-1}(z)$ is finite for each $z \in Z$, then P is covering map. **[5]**

c) Prove that there is no retraction of B^2 onto S^1. **[4]**

Q5) a) Define the following terms. **[4]**

i) Free group

ii) Wedge of the circles

b) If $G = G_1 * G_2$ show that $\frac{G}{[G,G]} \cong \left(\frac{G_1}{[G_1,G_1]}\right) \oplus \left(\frac{G_2}{[G_2,G_2]}\right)$. **[6]**

c) State Seifert - Van Kampen theorem. **[4]**

Q6) a) Let X be the wedge of circle S_α for $\alpha \in J$, then X is normal. **[5]**

b) Show that if X is an infinite wedge of circles, then X does not satify the first countability axiom. **[5]**

c) Prove that the fundamental group of the torus is a free abelian group of rank 2. **[4]**

Q7) a) Find spaces whose fundamental group is isomorphic to the following groups. **[8]**

i) $Zn \times Zm$

ii) $Zn * Zm$

b) Let $\pi : E \to X$ be a closed quotient map. If E is normal then so is X. **[6]**

Q8) a) Let $P:E\to B$ and $P':E'\to B'$ be covering maps, Let $p(e_0)=p'(e'o)=b_0$. There is an equivalence $h:E\to E'$ such that $h(e_o)=e'_o$ if and only if the groups $H_o=p_*(\pi_1(E,e_o))$ and $H'_o=p'_*(\pi_1(E',e'_o))$ are equal. If h exist, it is unique. **[6]**

b) Show that if n > 1, every continous map $f:S^n\to S^1$ is nulhomotopic.**[4]**

c) Find a continous map of the torus into S^1 that is not nulhomotopic. **[4]**

❍❍❍❍

Total No. of Questions : 8]

SEAT No. :

P273

[Total No. of Pages : 3

[5828]-406
M.A./M.Sc.
MATHEMATICS
MTUTO - 146 : Representation Theory of Finite Groups
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove triangle inequality. **[4]**

b) Verify whether following matrices are diagonalizable. **[6]**

i) $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii) $B \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

c) Define following terms : **[4]**

i) Decomposable representation.

ii) Unitary representation.

Q2) a) Let $\varphi : G \to G\,L\,(v)$ be a unitary representation of a group then show that φ is either irreducible or decomposable. **[6]**

b) Give an example of an indecomposible representation of Z which is not irreducible. **[4]**

c) Show that Z [L(G)] is a subspace of L(G). **[4]**

P.T.O.

Q3) a) Prove that a representation ρ is irreducible if and only if $< X_e, Y_\rho > = 1$. **[5]**

b) Let L be rectangular representation of G, then prove that the decomposition $L \sim d_1 \varphi^{(1)} \oplus d_2 \varphi^{(2)} \oplus$________ $\oplus ds\ \varphi^{(s)}$ holds **[5]**

c) Show that the formula

$|G| = d_1^2 + d_2^2 +$ _______ $+ ds^2$ holds **[4]**

Q4) a) Show that the set $B = \left\{ \sqrt{dk}\varphi_{(i)}^{(k)} | 1 \le (i.j) \le dk \right\}$ is an orthonormal basis for L (G) **[5]**

b) Prove that a finite group G is abelian if and only if it has |G| equivalence classes of irreducible representations. **[5]**

c) Define following terms : **[4]**

i) Character table

ii) Multiplicity

Q5) a) Prove that the set L(G) is a ring with addition taken pointwise and convolution as multiplication. **[4]**

b) Prove that the map $T = L(G) \rightarrow L(\hat{G})$ given by $Tf = \hat{f}$ is an invertible linear transformation. **[5]**

c) Prove that the linear map $T = L(G) \rightarrow L(\hat{G})$ given by $Tf = \hat{f}$ provides a ring isomorphism between (L(G), + ,*) and $(L(\hat{G}),+,*)$ **[5]**

Q6) a) Define the term completely reducible. **[2]**

b) Write an example of irreducible representation. **[2]**

c) Define Inner product space. **[2]**

d) State and prove Kayley Hamiton theorem. **[4]**

e) Show that $\varphi : Z_{/n} Z \rightarrow C^*$ defined by $\varphi(m) = e^{\frac{2\pi im}{n}}$ is a representation. **[4]**

Q7) a) Prove that every representation of a finite group is completely reducible. **[5]**

b) Define $\varphi : R \to T$ by $\varphi(t) = e^{2\pi it}$. Then prove that φ is unitary representation of the additive group of $\mathbb{R}$. **[5]**

c) Let $\varphi : G \to GL(v)$ be a non zero representation of a finite group. Then prove that φ is either irreducible or decomposible. **[4]**

