Total No. of Questions : 8]

SEAT No. : [ ]

PC3695

[Total No. of Pages : 4

[6334]-101
First Year M.A./M.Sc.
MATHEMATICS
MTUT-111 : Linear Algebra
(2019 Pattern) (Semester-I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let S be a subspace of a vector space V over a field F, such that S is generated by n vectors $a_1, \ldots, a_n$. Suppose $b_1, \ldots, b_m$ are vectors in S, with $m > n$ then prove that the vectors $b_1, \ldots, b_m$ are linearly dependent. **[7]**

b) Show that, the diagonals of parallelogram bisect each other. **[4]**

c) Test the linear dependence of the following set of vectors in R_3. **[3]**

(1, 1, 2), (3, 1, 2), (–1, 0, 0)

Q2) a) Let $T \in L(V,V)$ for some finite dimensional vector space V over F. Then prove that the following statements are equivalents: **[7]**

i) T is invertible.

ii) T is one to one.

iii) T is onto.

b) Let $S \in L(V,V)$ be given by **[7]**

$S(u_1) = u_1 - u_2$

$S(u_2) = u_1$

Where $\{u_1 = (1, 0), u_2 = (0,1)\}$ is a basis for V.

Let A be matrix of S with respect to the basis $\{u_1, u_2\}$ and B be the matrix of S with respect to $\{w_1, w_2\}$ given by

$w_1 = 3u_1 - u_2$

$w_2 = u_1 + u_2$

Find A, B and an invertible matrix X such that $X^{-1} A X = B$

P.T.O.

Q3) a) Let T$\in$L (V, V) be a linear transformation where V is a finite dimensional. Show that, the following statements are equivalents: **[7]**

i) T is orthogonal transformation.

ii) Inner product (T(u), T(v)) = Inner product (u,v) for all u, $v \in$ V.

iii) For some orthonormal basis $\{\mu_1,....,u_n\}$ of V the vectors $\{T(u_1), ...T(u_n)\}$ also form an orthonormal set.

b) Show that, the functions $f_n(x) = \cos nx$, $n = 1, 2,......$ form an orthonormal set in the vector space $C([-\Pi,\Pi])$ of continuous real valued functions on the closed interval $[-\Pi,\Pi]$ with respect to the inner product. **[7]**

$$(f \cdot g) = \frac{1}{\Pi}\int_{-\pi}^{\pi} f(x) \cdot g(x)dx$$

for continuous functions $f \cdot g \in C([-\Pi,\Pi])$.

Q4) a) Show that, a linear trasformation T$\in$L (V, V) is diagonable if and only if the minimal polynomial of T has the form $m(x) = (x-\xi_1).....(x-\xi_3)$ with distinct $\xi_1,....,\xi_3$ in F. **[7]**

b) Find the rational canonical form over the field of rational numbers of matrix A. **[4]**

Where $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

c) If for all u, $v \in$ V $\|u\| = \|v\| = 1$ then show that $|(u, v)| \le 1$. Where (u,v) represents an inner product of u and v. **[3]**

Q5) a) Let T be a linear transformation on a vector space over the complex number such that, **[7]**

$T(v_1) = -v_1 - v_2$

$T(v_2) = v_1 - 3v_2$

Where $\{v_1 = (1, 0), v_2 = (0, 1)\}$ is a basis for the vector space then find.

i) Characterstic polynomial of T.

ii) Minimal polynomial of T.

iii) Characteristic roots of T.

iv) Charcteristic vector of T.

b) Prove that, a linear transformation T preserves distances if and only if $||T(e_1)|| = ||T(e_2)|| = 1$ and $T(e_1) \perp T(e_2)$ **[4]**

c) Compute $A_1 \dot{X} B_1$, where

$$A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$ **[3]**

Q6) a) Let $T \in L(V, V)$, let $\{v_1,....,v_n\}$ be a basis of V and $\{f_1,...., f_n\}$ the dual basis of V*.
Let A be the matrix of T with respect to the base $\{v_1,....,v_n\}$. Then prove that, the matrix of T* with respect to the basis $\{f_1,....,f_n\}$ is the transpose matrix ${}^tA$. **[7]**

b) Let $T \in L(V, V)$. A mapping $T^* : V^* \to V^*$ defined by the rule **[5]**

$(T^* f)(x) = f(Tx), x \in V$

for all $f \in V^*$. Then prove that

$T^* \in L(V^*, V^*)$.

c) Find the companion matrix of $(x^2+1)^2$ over the rational field. **[2]**

Q7) a) Let U and V be the finite dimensional vector space with bases $\{u_1,....,u_k\}$, $\{v_1,....,v_k\}$ respectively then prove that

$U_1=\{(u,o) \mid u \in U\}$ and $V_1=\{(o,v) | v \in V\}$ are subspaces of U+V and U+V is the direct sum $U_1 \oplus V_1$ **[7]**

b) Let T be an orthogonal transformation on a real vector space V with an inner product then prove that V is a direct sum of irreducible invariant subspaces $\{W_1,...., W_s\}$ for $s \geq 1$, such that vectors belonging to distinct subspaces W_i and W_j are orthogonal. **[7]**

Q8) a) Let T be a normal transformation and let V and V′ be eigenvectors for T and it's adjoint T′ simultaneously such that V and V′ belongs to distinct eigenvalues for T. Then prove that (v, v') = 0 **[6]**

b) Let $\{E_1, \ldots, E_s\}$ be a set of linear transformation of V such that $1 = \Sigma E_i$ and $E_i E_j = o$ if $i \neq j$. Then prove that $\{E_i\}$ are self-adjoint for $1 \leq i \leq s$ if and only if the subspaces $\{E_1 V, \ldots, E_s V\}$ are mutually orthogonal. **[6]**

c) If $f(x_1, x_2) = x_1^2 - 8x_1 x_2 - 5x_2^2$

Find symmetric matrix A whose quadratic equation is $f(x_1, x_2)$. **[2]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC3696

[Total No. of Pages : 3

[6334]-102
M.A./M.Sc.-I
MATHEMATICS
MT UT-112 : Real Analysis
(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) i) Let A be the set of irrational numbers in [0,1] prove that m*(A) = 1. **[4]**

ii) Let f be a function defined on [–1,1] as **[3]**

$$f(x)=\begin{cases} x^2\cos\frac{1}{x}, & \text{if } -1\le x\le 1 \\ 0, & \text{if } x=0 \end{cases}$$

Show that f is of bounded variation.

b) i) Let $f(x)$ = sin$x$, $x\in\left[0,\frac{\pi}{2}\right]$ and $p=\left\{0<\frac{\pi}{4}<\frac{\pi}{2}\right\}$ be the partition on of $\left[0,\frac{\pi}{2}\right]$. Find V($f$, p) and Tv (f. p). **[4]**

c) State little wood's three principles. **[3]**

Q2) a) Prove that union of Countable collection of a measurable set is measurable. **[7]**

b) Prove that any set E of real numbers with positive outer measure contains a subset which fails to be measurable. **[7]**

P.T.O.

Q3) a) State and prove Egoroff's theorem. **[7]**

b) Let f be a simple function defined on E.Then prove that for each $\in > 0$, there is a continuous function g on R and a closed F contained in E for which $f \equiv g$ on F and m (E ~ F) < $\in$. **[7]**

Q4) a) Let f be the monotone function on closed and bounded interval $[a,b]$. Then prove that f is absolutely continuous on $[a,b]$ if and only if

$$\int_a^b f' = f(b) - f(a)$$ **[7]**

b) Let function f defined on [0,1] by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^2}\right), & for\ 0 < x \leq 1 \\ 0, & for\ \ x = 0 \end{cases}$

Show that f' is not integrable over $[a,b]$. **[7]**

Q5) a) Prove that every interval is measurable. **[7]**

b) Let $\{f_n\}_{n=1}^{\infty}$ be an increasing sequence of continuous functions on $[a,b]$ to function f on $[a,b]$. Show that $\{f_n\}_{n=1}^{\infty}$ converges uniformly on $[a,b]$.**[7]**

Q6) a) Let f be an extended real valued function on E. If f is measurable on E and $f \equiv g$ almost every where on E. Prove that g is measurable. **[7]**

b) Let f be a absolutely continuous function on closed, bounded interval $[a,b]$. prove that, f is differentiable almost every where on $[a, b]$ and $\int_a^b f' = f(b) - f(a)$. **[7]**

Q7) a) State and prove Borel-Cantelli theorem. **[7]**

b) Let f be a continuous function on closed and bounded interval $[a,b]$. The family divided difference functions $\{Diff_h f\}_{0<h\leq 1}$ is uniformly integrable over $[a, b]$ then prove that f is absolutely continuous on $[a,b]$. **[7]**

Q8) a) Let E be a measurable set of finite outer measure. For each $\in > 0$ there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^{\infty}$. If $\theta = \bigcup_{k=1}^{n} I_k$, then prove that $m^* (E \sim \theta) + m^* (\theta \sim E) < \in$. **[7]**

b) Prove that outer measure is translation invariant. **[7]**

.

❋ ❋ ❋

Total No. of Questions : 8]

SEAT No. :

PC3697

[Total No. of Pages : 3

[6334]-103

First Year M.A./M.Sc.

MATHEMATICS

MTUT-115 : Ordinary Differential Equations

(2019 Pattern) (Semester-I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) i) Explain the method of solving the equation $y' + ay = b(x)$. **[5]**

ii) Explain the difference between degree and order of differential equation. **[2]**

b) i) Show that the function $\phi(x) = \frac{2}{5} + e^{-5x}$ is the solution of the equation $y' + 5y = 2$. **[3]**

ii) If $\phi(x)$ is the solution of the equation $y' + iy = x$ such that $\phi(0)=2$ then find $\phi(\pi)$. **[4]**

Q2) a) If $a(x)$, $b(x)$ are continuous functions on an interval I and $A(x)$ is a function such that $A'(x) = a(x)$. Then show that, **[7]**

$$\psi(x) = e^{-A(x)} \int_{x_0}^{x} e^{A(t)} b(t)\, dt, \quad x_0 \varepsilon \mathrm{I} \text{ is}$$

a solution of the equation $y' + a(x)y = b(x)$.

b) Consider the equation $x^2 y' + 2xy = 1$ on $(0, \infty)$ then **[7]**

i) Show that every solution tends to zero as $x \to \infty$.

ii) Find solution $\phi(x)$ which satisfies $\phi(2) = 2\phi(1)$.

P.T.O.