Q8) a) Let the map $T = L(G) \to L(\hat{G})$ is given by $Tf = \hat{f}$. Then prove that T is an invertible linear transformation. **[4]**

b) Define the terms : **[5]**

Fourier transform and convolution. Further state where the fourier transform of cyclic group is used?

c) Prove that the class function form the center of L(G). **[5]**

❄ ❄ ❄

Total No. of Questions : 8]

SEAT No. :

P274

[Total No. of Pages : 3

[5828]-407
M.A./M.Sc.
MATHEMATICS
MTUTO147 : Coding Theory
(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) Prove that a code C is u - error-detecting if and only if $d(c) \geq u + 1$. **[5]**

b) For $S = \{101, 111, 010\} \subseteq F_2^3$, find F_2 - linear span $< S >$ and its orthogonal compliment $S^{\perp}$. **[5]**

c) Show that binary Hamming codes are perfect codes. **[4]**

Q2) a) Let V be a vector space over F_q. If dim (V) = K then prove that

i) V has q^k elements.

ii) V has $\frac{1}{K!}\prod_{i=0}^{k-1}(q^k - q^i)$ different bases. **[7]**

b) If $g(x) = (1+ x)(1 + x^2 + x^3) \in F_2(x) / (x^7-1)$ is a generator polynomial of cyclic code C then find C and dim (C) **[4]**

c) Let C be a binary linear code with parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find d(C). **[3]**

P.T.O.

Q3) a) For an integer q > 1 and itegers n, d such that $1 \le d \le n$, Prove that

$$\frac{q^n}{\sum_{i=0}^{d-1} \binom{n}{i}(q-1)^i} \le A_q(n,d)$$ **[5]**

b) Construct the incomplete maximum likelihood decoding table for binary code C = {000, 001, 010, 011} **[5]**

c) If C and D are two linear codes over F_q of same length then prove that $C \cap D$ is also a linear code over F_q. **[4]**

Q4) a) If $C = \{0000, 1011, 0101, 1110\} \subseteq F_2^4$ is a linear code then decode

i) w = 1101

ii) w = 0111

by using nearest neighbour decoding for linear code. **[6]**

b) Let $g(x)$ be the generator polynomial of an ideal of $F_q(x) \mid (x^n-1)$. If degree of $g(x)$ is $n-k$ then prove that dimension of cyclic code corresponding to the ideal is K. **[5]**

c) Is $C = \{(0,1,1,2), (2,0,1,1), (1,2,0,1), (1,1,2,0)\} \subseteq F_3^4$ cyclic code? Justify. **[3]**

Q5) a) Let S be a subset of F_q^n. Prove that dim (<S>) + dim ($S^\perp$) = n. **[5]**

b) Let $S = \{0100, 0101\} \subseteq F_2^4$. Verify that dim (<S>) + dim ($S^\perp$) = n. **[5]**

c) If q is a prime power then show that $Bq(n, n) = Aq(n, n) = q$. **[4]**

Q6) a) Prove that for all integers $\gamma \ge 0$,. a sphere of radius γ in A^n contains exactly $V_q^n(\gamma)$ vectors. where A is a alphabet of size q > 1. **[6]**

b) Suppose that codewords from the binary code {000, 100, 111} are being sent over a binary symmetric channel with crossover probability P = 0.03. Use maximum likelihood decoding rule to decode w = 011. **[4]**

c) Find dimension of the binary BCH code of length 15 with designed distance 3 generated by $g(x) = lcm\,(M^{(2)}(x), M^{(3)}(x))$ **[4]**

Q7) a) Find a generator matrix and a parity - check matrix for the linear code $C = <S>$ where $S = \{110000, 011000, 001100, 000110, 000011\} \subseteq F_3^6$. **[6]**

b) Let C be an $[n_1\ k]$ - linear code over Fq with generator matrix G then prove that $V \in C^{\perp}$ if and only if $V.G^T = 0$. **[5]**

c) Let $C = \{00000000, 11110000, 11111111\} \subseteq F_2^8$.

Exactly how many errors will C correct? **[3]**

Q8) a) Let C be an (n, k, d) - linear code over finite field F_q then prove that

i) Two cosets are either equal or they have empty intersection.

ii) For all u, v $\in F_q^n$, $u - v \in C$ if and only if u and v are in same coset. **[7]**

b) Consider (7, 4, 3) - binary Hamming code with generator polynomial $g(x) = 1 + x^2 + x^3$ and received word $w = 1011100$. Decode w. **[7]**

❄ ❄ ❄

Total No. of Questions : 8]

SEAT No. :

P275

[Total No. of Pages : 4

[5828]-408
S.Y. M.A./M.Sc.
MATHEMATICS
MTUTO-148: Probability and Statistics
(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of scientific calculator is allowed.***

Q1) a) Attempt following : **[4]**

i) State Baye's theorem.

ii) Define conditional probability of an event.

b) One bag contains 4 white balls and 3 black balls and second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from second bag is black? **[5]**

c) Suppose that the error in the reaction temperature in ºC for a controlled laboratory experiment is continuous random variable X having the probability density function

$$f(x) = \begin{cases} \dfrac{x^2}{3} & -1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

i) Verify that $f(x)$ is a density function

ii) Find $P(0 < x \leq 1)$

[5]

P.T.O.