Q3) a) Show that $\phi_1(x)=e^{r_1x}$ and $\phi_2(x)=e^{r_2x}$ are the solutions of the equation $L(y) = y'' + a_1 y' + a_2 y = 0$, where r_1 and r_2 are the distinct roots of the characteristic polynomial $P(r)=r^2+a_1r+a_2r$ such that a_1, a_2 are constants. **[7]**

b) Show that every solution of the constant coefficient equation $y''+a_1y'+a_2y=0$ tends to 0 as $x\to\infty$ if and only if the real parts of the roots of the characteristic polynomial are negative. **[7]**

Q4) a) Define Wronskian of ϕ_1 and ϕ_2, hence show that two solutions ϕ_1, ϕ_2 of $y''+a_1y'+a_2y=0$ are linearly independent on an interval I if and only if $W(\phi_1,\phi_2)(x)\neq 0, \forall x \varepsilon I$. **[7]**

b) i) Compute $W(\phi_1,\phi_2)$ where $\phi_1(x)=e^{\alpha x}\cos\beta x$, $\phi_2(x)=e^{\alpha x}\sin\beta x$. **[3]**

ii) Show that the functions $\phi_1(x)=x^2$ and $\phi_2(x)=x|x|$ are linearly independent on $(-\infty,\infty)$. **[4]**

Q5) a) If $\phi_1,\phi_2,.....\phi_n$ are n linearly independent solutions of the equation $y^{(n)}+a_1y^{(n-1)}+....+a_ny=0$ on an interval I. Then show that $\phi=c_1\phi_1+c_2\phi_2+....+c_n\phi_n$ is also solution on an interval I. Also, show that every solution is represented in this form. **[7]**

b) i) Find the $W(\phi_1,\phi_2,\phi_3)(x)$ for $\phi_1=e^x$, $\phi_2=xe^x$ and $\phi_3=x^2e^x$ at a point $x = 0$. **[3]**

ii) Compute three linearly independent solutions of the equation

$y''' - 4y' = 0$. **[4]**

Q6) a) Show that for given n^{th}-order homogeneous equation with constant coefficients L(y)=0 has n-linearly independent solutions on I. **[7]**

b) i) Find two linearly independent solutions of the equation **[5]**

$$(3x-1)^2 y'' + (9x-3)y' - 9y = 0 \text{ for } x > \frac{1}{3}.$$

ii) Explain the difference between trivial and non-trivial solution. **[2]**

Q7) a) Show that $\phi_1(x) = |x|^{r_1}$ and $\phi_2(x) = |x|^{r_2}, r_1 \neq r_2$ are the roots of $q(r) = r(r-1) + ar + b$ forms a basis for the solutions of second order Euler equation on any interval I not containing $x = 0$. **[7]**

b) Show that $\phi_1(x) = |x|^{i}$ and $\phi_2(x) = |x|^{-i}$ are linearly independent solutions of the equation $x^2 y'' + xy' + y = 0$. **[7]**

Q8) a) Explain the variables separable method for first order equation $y' = f(x, y)$. **[7]**

b) Show that $\phi(x) = \dfrac{y_0}{1 - y_0(x - x_0)}$ is a solution of the equation $y^1 = y^2$ which passes through the point (x_0, y_0). **[7]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC3698

[Total No. of Pages :2

[6334]-104

M.A./M.Sc. - I

MATHEMATICS

MTUT-114 : Advanced Calculus

(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If $f(x,y)=\dfrac{\sin(x^2+y^2)}{x^2+y^2}$, when $(x, y) \neq (0, 0)$. How must $f(0, 0)$ be defined, so as to make f continuous at origin? **[7]**

b) Find the gradient vector at each point for the scalar field defined by the equation $f(x,y,z)=x^{y^z}$. **[7]**

Q2) a) If the partial derivatives D, f, ..., Dnf exist in some n - ball B(a) are continuous at a. Then show that f is differentiable at a. **[10]**

b) For the following scalar field f determine the set of points (x, y) at which f is continuous, $f(x,y)=\arccos\sqrt{\dfrac{x}{y}}$. **[4]**

Q3) a) If $\overline{f}$ is constant force, say $\overline{f}=\overline{c}$, then show that the work done by $\overline{f}$ in moving a particle from $\overline{a}$ to $\overline{b}$ along any piecewise smooth path joining $\overline{a}$ to $\overline{b}$ is $\overline{c}.\left(\overline{b}.\overline{a}\right)$. **[4]**

b) Evaluate the integral $\int_C (x+y)\,ds$, where C is the triangle with vertices (0, 0), (1, 0), (0, 1) traversed in a counter clockwise direction. **[10]**

P.T.O.

Q4) a) Consider a uniform semicircular wire of radius 'a' show that the centroid lies on the axis of symmetry at a distance $\frac{2a}{\pi}$ from the center. **[7]**

b) Consider $\int_C \frac{(x+y)dx-(x-y)dx}{x^2+y^2}$, where C is the circle $x^2 + y^2 = a^2$, traversed once in a counter clockwise direction. Evaluate the above integral. **[7]**

Q5) a) Evaluate the double integral $\iint_Q xy(x+y)\,dxdy$, where $Q=[0,1]\times[0,1]$. **[7]**

b) Find the Jacobian for spherical transformation. **[7]**

Q6) a) Prove that, transformation formula in general case is $\iint_S f(x,y)dxdy = \iint_T f[x(u,v), y(u,v)]\,|J(u,v)|\,du\,dv$. **[10]**

b) Define **[4]**

i) Rectifiable curve

ii) Jorden curve

Q7) a) Define **[7]**

i) Regular point

ii) Singular point

b) Show that, the area of hemispher is $2\pi a^2$. **[7]**

Q8) a) If $f(x,y,z) = xy^2z^2\bar{i} + z^2 \sin y\,\bar{j} + x^2e^y\bar{k}$. Find div $\bar{f}$. **[4]**

b) State and prove stokes' theorem. **[10]**

Total No. of Questions : 8]

SEAT No. :

PC3699

[Total No. of Pages :3

[6334]-105

M.A./M.Sc.-I

MATHEMATICS

MTUT113 : Group Theory

(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let G be a group and $a \in$ G, c(a) denote the centralizer of a in G. Prove that C(a) is a subgroup of G. **[5]**

b) Let G be a group and $a \in$ G. Prove that if a has infinite order than $a^i = a^j$ if and only if $i = j$. Also prove that if a is of finite order n then $a^i = a^j$ if and only if n divides $i - j$. **[5]**

c) Give an example of a non-Abelian group all of whose subgroups are Abelian. **[4]**

Q2) a) Prove that every subgroup of a cyclic group is cyclic. Moreover prove that if $|< a >| = n$, then order of any subgroup of $< a >$ is a divisor of n. **[5]**

b) Let S_n be a permutation group on n symbols and $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$, $\beta = (\beta_1, \beta_2, ..., \beta_n)$ be disjoint cycles in S_n then prove that $\alpha\beta = \beta\alpha$.**[5]**

c) If $\sigma = (1\ 5\ 4)\ (2\ 7)$, $\rho = (1\ 2\ 8\ 7\ 5\ 6)\ (3\ ,\ 4)$ are permutations of S_8. Calculate $\sigma^{-1}\rho$ and $\rho^{-1}\sigma$. **[4]**

Q3) a) Prove that $\mathrm{Aut}(\mathbb{Z}_n)$ is isomorphic to $u(n)$, for every positive integer n. **[5]**

b) Let A_n denote the set of even permutations of the group of permutations S_n then prove that A_n is subgroup of S_n of order $\frac{n!}{2}$. **[5]**

c) Let G = SL(2, 1R), the group of 2 × 2 real matrices with determinant 1. Prove that $\varphi_M(A) = MAM^{-1}$ for all A in G and fix M in G is an isomorphism of SL (2, 1R). **[4]**

P.T.O.

Q4) a) Let G be a finite group of permutations of a sets and $i \in$ S. Prove that $|G| = |orb_G(i)|\,|stab_G(i)|$. **[5]**

b) Justify with an example that the converse of Lagrange's theorem is not true. **[5]**

c) Let |G| = 15. If G has only one subgroup of order 3 and only one subgroup of order 5, prove that G is cyclic. Generalise to $|G| = pq$ where p and q are prime. **[4]**

Q5) a) Prove that the group of complex number under addition is isomorphic to $\mathbb{R} \oplus \mathbb{R}$. **[5]**

b) Determine the number of cyclic subgroup of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$. **[5]**

c) Express U(165) as an external direct product of U - groups in four different ways. **[4]**

Q6) a) Let G be a group and H be a normal subgroup of G, prove that $G/H = \{aH \neq a \in G\}$ is a group under the operation $(aH)(bH) = abH$. **[5]**

b) Let H be a normal subgroup of G and K is a subgroup of G that contains H. Prove that K is normal in G if and only if K/H is normal in G/H. **[5]**

c) Let $G = \mathbb{Z}_{18}$ and H = <6>. Determine the factor group G/H. **[4]**

Q7) a) Let φ be a homomorphism from a group G to a group $\overline{G}$ and let H be a subgroup of G. Then prove that: **[5]**

i) If $\overline{k}$ is a subgroup of $\overline{G}$ then $\varphi^{-1}(\overline{k})$ is a subgroup of G.

ii) If $\overline{k}$ is a normal subgroup of $\overline{G}$ then $\varphi^{-1}(\overline{k}) = \{k \in G \mid \varphi(k) \in \overline{k}\}$ is a normal subgroup of G.

iii) If φ is onto and ker φ = {e}, then φ is an isomorphism from G to $\overline{G}$.

b) Let H be a subgroup of G, N(H) denote the normalizer of G and C(H) denote the centralizer of H. Then prove that N(H)/C(H) is isomorphic to a subgroup of Aut(H). **[5]**

c) Suppose that φ is a homomorphism from U(30) to U(30) with kerφ = {1, 11}. If φ(7) = 7 find all elements of U(30) that map to 7. **[4]**

Q8) a) Let G be a group, $|G| = p^n$, $n \geq 1$ then prove that $|Z(G)| > 1$, where Z(G) denotes the center of g. **[5]**

b) Prove that $\mathbb{Z}_{255}$ is the only group of order 255. **[5]**

c) Let G be a group of order 60. If the sylow 3 - subgroup is normal, show that sylow 5 - subgroup is normal. **[4]**

Total No. of Questions : 8]

SEAT No. :

PC3700

[Total No. of Pages :3

[6334]-201

F.Y.M.A./M.Sc.

MATHEMATICS

MTUT-121: Complex Analysis

(2019 Pattern) (Semester- II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions out of eight questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let $P=\sum_{n\geq 0} a_n t^n$ be a power series over $\mathbb{C}$. Put L=lim $\sup\limits_{\cap} \sqrt[n]{|a_n|}$ and

$R=\frac{1}{L}$ with the convention $\frac{1}{0}=\infty;\frac{1}{\infty}=0$ Then prove that **[5]**

i) For all $0 < r < R$, the series $P(t)$ is absolutely and uniformly convergent in $|z|\leq r$ and

ii) For all $|z|>R$, the series is divergent.

b) Prove that non constant polynomial in one variable with coefficients in $\mathbb{C}$ has atleast one root in $\mathbb{C}$ **[5]**

c) For any two complex numbers z_1 and z_2 show that

$$|1-\overline{z}_1 z_2|^2-|z_1-z_2|^2=\left(1-|z_1|^2\right)\left(1-|z_2|^2\right).$$ **[4]**

Q2) a) Let U be a convex open subset of $\mathbb{R}^2$ and $f:U\rightarrow\mathbb{R}$ be differentiable function such that $D(f)_z=0$ for all $z\in U$. Then show that $f(z)=c$, a constant on U. **[5]**

b) Discuss the continuity of the following function **[4]**

$$f(x,y)=\begin{cases}\dfrac{x^3y}{x^2-y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

P.T.O.

c) Show that the functions $f : \mathbb{C} \to \mathbb{C}$ and $g: \mathbb{C} \to \mathbb{C}$ defined by $f(z) = \text{Re}(z)$ and $g(z) = \text{Im}(z)$ are not complex differentiable any where. **[3]**

d) Compute the value of $\int_{|z-a|=r} (z-a)^n dz$ where n is a positive integer. **[2]**

Q3) a) Prove that for a continuous complex valued function f defined in a region Ω, the integral $\int_w f dz = 0$ for all closed contours w if and only if f is the derivative of a holomorphic function on Ω. **[5]**

b) Let & be a complex differentiable function on an open set containing the closure of a disc D. Then prove that $f(z) = \frac{1}{2\pi i} \int_{\partial D} \frac{f(\xi)}{(\xi - z)} d\xi, z \in \text{D}.$ **[5]**

c) Evaluate $\int_C \frac{z^2 - z + 2}{z^3 - 2z^2} dz$ where C is the boundary of the rectangle with vertices $3 \pm i, -1 \pm i$ traversed clockwise. **[4]**

Q4) a) Prove that a bounded and entire function is a constant function. **[5]**

b) Let f be a (non constant) holomorphic function in a domain U. Suppose $\{z \in \text{D} : |f(z)| = k\}$ is the entire boundary of a domain $\Omega \subset \text{U}$. Show that & must vanish at some point in Ω. **[4]**

c) Identify the singularity and its nature for the function $f(z) = \frac{\sin z}{z}, z \neq 0$ **[3]**

d) Determine the points at which the function $f(z) = xy + iy$ is complex differentiable **[2]**

Q5) a) Prove that a non constant holomorphic function on an open set is an open mapping. **[5]**

b) Let f be a holomorphic function on A=(r_1, r_2). Let $r_1 < \rho_1 < \rho_2 < r_2$. Then for $\rho_1 < |z| < \rho_2$ prove that $f(z) = \frac{1}{2\pi i} \int_{|w|=\rho_2} \frac{f(w)}{w - z} dw - \frac{1}{2\pi i} \int_{|w|=\rho_1} \frac{f(w)}{w - z} dw.$ **[5]**

c) Compute the residues at all singular points of the function $f(z) = \cot z$ in $|z| = 1$. **[4]**

Q6) a) Let *f* be a holomorphic function in $B_r(a)\setminus\{a\}$ and let a be an essential singularity of *f*. Then prove that *f* takes values arbitrarily close to any arbitrary complex number inside any arbitrary neighborhood of a. **[5]**

b) Find the Lourent series expansion of the function $f(z)=\dfrac{1}{z(z-a)(z-b)}$ for some $0<|a|<|b|$ in the annulus $A=\{0<|z|<|a|\}$ **[5]**

c) Prove that $\int_0^{\pi} e^{-R\sin\theta}d\theta<\dfrac{\pi}{R}, R>\theta$ **[4]**

Q7) a) Use the complex method to prove that $\int_0^{2\pi}\dfrac{d\theta}{1+a\sin\theta}=\dfrac{2\pi}{\sqrt{1-a^2}}, -|<a<|$ **[7]**

b) Evaluate $\int_0^{\infty}\dfrac{cosx}{x^2+a^2}dx, a>0$ **[7]**

Q8) a) Let $f:D\to\bar{D}$ be a holomorphic function such that $f(0)=0$ Then prove that $|f(z)|\leq|z|$ and $|f'|(0)|\leq|$. Further show that the following are equivalent. **[6]**

i) There exists $z_0\neq 0$ with $|z_0|<|$ and $|f(z_0)|=|z_0|$.

ii) $|f'(0)|=1$.

iii) $f(z)=cz$ for some $|c|=1$

b) Let *f* be a holomorphic in a neighborhood of z_0, $f(z_0)=w_0$ and let have $f(z)-w_0$ have a zero of order *n* at z_0 Then prove that there exists $\varepsilon>0$ and $\delta>0$ such that if $0<|a-\varepsilon|<\delta$ then $f(z)=a$ has exactly *n* simple solutions in the disc $|z-z_0)<\varepsilon$. **[5]**

c) Let $f(z)=\dfrac{2z^2-1}{(z^2+1)(z^2+4)}$. Γ_R be the upper half circle $|z|<R$ oriented in counter clockwise direction and $|f(z)|<A$ Find the value of A. **[3]**

Total No. of Questions : 8]

SEAT No. :

PC3701

[Total No. of Pages : 2

[6334]-202

First Year M.A./M.Sc.

MATHEMATICS

MTUT - 122 : General Topology

(2019 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If A is countable and B is countable then show that $A \times B$ is countable. **[5]**

b) Show that there is bijective correspondence of $A \times B$ with $B \times A$. **[5]**

c) Let $A = \{F : \{0,1\} \to \mathbb{Z}_+ / f \text{ is function}\}$ be a set. Show that A is countable. **[4]**

Q2) a) Show that the collection $C = \{[a,b) / a < b, a \text{ and } b \text{ rational}\}$ is a basis that generate a topology different from the lower limit topology on $\mathbb{R}$. **[5]**

b) Let $\pi_1 : x \times y \to x$ be defined by $\pi_1(x, y) = x$. Show that π_1 is open and continuous map. **[5]**

c) Let Y be a subspace of X; Let A be a subset of Y : let $\bar{A}$ denote the closure of A in X. Then show that the closure of A in Y equals $\bar{A} \cap Y$. **[4]**

Q3) a) State and prove the pasting lemma. **[5]**

b) Let $f : A \to B$ and $g : C \to D$ be continuous functions. Let $f \times g : A \times C \to B \times D$ be a function defined by $(f \times g)(a \times c) = f(a) \times g(c)$. **[4]**

c) Show that the map $f : \mathbb{R} \to \mathbb{R}^w$ defined by $f(t) = \left(t, \frac{t}{2}, \frac{t}{3}, ...\right)$ is not continuous if $\mathbb{R}^w$ is given box topology and $\mathbb{R}$ is given standard topology. **[5]**

P.T.O.

Q4) a) If X and Y are connected spaces then show that $X \times Y$ is also connected. **[5]**

b) Show that a subspace of a Hausdorff space is Hausdorff. **[5]**

c) Let A be a connected subspace of X. If $A \subset B \subset \overline{A}$ then show that B is also connected. **[4]**

Q5) a) Prove that every compact subspace of a Hausdorff space is closed. **[5]**

b) Show that every path connected space is connected. **[5]**

c) Show that every closed subspace of a compact space is compact. **[4]**

Q6) a) Show that a subspace of a Lindelöf space need not be Lindelöf. **[5]**

b) Show that metrizable space is first countable. **[5]**

c) Define : **[4]**

i) Lindelöf space.

ii) Normal space.

Q7) a) Show that $\mathbb{R}_l$ is normal. **[5]**

b) Show that every order topology is regular. **[5]**

c) Show that every metrizable Lindelöf space is second countable. **[4]**

Q8) a) Show that a connected normal space having more than one point is uncountable. **[5]**

b) Show that compact Hausdorff space is normal. **[5]**

c) Show that every regular Lindelöf space is normal. **[4]**

☯☯☯☯

Total No. of Questions : 8]

SEAT No. :

PC-3702

[Total No. of Pages : 2

[6334]-203
F.Y. M.A./M.Sc.
MATHEMATICS
MTUT - 123 : Ring Theory
(2019 Pattern) (CBCS) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) The natural multiplication of cosets of I, namely, (a + I) (b + I) = ab + I make R/I into a ring if and only if I is a 2 - sided ideal. **[6]**

b) Show that the intersection of two prime ideals is prime if and only if one of them is contained in the other. **[6]**

c) Define simple ring and give an example of simple ring. **[2]**

Q2) a) If R is commutative ring with 1 then $A \in M_n(R)$ is unit if and only if its determinant det (A) is a unit in R. **[7]**

b) Show that ring R is an integral domain if and only if $R \neq (0)$, R has no non-trivial nilpotent elements and (0) is prime ideal in R. **[7]**

Q3) a) For $2 \leq n \in N$, then prove that any ring Z/nZ is a field if and only if $\frac{Z}{nZ}$ is an integral domain if and only if n is a prime number. **[6]**

b) Prove that the characteristic of a local ring is either o or a power of a prime. **[5]**

c) Define semi - local ring. Give an example of semi - local ring. **[3]**

P.T.O.

Q4) a) If $I \subseteq J$ are both 2 - sided ideal in R then (R/I)/(J/I) is normally isomorphic to R/J. **[7]**

b) Prove that the ring End_k (v) is a simple ring if and only if V is a finite dimensional vector space over field K. **[7]**

Q5) a) Let I be an ideal in a ring R. Then I is a 2-sided ideal in R if and only if is the kernel of some homomorphism $f : R \rightarrow S$ for a suitable rings . **[7]**

b) State chinese Remainder theorem for a commutative ring R with 1. **[5]**

c) Prove or disprove $7 - 5\sqrt{2}$ is unit in $Z\left[\sqrt{2}\right]$. **[2]**

Q6) a) Prove that a Euclidean domain R has unity and whose group of units is given by $u(R) = \{a \in R^* \mid d(a) = d(1)\}$. **[7]**

b) Prove that every Euclidean domain is a PID. **[5]**

c) Prove or disprove : The polynomial $2 + 2x + 3x^3$ is irreducible in $Q[x]$. **[2]**

Q7) a) If d is a positive integer, then the ring $Z\left[i\sqrt{d}\right]$ is a factorization domain. **[5]**

b) Show that $1 + i\sqrt{3}$ is irreducible but not a prime element. **[5]**

c) To show that (Q. +) has no maximal subring. **[4]**

Q8) a) Show that Direct sum of free modules is a free module and give an example of a non-free module. **[7]**

b) Show that vector space is free module. **[7]**

♦ ♦ ♦

Total No. of Questions : 8]

SEAT No. : [ ]

PC3703

[Total No. of Pages : 4

[6334]-204

First Year M.A./M.Sc.