Q2) a) Prove that the covariance of two random variables X and Y with means μ_x and μ_y respectively is given by $\sigma_{xy} = E(XY) - \mu_x\mu_y$. **[4]**

b) The fraction X of male runners and the fraction Y of female runners who complete in Marathon races are described by the joint density function.

$$f(x,y) = \begin{cases} 8xy & 0 \le y \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$ **[5]**

c) Suppose that the number of cars X that pass through the car wash between 4.00 p.m. to 5.00 p.m. on Friday has the following probability distribution.

x	4	5	6	7	8	9
P(x)	1/12	1/12	1/4	1/4	1/6	1/6

Let $g(x) = 2x - 1$ represent the amount of money paid to the attendant by the manager. Find the attendants expected earning for this particular time period. **[5]**

Q3) a) Prove that the mean and variance of a variable following the geometric distribution are $\mu = \frac{1}{p}$ and $\sigma^2 = \frac{1-p}{p^2}$. **[4]**

b) A home owner plants 6 bulbs selected at random from a box containing 5 tulip bulbs and 4 daffodil bulbs. What is the probability that he planted two daffodil bulbs and 4 tulip bulbs. **[5]**

c) On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection. **[5]**

i) Exactly five accidents will occur

ii) At least two accidents will occur

Q4) a) Prove that the mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2 respectively. Further show that the Standard deviation is σ. **[4]**

b) Suppose that a system contains a certain type of component whose time in years, to failure is given by T. The random variable T is modelled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years. **[5]**

c) Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$. Find the probability that X assumes the value greater than 362. **[5]**

Q5) a) Prove that an unbiased estimate of σ^2 is $S^2 = \dfrac{Syy - b_1 5xy}{n-2}$. **[4]**

b) Compute and interpret the correlation coefficient for the following data. **[5]**

Mathematics grade	English grade
70	74
92	84
80	63
74	87
65	78
83	90

c) The life in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. If 100 of these switches are installed in different systems. What is the probability that at most 30 fail during the first years. **[5]**

Q6) a) A manufacturing firm employes three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact plant 1, 2, and 3 are used for 30%, 20% and 50% of the products respectively. The defect rate is different for 3 procedures as follows. $P(D|P_1) = 0.01$, $P(D|P_2) = 0.03$, $P(D|P_3) = 0.02$ where $P(D|P_i)$ is the probability of the defective product given plan i. If a random product was observed and found to be defective, which plan was observed most likely used and thus responsible. **[6]**

b) If $X_1, X_2, \ldots X_n$ are mutually independent random variables that have respectively Chisquared distributions with $V_1, V_2, \ldots, V_n$ degrees of freedom then prove that the random variable $Y = X_1 + X_2 + \ldots + X_n$ has a Chi squared distribution with $V = V_1 + V_2 + \ldots + V_n$ degrees of freedom. **[4]**

c) In how many ways can 7 graduates be assigned to one triple and two double hotel rooms during a conference. **[4]**

Q7) a) Let X and Y be two random variable with moment generating functions $M_x(t)$ and $M_y(t)$ respectively. If $M_x(t) = M_y(t)$ for all values of t then prove that X & Y have the same probability distribution. **[4]**

b) Show that the moment generating function of a random variable X having normal probability distribution with mean μ and variance σ^2 is given by $M_x(t) = e^{(\mu t + \frac{1}{2}\sigma^2 t^2)}$. **[5]**

c) A group of 10 individuals is used for a biological case study. The group contains 3 people with blood type 0, 4 with blood type A, 3 with blood type B. What is the probability that a random sample of 5 will contain 1 person with blood type 0, 2 people with blood type A and two people with blood type B. **[5]**

Q8) a) Prove that mean and variance of gamma distribution are μ = αβ and $\sigma^2 = \alpha\beta^2$. **[4]**

b) What are the implications of a transformed model. **[5]**

c) In an National Basketball Association NBA championship series, the team that wins 4 games out of seven is the winner. Suppose that teams A and B face each other in the championship games and that team A has probability 0.55 of winning a game over team B.

i) What is the probability that team A will win the series in 6 games?

ii) What is the probability that team A will win the series?

[5]

✷ ✷ ✷