MATHEMATICS

MTUT-124 : Advanced Numerical Analysis

(2019 Pattern) (Semester-II)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let f be a continuous functions with an continuous derivatives. The equation $f(x) = 0$ has a root of multiplicity m at $x = p$ if and only if

$$f(p) = f'(p) = f''(p) = ... = f^{(m-1)}(p) = 0 \text{ but } f^{m}(p) \neq 0.$$ **[5]**

b) The function $f(x) = x^3 - x^2 - 10x + 7$ has root on the interval (0, 1). Approximate this root within an absolute toteranic of 10^{-6} has a stopping condition by using Newton's method. **[5]**

c) Compute the following limit & determine the corresponding rate of convergence. **[4]**

$$\lim_{n\to\infty} \frac{n+3}{n+7}$$

Q2) a) Prove that the order of convergence of secant method is approximately 1.618 (α = 1.618) and asymptotic error constant. **[5]**

$$\lambda \approx C^{y\alpha} = \left(\frac{f''(p)}{\alpha f'(p)}\right)^{(\alpha-1)}$$

b) Solve the following system of equation using Gaussian elimination with partical pivoting. **[5]**

$0.25x_1 + 0.35x_2 + 0.15x_3 = 0.60$
$0.20x_1 + 0.20x_2 + 0.25x_3 = 0.90$
$0.15x_1 + 0.20x_2 + 0.25x_3 = 0.70$

c) Show that the convergence of the sequence generated by the formula. **[4]**

$$x_{n+1} = \frac{x_n^3 + 3x_n a}{3x_n^2 + a}$$

toward $\sqrt{a}$ is third order. What is the asymptotic error constant.

P.T.O.

Q3) a) Determine the crout decomposition of the given matrix and then solve the system $Ax=b$ for the right hand side vector. **[5]**

$$\text{Where } A=\begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}, \quad b=\begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$$

b) Construct Housholder matrix H for $W=\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T$. **[5]**

c) Define the term: **[4]**

i) Orthogonal matrix

ii) Degree of precision

Q4) a) Solve the following system of linear equations by SOR method start with $X^{(0)} = [0, 0, 0]^T$ ant $\omega = 0.9$ **[5]**

(Perform 2 iterations)

$$5x_1 + x_2 + 2x_3 = 10$$
$$-3x_1 + 9x_2 + 4x_3 = -14$$
$$x_1 + 2x_2 - 7x_3 = -33$$

b) Solve the following system of linear equations by Gauss-Seidel method start with $X^{(0)} = [0, 0, 0]^T$ **[5]**

(Perform 3 iterations)

$$4x_1 - x_2 = 2$$
$$-x_1 + 4x_2 - x_3 = 4$$
$$-x_2 + 4x_3 = 10$$

c) Compute the condition number (K_α) for the matrix $A=\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}$ **[4]**

Q5) a) Use the QR factorization of a symmetric tridiagonal matrix. **[5]**

$$A=\begin{bmatrix} 4 & 3 & 0 \\ 3 & 1 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Find product $R^{(0)} \; Q^{(0)}$.

b) Find the matrix $A = \begin{bmatrix} -2 & -2 & 3 \\ -10 & -1 & 6 \\ 10 & -2 & -9 \end{bmatrix}$

With initial vector $X^{(0)} = [1, 0, 0]^T$

Perform three iteration of power method to find dominant eigenvalue and corresponding eigenvector. **[5]**

c) Let Q_1 and Q_2 be orthogonal matrices show that the matrices Q_1Q_2 and Q_2Q_1 are orthogonal. **[4]**

Q6) a) Derive the closed Newton cotes with n = 3

$$\int_a^b f(x)dx = \left(\frac{b-a}{8}\right)\left[f(a) + 3f(a+\Delta x) + 3f(a+2\Delta x) + f(b)\right]$$ **[5]**

b) Derive composite Teapezoidal Rule with error term.

$$\int_a^b f(x)dx = \frac{h}{2}\left[f(x_0) + 2\sum_{j=1}^{n-1} f(x_j) + f(x_n)\right] - \frac{(b-a)h^2}{12} f''(\xi)$$

Where $h = \dfrac{b-a}{n}$, $x_j = a + jh, 0 \le j \le n$ and $\xi \in [a,b]$. **[5]**

c) If $f(x) = ln(x)$ find $f'(\alpha)$ for h = 1.0, 0.01. **[4]**

Q7) a) Apply Euler's method to approximate solution of initial value problem.

$$\frac{dx}{dt} = x - t,\ 0 \le t \le 2,\ x(0) = 2,\ \text{N} = 4$$ **[5]**

b) Find solution of initial value problem $\dfrac{dx}{dt} = 1 + \dfrac{x}{t}\ \ (1 \le t \le 6)$

$x(1) = 1$ using second order Runge-Kutta method with $n = 0.5$ **[5]**

c) For the following differential equation identify the function $f(t,x)$ and Calculate $\dfrac{df}{dt}, \dfrac{d^2f}{dt^2}, \dfrac{d^3f}{dt^3}$ **[4]**

$x^1 + 2x^2 = t^2 - 1$

Q8) a) Solve the initial value problem.

$\frac{dx}{dt} = 1 + \frac{x}{t}$; $1 \le t \le 1.5,\ x(1) = 1\ \ h = 0.25$ by using Taylor method of order N = 2. **[5]**

b) Evaluate $\int_{-2}^{2} \frac{x}{5+2x}dx$ by using Trapezoidal rule by dividing [–2, 2] into five equal subintervals. **[5]**

c) Define the term. **[4]**

i) Relative error

ii) Triangular Matrix

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC-3704

[Total No. of Pages : 3

[6334]-205
M.Sc
MATHEMATICS
MTUT 125 : PARTIAL DIFFERENTIAL EQUATIONS
(2019 Pattern) (Semester - II) (CBCS)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If $u_1 = \frac{\partial u}{\partial x}, u_2 = \frac{\partial u}{\partial y}, u_3 = \frac{\partial u}{\partial z}$, show that the equation $f(x,y,z,u_1,u_2,u_3) = 0$ and $g(x,y,z,u_1,u_2,u_3) = 0$ are compatible if

$$\frac{\partial(f,g)}{\partial(x,u_1)} + \frac{\partial(f,g)}{\partial(y,u_2)} + \frac{\partial(f,g)}{\partial(z,u_3)} = 0$$ **[5]**

b) Attempt the following.

i) Solve the partial differential equation $(z^2 - 2yz - y^2)\,p + (xy + zx)\,q = xy - zx$ **[4]**

ii) Find the complete integral of partial differential equation $f(x,y,z,p,q) = 2\,(xp + yq + z) - yp^2 = 0$ by using charpit's method. **[5]**

Q2) a) Explain charpits method for solution of Non - linear partial differential equation of first order. **[6]**

b) Attempt the following.

i) Verify that the equation $Z = \sqrt{2x+a} + \sqrt{2y+b}$ is solution of partial differential equation $z = \frac{1}{p} + \frac{1}{q}$ **[3]**

ii) Find the complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + 2p_2^2p_3 = 0$ by using Jacobis method **[5]**

P.T.O.

Q3) a) If $\alpha_r D+\beta_r D' +\gamma_r$ is a factor of F(D, D') and ϕ_r (ξ) is an arbitrary function of a single variable ξ, then if

$\alpha r \neq 0, u_r = exp\left(\frac{-\gamma_r x}{\alpha_r}\right) \phi_r(\beta_r x - \alpha_r y)$ is a solution of the equation F(D, D') z = 0 **[6]**

b) Attempt the following

i) Solve $(D^2 - 2DD' - 15D'^2)\, z = 12\, xy$ **[4]**

ii) Find the complete solution of $(D^2 - 2DD' + D'^2)\, z = e^{x+2y} + x^3$ **[4]**

Q4) a) If 'f' and 'g' be arbitrary functions of their respective arguments show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{C^2}\frac{\partial^2 u}{\partial t^2}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$ **[4]**

b) Attempt the following

i) Prove that if $u_1, u_2, \ldots\ldots, u_n$ are solutions of the homogeneous linear partial differential equation $F(D,D')\, z = 0$ then $\sum_{r=1}^{n} c_r u_r$, where C_r's are arbitrary constants, is also solution **[4]**

ii) Find the solution of the equation $\nabla_1^2 z = e^{-x}$ cosy, which tends to zero as $x \to \infty$ and has the value cosy when $x = 0$ **[6]**

Q5) a) Derive the laplace equation of second order partial differential equation. **[4]**

b) Attempt the following

i) Find the steady state temperature in a rectagular plate bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$ if the edge $y = 0$ is insulated, the edge $x = 0$ and $x = a$ are kept at 0°c and the edge $y = b$ is kept at temperature $f(x)$ **[6]**

ii) Slove the boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ with $u(0, y) = 8e^{-3y}$ by the method of separation of variables . **[4]**

Q6) a) Derive one dimensional wave equation . **[5]**

b) Attempt the following.

i) Find the deflection *u (x,t)* of the vibrating string whose length is π^2 and $c^2 = 1$ corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$ **[4]**

ii) Solve the Wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$, where $u = p_0 \cos pt$ (p_0 is a constant) when $x = 1$ and $u = 0$ when $x = 0$ **[5]**

Q7) a) Derive One dimensional heat equation. **[6]**

b) Attempt the following.

i) Find by method of separation of variables the solution *u (x, t)* of the boundry value problem. **[6]**

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}\ t > 0, 0 < x < 2\ u(0,t) = 2,\ u(2,t) = 0, t > 0, u(x,0) = x, 0 < x < 2$$

ii) Classify the equations **[2]**

a) $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$

b) $\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$

Q8) a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to cannonical from, and hence solve it. **[7]**

b) Attempt the following.

Solve

$$K\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial u}{\partial t}\ \text{for}\ 0 < x < \pi, t > 0\ \text{if}\ u_x(0,t) = u_x(\pi,t) = 0\ \text{and}\ u(x, 0) = \sin x$$

[7]

Total No. of Questions : 8]

SEAT No. :

PC3705

[Total No. of Pages : 3

[6334]-301

Second Year M.A./M.Sc.

MATHEMATICS

MTUT-131 : Functional Analysis

(2019 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Symbols have their usual meanings.*

Q1) a) i) Define Hilbert space. **[2]**

ii) Let I be any set and let l^2 (I) denote the set of all functions x : I$\rightarrow$F such that x (i) = 0 for all but countable number of i's and

$$\sum_{i\in I} |x(i)|^2 < \infty, \forall\ x, y \in l^2(I)$$

Define $\langle x, y\rangle = \sum_{i\in I} x(i)\overline{y(i)}$ then show that l^2(I) is Hilbert space. **[5]**

b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **[7]**

Q2) a) Let H be a Hilbert space and let f be an arbitrary functional in H*. Then there exists a unique vector y in H such that **[7]**

$f(x) = (x, y)$ for every $x \in$ H

b) The adjoint operation T$\rightarrow$T* on B(H), then prove the following properties. **[7]**

i) $(T_1+T_2)^* = T_1^* + T_2^*$

ii) $(\alpha T)^* = \overline{\alpha}\ T^*$

P.T.O.

iii) $(T_1 T_2)^* = T_2^* T_1^*$

iv) $T^{**} = T$

v) $\|T^*\| = \|T\|$

vi) $\|T^*T\| = \|T\|^2$

Q3) a) State and prove the open mapping theorem. **[7]**

b) Define normed linear space with suitable example. **[4]**

c) For $1 \le P < \infty$, show that the space l^p is separable. **[3]**

Q4) a) i) Define Banach space. **[2]**

ii) Show that set of real numbers $\mathbb{R}$ is a banach space under the norm defined by $\|x\| = |x|,\ x \in \mathbb{R}$. **[5]**

b) If N and N′ are normed linear spaces then prove that set B(N, N′) of all continuous linear transformation of N into N′ is itself a normed linear space with respect to the pointwise linear operations and the norm defined as

$\|T\| = \sup \{\|T(x)\| / \|x\| \le 1\}$

Further prove that if N′ is a Banach space then B(N, N′) is also a Banach space. **[7]**

Q5) a) If M is a closed linear subspace of a normed linear space N and x_o is a vector not in M, then prove that there exists a functional f_o in N* such that $f_o(M) = 0$ and $f_o(x_0) \neq 0$. **[7]**

b) Define Bounded linear transformation. **[2]**

c) Let T be a Bounded linear transformation from a normed linear transformation N onto N′, if there exist a positive number K such that $\| Tx \| \ge K \| x \|, \forall x \in X$. Then show that $T^{-1} : N' \to N$ exists and it is bounded. **[5]**

Q6) a) State and prove the closed graph theorem. **[7]**

b) i) Let H be two dimensional Hilbert space with B = $\{e_1, e_2\}$ as a basis, then find the spectrum of T on H which is defined by $T(e_1) = e_2$ and $T(e_2) = e_1$. **[3]**

ii) Let T be an operator on finite dimensional complex Hilbert space H prove that **[4]**

1) T is singular if and only if $0 \in \sigma(T)$

2) If T is non-simgular then $\lambda \in \sigma(T)$ if and only if $\lambda^{-1} \in \sigma(T^{-1})$.

Q7) a) If B and B^1 are Banach spaces and it T is a continuous linear transformation of B onto B', then prove that the image of each open sphere centered on the origin in B contains an open sphere centered on the origin in B'. **[7]**

b) If P and Q are projections on a closed linear subspace M and N on Hilbert space H, then prove that PQ is a projection if and only if PQ=QP and in this case PQ is the projection on $M \cap N$. **[7]**

Q8) a) State and prove Bessel's inequality. **[7]**

b) Show that if T is non-singular, then T* is also non-singular and that in this case $(T^*)^{-1} = (T^{-1})^*$. **[4]**

c) If T is an operator on a Hilbert space H then prove that T is self adjoint if and only if $\langle Tx, x \rangle$ is real for all $x \in H$. **[3]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC3706

[Total No. of Pages : 2

[6334]-302
S.Y.M.A./M.Sc.
MATHEMATICS
MT UT-132 : Field Theory
(2019 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions of the following.***
2) ***Figures to the right indicate full marks.***

Q1) a) If E is splitting field of a polynomial of degree *n* over a field F then show that [E : F} $\leq$ *n!* **[7]**

b) Is $\mathbb{C} \backslash \mathbb{R}$ a normal extension? Justify your answer. **[5]**

c) Define separable extension. **[2]**

Q2) a) State and prove Gauss lemma **[7]**

b) Prove that it is impossible to construct a square equal in area to the area of circle radius 1. **[5]**

c) Find minimal polynomial of $\alpha = \sqrt{2} - 1$ over Q. **[2]**

Q3) a) Prove that the multiplicative group of non-zero elements of a finite field is cyclic. **[7]**

b) State and prove Kronecker theorem. **[5]**

c) Construct a field with 4 elements. **[2]**

P.T.O.

Q4) a) Prove or Disprove every algebraic extension is a finite extension. **[7]**

b) Find splitting field of polynomial x^2+1 over $\mathbb{R}$. **[5]**

c) Define prime and perfect field. **[2]**

Q5) a) If $f(x) \in f[x]$ is a polynomial of degree ≥ 1 with α as a root. Then prove that α is a multiple root if and only if $f'(\alpha) = 0$ **[7]**

b) State fundamental theorem of Galois Theory. **[5]**

c) With usual notations find $\Phi_3(x), \Phi_4(x)$. **[2]**

Q6) a) If p(x) is an irreducible polynomial of degree 'n' in F(x) and u is a root of p(x) in an extension E of F then prove that $\{1, u, u^2, ..., u^{n-1}\}$ forms a basis of F(u) over F. **[7]**

b) Find $G(Q(3\sqrt{2}) \backslash Q)$ **[5]**

c) Prore that $x^7 + 3x + 6$ is irreducible over Q. **[2]**

Q7) a) Find Galois group of polynomial $x^3 - 2$ over Q. **[7]**

b) If $F \subseteq E \subseteq K$ are field and $[K:E] < \infty$,
$[E:F] < \infty$ then prove that

i) $[K:F] < \infty$

ii) $[K:F] = [K:E]\,[E:F]$ **[7]**

Q8) a) Prove that normal extension of a normal extension need not to be a normal. **[7]**

b) If a, b are constructible number's then prove that $a \pm b$, ab are also constructible. **[7]**

.

❋ ❋ ❋

Total No. of Questions : 4]

SEAT No. :

PC3707

[Total No. of Pages :1

[6334]-303

M.A./M.Sc.-II

MATHEMATICS

MTUT-133 : Programming with Python

(2019 Pattern) (Semester- III)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***Question 1 is compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Attempt any two questions from Q.2, 3 and 4.***

Q1) Attempt the following:

a) What are the advantages of using python over other languages? **[4]**

b) Write any six development environments which are availabe in Python.**[3]**

Q2) Attempt the following:

a) Write a program to generate the following pattern in Python. **[5]**

1
2 3
4 5 6
7 8 9 10

b) Write a program to swap two numbers. **[4]**

c) Write a note on for loop in python. **[5]**

Q3) Attempt the following:

a) Write a program to find the greatest of the three numbers entered by the user. **[5]**

b) Write a note on file handling in Python. **[6]**

c) Explain how an object is created in Python? **[3]**

Q4) Attempt the following:

a) What is a class? What are the essential components of a class? Define attributes and functions of a class. **[7]**

b) Write a generator to produce arithmetic progression where, the first term, the common difference and the number of terms is entered by the user.

[7]

Total No. of Questions : 8]

SEAT No. :

PC-3708

[Total No. of Pages : 3

[6334]-304
M.A./M.Sc.
MATHEMATICS
MTUTO134 : Discrete Mathematics
(2019 Pattern) (Semester - III) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that a graph G is Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **[7]**

b) How many ways can a committee be formed from four men and six women with at least two men and at least twice as many women as men? **[5]**

c) Find the number of arrangements of the letters in the word 'PROBABILITY'. **[2]**

Q2) a) Prove that isomorphism relation defined on set of simple graphs is an equivalence relation. **[5]**

b) Prove that the number of selections with repetition of r objects chosen from n types of objects is C(r + n – 1, r) **[5]**

c) Find a generating function for ar, where ar is the number of ways n distinct dice can show a sum r. **[4]**

Q3) a) Let T be a tree with average vertex degree a. Determine n(T) in terms of a. **[5]**

b) If two vertices are nonadjacent in the Petersen graph then prove that they have exactly one common neighbour. **[5]**

c) Solve the following recurrence relation. $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = 1$, $a_1 = 1$ **[4]**

P.T.O.

Q4) a) State and prove Inclusion-Exclusion formula. **[7]**

b) Prove that X, Y - bi graph G has a matching that saturates X if and only if $|N(S)| \geq |S|$ for all $S \subseteq X$. **[7]**

Q5) a) Find a recurrence relation for the number of ways to arrange cars in a row with n spaces if we can use cadillacs, Hummers or Fords. A Hummer requires two spaces, whereas cadillac or Ford requires just one space. **[5]**

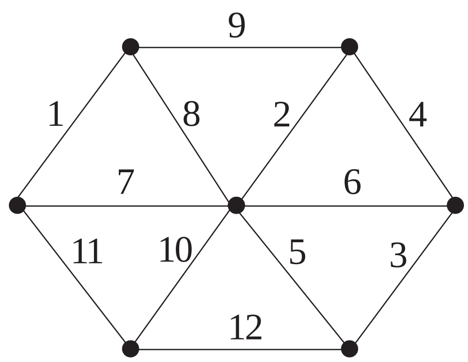
b) Prove that deleting a leaf from an n-vertex tree produces a tree with n-1 vertices. **[5]**

c) Find the coefficient of x^{16} in $(x^2 + x^3 + x^4 + ...)^5$ **[4]**

Q6) a) What is the probability that n people randomly reach into dark closet to retrive their hats, no person will pick his own hat? **[7]**

b) Let $S \subseteq N$ of size n. Prove that there are n^{n-2} trees with vertex set S. **[7]**

Q7) a) Use Kruskal's Algorithm to find the minimum spanning tree for the following weighted graph. **[6]**



b) Show by a combinatorial argument that **[6]**

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + ... + \binom{n}{n} = z^n$$

c) Prove or disprove the following statement.

If every vertex of a simple graph G has degree Z then G is cycle. **[2]**

Q8) a) Prove that a graph is bipartite if and only if it has no odd cycle. **[7]**

b) Show that number of partitions of an integer r as a sum of m positive integers is equal to the number of partitions of r as a sum of positive integers in which the largest is m. **[5]**

c) Show that $\sum_{k=0}^{n} 2^k \binom{n}{k} = 3^n$ **[2]**

Total No. of Questions : 8]

SEAT No. :

PC3709

[Total No. of Pages : 2

[6334]-305
S.Y.M.A./M.Sc.
MATHEMATICS
MTUTO-135 : Mechanics
(2019 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Explain D' Alemberts principle. **[5]**

b) Use D' Alemberts principle to determine equation of motion of simple pendulum. **[5]**

c) Show that the total energy of a particle moving in a conservative force field remains constant if the potential energy is not a explicit function of time. **[4]**

Q2) a) Explain Atwood machine and discuss its motion. **[7]**

b) If the Lagrangian function does not contain time *t* explicitly, show that the total energy of the conservative system is conserved. **[7]**

Q3) a) A particle slides down a curve in the vertical plane under gravity. Find the curve such that it reaches the lowest point in shortest time. **[7]**

b) Find the external of the functional **[7]**

$$I\left[y(x)\right] = \int_0^{\log 2}\left(e^{-x}y'^2 - e^x y^2\right)dx$$

Q4) a) Show that the two lagrangians **[7]**

$L_1 = (q+\dot{q})^2$, $L_2 = \left(q^2+\dot{q}^2\right)$ are equivalent.

b) Write a note on brachistochrome problem. **[7]**

P.T.O.

Q5) a) Prove that field force motion is always motion in plane. **[5]**

b) Prove Kepler's second law of planetary motion. **[5]**

c) The lagrangian of an harmonic oscillator of unit mass is

$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta x \dot{x}$. Find Hamiltonian. **[4]**

Q6) a) Show that the Euler-Lagranges equation of the functional.

$$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\, dx$$ **[5]**

has the first integral $f - y' \frac{\partial f}{\partial y'} = \text{constant}$

b) Find the extremals of the functional **[5]**

$$I(y(x)) = \int_{x_0}^{x_1} \frac{(1+y^2)}{y'^2}\, dx$$

c) Explain basic Lemma. **[4]**

Q7) a) Prove that Keplers first law of planetary motion. **[7]**

b) Derive Hamilton's canonical equations from Hamilton's principle. **[7]**

Q8) a) Deduce Newton's second law of motion from Hamilton's principle. **[7]**

b) Set up Hamiltonian for the Lagrangian

$$L(q, \dot{q}, t) = \frac{m}{2}\left[\dot{q}^2 \sin^2 \omega t + q\dot{q} \sin^2 \omega t + q^2 \omega^2\right]$$

Derive the Hamilton's equation of motion. **[7]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC3710

[Total No. of Pages : 3

[6334]-306

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-136 : Advanced Complex Analysis

(2019 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove the Schwarz reflection principle. **[5]**

b) If $\{f_n\}_{n=1}^{\infty}$ is a sequence of holomorphic functions that converges uniformly to a function f on every compact subset of Ω, then prove that f is holomorphic in Ω. **[5]**

c) Prove that $\int_0^{\infty} \frac{1-\cos x}{x^2} dx = \frac{\pi}{2}$. **[4]**

Q2) a) State and prove Morera's theorem. **[5]**

b) Suppose f is holomorphic in an open set Ω. If D is a disc centered at z_0 and whose closure is contained in Ω, then prove that f has a power series expansion at z_0: **[5]**

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n \quad \text{for all } z \in \mathrm{D},$$

and the coefficients are given by

$$a_n = \frac{f^{(n)}(z_0)}{n!} \text{ for all } n \geq 0.$$

c) Prove that the map $f(z) = \frac{1+z}{1-z}$ takes the upper half-disc $\{z = x + iy : |z| < 1 \text{ and } y > 0\}$ conformally to the first quadrant $\{w = u + iv : u > 0 \text{ and } v > 0\}$ **[4]**

P.T.O.

Q3) a) State and prove Montel's theorem. **[7]**

b) Show that the map $f(z)=\sin z$ takes the upper half-plane conformally onto the half-strip $\left\{w=x+iy:-\frac{\pi}{2}<x<\frac{\pi}{2}, y>0\right\}$. **[5]**

c) State the Riemann mapping theorem. **[2]**

Q4) a) Prove that the only automorphisms of the unit disc that fix the origin are the rotations. **[5]**

b) Prove that any two proper simply connected open subsets in $\mathbb{C}$ are conformally equivalent. **[5]**

c) Let Ω be an open subset of $\mathbb{C}$.

i) Define a uniformly bounded family $\mathcal{F}$ on compact subsets of Ω. **[2]**

ii) Define an equicontinuous family $\mathcal{F}$ on a compact set $K\subset\Omega$ **[2]**

Q5) a) Show that the two series. $\sum_{(n,m)\neq(0,0)}\frac{1}{(|n|+|m|)^r}$ and $\sum_{n+m\tau\in\Lambda^*}\frac{1}{|n+m\tau|^r}$

Where Λ^* denotes the lattice minus the origin, that is $\Lambda^*=\Lambda-\{(0,0)\}$, converge if $r>2$. **[7]**

b) Show that the Weierstrass $\wp$ function is an elliptic function that has periods 1 and τ and double poles at the lattice points. **[7]**

Q6) a) If F is a conformal map from the upper half-plane to the polygonal region P and maps the points $A_1,\ldots, A_{n-1},\infty$ to the vertices of boundary polygon p, then prove that there exist constants C_1 and C_2 such that

$$F(z)=C_1\int_0^z\frac{d\zeta}{(\zeta-A_1)^{B_1}\ldots(\zeta-A_{n-1})^{B_{n-1}}}+C_2.$$ **[7]**

b) Show that the function **[5]**

$$\int_0^z \frac{d\zeta}{\sqrt{\zeta(\zeta-1)(\zeta-\lambda)}},$$

with $\lambda \in \mathbb{R}$ and $\lambda \neq 1,$ maps the upper half-plane conformally to a rectangle, one of whose vertices is the image of the point at infinity.

c) Define the general Schwarz-Christoffel integral. **[2]**

Q7) a) Let z_0 be a point on the unit circle and if $F : \mathbb{D} \to \mathbb{P}$ is a conformal map, then prove that $F(z)$ tends to a limit as z approaches z_0 within the unit disc. **[7]**

b) Prove that there exist complex numbers c_1 and c_2 so that the conformal map F of the half-plane $\mathbb{H}$ to polygonal region $\mathbb{P}$ is given by $F(z)=c_1S(z)+c_2$, where S is the Schwarz-Christoffel integral. **[7]**

Q8) a) Let $F: \mathbb{H} \to \mathbb{C}$ be a holomorphic function that satisfies $|F(z)| \leq 1$ and $F(i)=0$.

Prove that $|F(z)| \leq \left|\frac{z-i}{z+i}\right|$ for all $z \in \mathbb{H}$. **[5]**

b) Show that if $f: D(0,R) \to \mathbb{C}$ is holomorphic, with $|f(z)| \leq M$ for some $M>0$,

then $\left|\frac{f(z)-f(0)}{M^2 - \overline{f(0)}f(z)}\right| \leq \frac{|z|}{MR}.$ **[5]**

c) If $f: U \to V$ is holomorphic and injective, then prove that $f'(z) \neq 0$ for all $z \in U$. **[4]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC3711

[Total No. of Pages : 3

[6334]-307

M.A./M.Sc.- II

MATHEMATICS

MTUTO 137 : Integral Equations

(2019 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any 5 questions from the following.*
2) *Figures to the right indicate full marks.*

Q1) a) Explain the Adomian Decomposition method to find the solution of Fredholm integral equation. **[5]**

b) Convert the following initial value problem to an equivalent Volterra integral equation. $y''' - 3y'' - 6y' + 5y = 0$ subject to the initial conditions $y(0) = y'(0) = y''(0) = 1$. **[5]**

c) If $u(x) = e^{-x^2}$ is a solution of Volterra integral equation $u(x) = 1 - \alpha\int_0^x tu(t)dt$ then find α. **[4]**

Q2) a) Derive an equivalent Fredholm integral equation from the following boundary value problem. $y''(x) + y(x) = x$, $0 < x < \pi$ subject to the boundary conditions $y(0) = 1$, $y(\pi) = \pi - 1$. **[5]**

b) Solve the Fredholm integral equation $u(x) = \frac{23}{6}x + \frac{1}{8}\int_0^1 xtu(t)dt$ by using method of successive substitutions. **[5]**

c) Solve the Volterra integral equation $u(x) = \cos x + \sin x - \int_0^x u(t)dt$ by using Modified Decomposition method. **[4]**

P.T.O.

Q3) a) Solve the Volterra integral equation $u(x)\, e^{x} + \int_0^x (t-x)\,u(t)\,dt$ by converting it to an equivalent initial value problem. **[5]**

b) Explain the method of successive approximations to find the solution of Volterra integral equation. **[5]**

c) Use direct computation method to find the solution of following Fredholm integral equation $u(x) = \frac{5}{6}x + \frac{1}{2}\int_0^1 xtu(t)dt$. **[4]**

Q4) a) Solve the following Homogeneous Fredholm integral equation. **[5]**

$$u(x) = \frac{2}{\pi}\lambda\int_0^{\pi} \cos(x+t)\,u(t)dt.$$

b) Find the solution of following Volterra integral equation of the first kind by converting it to second kind. $x^2 + \frac{1}{6}x^3 = \int_0^x (2+x-t)\,u(t)dt.$ **[5]**

c) Solve the following first order Fredholm integro-differential equation by direct computation method $u'(x) = 1 - \frac{1}{3}x + x\int_0^1 tu(t)\; dt, \mathrm{u}(0) = 0$. **[4]**

Q5) a) Find the solution of following Volterra integro-differential equation by using Adomian Decomposition method. **[5]**

$$u''(x) = x + \int_0^x (x-t)u(t)dt, u(0) = 0, \mathrm{u}'(0) = 1$$

b) Solve the following Fredholm integro-differential equation by converting it to the standard Fredholm integral equation. **[5]**

$$u''(x) = e^{x} - x + x\int_0^1 tu(t)dt, u(0) = 1, u'(0) = 1.$$

c) Explain Noise Term Phenomenon for Fredholm integral equation. **[4]**

Q6) a) Solve the following volterra integral equation by using Adomian decomposition method. **[5]**

$$u(x) = 1 + x - \int_0^x (x-t)u(t)dt.$$

b) Find the solution of following Fredholm integro-differential equation. **[5]**

$$u'(x) = 2\sec^2 x.\tan x - x + \int_0^{\pi/4} xu(t)dt, u(0) = 1$$

c) Explain volterra integro-differential equation with separable kernels with examples. **[4]**

Q7) a) Explain Leibniz rule and hence convert the following Volterra integral equation to an equivalent initial value problem. **[7]**

$$u(x) = x + \int_0^x (t-x)u(t)\,dt$$

b) Solve the following Volterra integral equation **[7]**

$u(x) = 1 + \int_0^x u(t)\,dt$ by using

i) Adomian decomposition method

ii) Successive approximation method

Also write the conclusion

Q8) a) Explain Adomian Decomposition method for Fredholm integro-differential equation. Also explain its types. **[7]**

b) Explain series solution method for Volterra integro-differential equation. Hence find the solution of equation. **[7]**

$$u''(x) = x\cosh x - \int_0^x tu(t)\,dt,\ u(0) = 0,\ u'(0) = 1$$

.

❋ ❋ ❋

Total No. of Questions : 8]

SEAT No. :

PC3712

[Total No. of Pages :3

[6334]-308

S.Y.M.A./M.Sc.

MATHEMATICS

MTUTO 138 : DIFFERENTIAL MANIFOLDS

(2019 Pattern) (Semester- III)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) Let X be an n by k matrix with $k \leq n$. Then prove that $V(X) = \left[\sum_{[I]} \det^2 X_I\right]^{1/2}$, where the symbol $[I]$ indicates that the summation extends over all ascending k - tuples from the set $\{1, 2, ..., n\}$. **[7]**

b) Let M be a manifold in $\mathbb{R}^n$ and let $\alpha : U \rightarrow V$ be a co-ordinate patch on M. If U_0 is subset of U that is open in U, then show that the restriction of α to U_0 is also a co-ordinate patch on M. **[4]**

c) Is $I \times I$ a 2 - manifold in $\mathbb{R}^2$? Justify. **[3]**

Q2) a) Show that solid torus is a 3 - manifold. **[5]**

b) Let W be a k - dimensional linear subspace of R^n. Show that there is an orthogonal transformation $h : \mathbb{R}^n \rightarrow \mathbb{R}^n$ that carries W onto subspace $R^k \times O$ of $\mathbb{R}^n$. **[5]**

c) Let $X = [a \;\; b]$ **[4]**

i) Find $X^{tr} X$

ii) Find $V(X)$

P.T.O.

Q3) a) Let f, g, h be tensors on V. Then show that **[6]**

i) $f \otimes (g \otimes h) = (f \otimes g) \otimes h$

ii) $(cf) \otimes g = c(f \otimes g) = f \otimes (cg)$

iii) Suppose f and g have same order then $(f+g) \otimes h = f \otimes h + g \otimes h$

b) Let f be a k - tensor on V; let $\sigma, \tau \in S_k$ prove that the tensor f is alternating if and only if $f^{\sigma} = (Sgn\ \sigma) f$ for all σ. **[4]**

c) Which of the following are alternating tensors in $\mathbb{R}^4$ **[4]**

i) $f(x, y) = x_1 y_2 - x_2 y_1 + x_1 y_1$

ii) $g(x, y) = x_1 y_3 - x_3 y_2$

Q4) a) Let V be a vector space. If T : V → W is linear transformation, and if f and g are alternating tensor on W, then show that $T^*(f \wedge g) = T^* f \wedge T^* g$. **[5]**

b) Prove that the operator d is linear on 0 - form S. **[4]**

c) Give an example of a closed form which is not exact. **[5]**

Q5) a) Let 0 be open in $\mathbb{R}^n$; let $f : 0 \to \mathbb{R}$ be of class C^r. Let M be the set of points x for which $f(x) = 0$; Let N be the set of points for which $f(x) \geq 0$. Suppose M is nonempty and $Df(x)$ has rank 1 at each point of M. Then prove that N is an n - manifold in $\mathbb{R}^n$ and $\partial N = M$. **[6]**

b) If the support of f can be covered by single co-ordinate patch, then show that $\int_M f dV$ is well defined, independent of the choices of co-ordinate patch. **[5]**

c) Let $\alpha : \mathbb{R} \to \mathbb{R}^2$ be the map $\alpha(x) = (x, x^2)$; Let M be the image set of α. Show that M is a 1 - manifold in $\mathbb{R}^2$ covered by the single co-ordinate patch α. **[3]**

Q6) a) Let A be open in $\mathbb{R}^k$ or $\mathbb{H}^k$; Let $\alpha \to \mathbb{R}^m$ be of class C^r. Let B be an open set of $\mathbb{R}^m$ or $\mathbb{H}^m$ containing $\alpha(A)$; Let $\beta : B \to \mathbb{R}^n$ be of class C^r. Then prove that $(\beta o \alpha)_* = \beta_* o \alpha_*$. **[5]**

b) Let A be open in $\mathbb{R}^k$; Let $\alpha : A \to \mathbb{R}^n$ be a C^∞ map. Let x denote general point of $\mathbb{R}^k$; let y denote the general point of R^n. Then dx_i and dy_i denote elementary 1 - forms in $\mathbb{R}^k$ and $\mathbb{R}^m$, respectively. Show that $\alpha^*(dy_i) = d\alpha_i$. **[5]**

c) Let $A = \mathbb{R}^2 - \{0\}$, if $W = \dfrac{-ydx + xdy}{x^2 + y^2}$, then show that W is not exact in A. **[4]**

Q7) a) Let A be open in $\mathbb{R}^k$; Let $\alpha : A \to \mathbb{R}^n$ be of class C^∞; let Y = α(A). Let x denote the general point of A; and let z denote the general point of $\mathbb{R}^n$. If $w = f\, dz_I$ is a k - form defined in an open set containing Y, then show that $\int_{Y_\alpha} w = \int_A (f o \alpha) \det(\partial \alpha_I / \partial x)$. **[5]**

b) Let M be a compact orientable 2 - manifold in $\mathbb{R}^3$. Let N be a unit normal field to M. Let F be a C^∞ vector field defined in an open set about M. If ∂M is empty, then prove that $\int_M \langle \text{curl}\, F, N \rangle dV = 0$. **[5]**

c) Let w be the 1 - form $w = x_2 dx_1 + 3x_1 dx_3$ Evaluate $\int_{\partial M} w$. **[4]**

Q8) a) Show that torus T is orientable 2 - manifolds in $\mathbb{R}^3$. **[5]**

b) If M is an oriented $n - 1$ manifold in $\mathbb{R}^n$, then prove that the unit normal vector N(P) corresponding to the orientation of M is a C^∞ function of P. **[5]**

c) Give an example of alternating tensors f and g such that $f \otimes g$ is not alternating. **[4]**

Total No. of Questions : 8]

SEAT No. :

PC3713

[Total No. of Pages :3

[6334]-401
S.Y.M.A./M.Sc.
MATHEMATICS
MTUT - 141 : Fourier Series and Boundary Value Problems
(2019 Pattern) (Semester- IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If $f \in c_p(0,\pi)$, then prove that the fourier cosine series coefficient a_n tends to zero as n tends to infinity. **[7]**

b) Find the fourier series corresponding to the function $f(x)$ defined on the fundamental internal $(-\pi < x < \pi)$

$f(x) = 0, -\pi < x < 0$

$= x, 0 < x < \pi$ **[5]**

c) Show that $\int_0^{\pi} \cos mx \cos nx\, dx = \begin{cases} 0, & m \neq n \\ \pi/2, & m = n \end{cases}$ if m & n are positive integer's. **[2]**

Q2) a) Suppose that a function $g(u)$ is piecewise continuous on the interval $0 < u < \pi$ and that the right hand derivative $g_R'(0)$ exists. Then prove that

$$\lim_{N \to \infty} \int_0^{\pi} g(u) D_N(u)du = \frac{\pi}{2} g(0,t).$$ **[6]**

b) Find the fourier cosine series for the function $f(x) = x^2$ $(0 < x < \pi)$. **[5]**

c) Find the fourier sine series for the function $f(x) = x$. **[3]**

P.T.O.

Q3) a) If f is piecewise continuous on the interval $-\pi < x < \pi$, then prove that

$$\int_{-\pi}^{\pi} f(s)ds = \frac{a_0}{2}(x+\pi) + \sum_{n=1}^{\infty}\frac{1}{n}\left[a_n \sinh x - b_n(\cosh x + (-1))^{n+1}\right].$$ **[7]**

b) Find the fourier sine series for the function $f(x) = x(\pi^2 - x^2)$ $(0 < x < \pi)$. **[5]**

c) Let $f(x) = \frac{e^x - 1}{x}$, $x \neq 0$. Find $f(0, t)$ and $f_R'(0)$. **[2]**

Q4) a) Solve the following boundary value problem. **[7]**

$y_{tt}(x, t) = a^2 y_{xx}(x, t)$ $(0 < x < c, t > 0)$

$y(0, t) = 0, y(c, t) = 0,$ $y_t(x, 0) = 0$

$y(x, 0) = f(x)$

b) Solve the boundary value problem. **[7]**

$u_{xx}(x, y) + u_{yy}(x, y) = 0$ $(0 < x < \pi, 0 < y < 2)$

$u_x(0, y) = u_x(\pi, y) = 0, u(x, 0) = 0$

With initial condition $u(x, 2) = f(x)$

Q5) a) Solve the following boundary value problem. **[7]**

$$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_\rho(\rho,\phi) + u_{\phi\phi}(\rho,\phi) = 0 \qquad (1 < \rho < b, 0 < \phi < \pi)$$

With boundary condition's

$u(\rho, 0) = 0, u(\rho, \pi) = 0$ $(1 < \rho < b)$

$u(1, \phi) = 0, u(b, \phi) = u_0$ $(0 < \phi < \pi)$

Where u_0 is a constant.

b) Solve the boundary value problem. **[7]**

$u_t(x, t) = ku_{xx}(x, t) + q(t)$ $(0 < x < \pi, t > 0)$

$u(0, t) = 0, u(\pi, t) = 0$ and $u(x, 0) = f(x)$

Q6) a) Let $C_n (n = 1, 2, 3, ...)$ be the fourier constants for a function f in $C_p(a, b)$ with respect to an orthonormal set $\{\phi_n(x)\} (n = 1, 2, 3, ...)$ in that space. Then prove that all possible linear combination's of the function's $\phi_1(x)$, $\phi_2(x),..., \phi_N(x)$ the combination's $C_1\phi_1(x) + C_2\phi_2(x) + ... + C_N\phi_N(x)$ is the best approximation in the mean to $f(x)$ on the fundamental interval $a < x < b$. **[6]**

b) Solve the boundary value problem. **[6]**

$u_{xx}(x, y) + u_{yy}(x, y) = 0$ $(0 < x < \pi, y > 0)$

$u_x(0, y) = 0, u(\pi, y) = 0$ $(y > 0)$

$-ku_y(x, 0) = f(x)$ $(0 < x < \pi)$

c) If $L = x$ and $M = \frac{\partial}{\partial x}$ are linear operator's on $c_p(a, b)$, then show that the product LM and ML are not always the same. **[2]**

Q7) a) If $c_n(n = 1, 2, 3, ...)$ are the fourier constant's for a function f in $c_p(a, b)$ with respect to an orthonormal set in that space, then prove that $\lim_{N\to\infty} c_n = 0$. **[7]**

b) Prove that a necessary and sufficient condition for an orthonormal set $\{\phi_n\}(n = 1, 2, ...)$ to be complete is that for each function F in space considered, parseval equation $\sum_{n=1}^{\infty} C_n^2 = \|f\|^2$, where c_n are the fourier constant, $c_n = (f, \phi_n)$ be satisfied. **[5]**

c) Show that $\psi_1(x) = 1$ and $\psi_2(x) = 1 - 3x^2$ are orthogonal on the internal $-1 < x < 1$. **[2]**

Q8) a) Let λ be an eigenvalue of the regular sturm-liouville problem **[7]**

$(rX')' + (g + \lambda p)\, X = 0 \qquad (a < x < b)$

$a_1X(a) + a_2\, X'(a) = 0$

$b_1X(b) + b_2\, X'(b) = 0$

If the condition's $g(x) \le 0$ $(a \le x \le b)$ and $a_1a_2 \le 0$, $b_1b_2 \ge 0$ are satisfied then prove that $\lambda \ge 0$.

b) Find the eigenvalues and normalized eigen function of sturm-liouville problem. $X'' + \lambda X = 0$, $X(0) = 0$, $hX(1) + X'(1) = 0 \quad h > 0$. **[5]**

c) Show that each of the function's $y_1 = \frac{1}{x}$ and $y_2 = \frac{1}{1+x}$ satisfies non-linear differential equation $y^1 + y^2 = 0$. Also show that if c is constant where $c \ne 0$ & $c \ne 1$ neither cy_1 nor cy_2 satisfies the equation. **[2]**

Total No. of Questions : 8]

SEAT No. :

PC3714

[Total No. of Pages : 3

[6334]-402
S.Y. M.A./M.Sc.
MATHEMATICS
MTUT - 142 : Differential Geometry
(2019 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let U be an open set in $\mathbb{R}^{n+1}$ and let $f : \mathrm{U} \to \mathbb{R}$ be smooth. Let $p \in \mathrm{U}$ be a regular point of f and let $c = f(p)$. Show that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$. **[5]**

b) Find the spherical image of the sphere $x_1^2 + x_2^2 + ... + x_{n+1}^2 = r^2 (r > 0)$ for $n = 1$ and $n = 2$. **[5]**

c) State the properties of Levi-Civita parallelism. **[4]**

Q2) a) Let X be a smooth vector field on an open set $\mathrm{U} \subset \mathbb{R}^{n+1}$ and let $p \subset \mathrm{U}$. Show that there exists an open interval I containing 0 and an integral curve $\alpha : \mathrm{I} \to \mathrm{U}$ of X such that **[5]**

i) $\alpha(0) = p$

ii) If $\beta : \tilde{\mathrm{I}} \to \mathrm{U}$ is any other integral curve of X with $\beta(0) = p$ then $\tilde{\mathrm{I}} \subset \mathrm{I}$ and $\beta(t) = \alpha(t)$ for all $t \in \tilde{\mathrm{I}}$.

b) Show that if S is a connected n-surface in $\mathbb{R}^{n+1}$ and $g : \mathrm{S} \to \mathbb{R}$ is smooth and takes only the values +1 and –1 then g is constant. **[5]**

c) Sketch the level sets $f^{-1}(0)$ and $f^{-1}(1)$ for n=1 of the function $f(x_1,....,x_{n+1}) = 0x_1^2 + x_2^2 + ... + x_{n+1}^2$. **[4]**

P.T.O.

Q3) a) Let S be an n-surface in $\mathbb{R}^{n+1}, \alpha : \mathrm{I} \to \mathrm{S}$, be a parametrized curve in S, let to $\in$I and $v \in \mathrm{S}_{\alpha(t0)}$. Show that there exists a unique vector field V tangent to S along α which is parallel and has $V_{(t0)} = v$. **[7]**

b) Define normal curvature of surface. Let S be sphere of radius r

$x_1^2 + \ldots\ldots + x_{n+1}^2 = r^2$ and is oriented by inward normal $\mathrm{N}(p) = \left(p, \frac{-p}{\|p\|}\right)$.

Find the normal curvature of S at $p \in$S in the direction of $v \in \mathrm{S}_p$. **[5]**

c) State Langrange's multiplier theorem. **[2]**

Q4) a) Let $a, b, c \in \mathbb{R}$ be such that $ac - b^2 > 0$. Show that the maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse

$ax_1^2 + 2bx_1x_2 + cx_2^2 = 1$ are of the form $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ where λ_1 and λ_2 are the

eigenvalues of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. **[7]**

b) Let L_p be a Weingarten map. Show that $L_p(v) \cdot w = v \cdot L_p(w)$ for all $v, w \in S_p$. **[5]**

c) Define the convariant derivative. **[2]**

Q5) a) Find global parametrization of the circle C : $(x_1 - a)^2 + (x_2 - b)^2 = r^2$. Also find curvature K for the circle oriented by inward normal. **[7]**

b) Show that a parametrized curve $\alpha(t) = (\cos at)e_1 + (\sin at)e_2$ is a geodesic on the sphere $x_1^2 + x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$ for each pair of orthogonal unit vectors $\{e_1, e_2\}$ in $\mathbb{R}^3$ and each $a \in \mathbb{R}$. **[5]**

c) Is möbius band an n-surface? Justify. **[5]**

Q6) a) Show that the 1-form η on $\mathbb{R}^3 \setminus \{0\}$ defined by

$$\eta = \frac{-x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2 \text{ is not exact.}$$ **[7]**

b) Let V be a finite dimensional vector space with dot product and let L : V → V be a self-adjoint linear transformation on V. Let S = $\{v \in V | v.v = 1\}$ and define $f : S \to \mathbb{R}$ by $f(v) = L(v).v$. Suppose f is stationary at $v_0 \in S$. Then show that $L(v_0) = f(v_0).v_0$. **[5]**

c) Show that if $\alpha : I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$ for all $t \in I$. **[2]**

Q7) a) Prove that on each compact oriented n-surface in $\mathbb{R}^{n+1}$ there exists a point P such that the second fundamental form at P is definite. **[7]**

b) Let C be a connected oriented plane curve and let β : I → C be a unit speed global parametrization of C. Then β is either one to one or periodic. Also, β is periodic if and only if C is compact. **[7]**

Q8) a) Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$ where a, b, c are all non-zero and is oriented by the outward normal. Show that the Gaussian curvature of S is $K(p) = \dfrac{1}{a^2 b^2 c^2 \left(\dfrac{x_1^2}{a^4} + \dfrac{x_2^2}{b^4} + \dfrac{x_3^2}{c^4} \right)^2}$. **[7]**

b) Let S be a compact connected oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f : \mathbb{R}^{n+1} \to \mathbb{R}$ with $\nabla f(p) \neq 0$ for all $p \in S$. Then show that the Gauss map maps S onto the unit sphere S^n.

[7]

☯☯☯☯

Total No. of Questions : 4]

SEAT No. :

PC-3715

[Total No. of Page : 1

[6334]-403

M.A./M.Sc.

MATHEMATICS

MTUT-143 : Introduction to Data Science

(2019 Pattern) (Credit System) (Semester - IV)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates :

1) ***Questions 1 is compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Attempt any two questions from Q.2, 3 and 4.***

Q1) Explain all steps involved in the process of data science. **[7]**

Q2) a) What is machine learning? Explain the types of machine learning. **[5]**

b) Give any five forms of data. **[5]**

c) State details to be covered in project charter. **[4]**

Q3) a) State any five techniques used to handle the missing data. **[5]**

b) State problems occuring in handling large data. **[5]**

c) State the phases involved in the modeling of data science process. **[4]**

Q4) a) Explain the concept of 'Bag of words' in text mining techniques. **[5]**

b) Explain in detail concept of Hadoop and its components. **[5]**

c) State the reasons why we create our own data visualisation techniques instead of using previously known data techniques. **[4]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

PC3716

[Total No. of Pages : 3

[6334]-404

S.Y.M.A./M.Sc.

MATHEMATICS

MTUTO-144 : Number Theory

(2019 Pattern) (Semester-IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If a, b, m$\in \mathbb{Z}$ and $m \neq 0$. Then prove that. **[7]**

i) $a \equiv a$ (mod m)

ii) $a \equiv b$ (mod m) implies that $b \equiv a$ (mod m)

iii) If $a \equiv b$ (mod m) and $b \equiv c$ (mod m). Then $a \equiv c$ (mod m)

b) If p is a prime and $a \not\equiv 0$ (mod p). Then prove that $ax \equiv b$ (mod p) has one and only one solution. **[4]**

c) Let $f(x) = 6x^3 + 2x^2+1$. Find degree of $f(x) \equiv 0$ (mod 7) **[3]**

Q2) a) Let K [x] denotes ring of polynomials with coefficients in a field K and f, g $\in$K[x]. If g$\neq$0, then prove that there exist polynomials h, r $\in$K[x] such that f = hg+r. Where either r=0 or r$\neq$0 and deg(r) < deg (g). **[6]**

b) Show that $41|2^{20}-1$. **[4]**

c) Find remainder of 7^{486} when divided by 13. **[4]**

Q3) a) Prove that every non-zero non unit of an integral domain R is a product of irreducibles. **[8]**

b) State and prove de-polignac's formula. **[6]**

P.T.O.

Q4) a) State and prove Fermat's theorem. **[5]**

b) If p is an odd prime then prove that **[5]**

$$\left(\frac{a}{p}\right) \equiv (a)^{\left(\frac{p-1}{2}\right)} \text{ (mod p)}$$

c) Find $\Omega^{(12)}$, d(12) **[4]**

Q5) a) Suppose that Q and Q^1 are odd and positive. Then prove that. **[7]**

i) $\left(\frac{P}{Q}\right)\left(\frac{P}{Q'}\right)=\left(\frac{P}{QQ'}\right)$

ii) $\left(\frac{P}{Q}\right)\left(\frac{P'}{Q}\right)=\left(\frac{PP'}{Q}\right)$

b) Find all integers that satisfy the following congruences simultaneously. **[5]**

$x \equiv 1$ (mod 4)

$x \equiv 0$ (mod 3)

$x \equiv 5$ (mod 7)

c) Let x and y be real numbers. Then prove that, **[2]**

$$[x]+[-x]=\begin{cases} 0 \ , & \text{if } x \text{ is an integer} \\ -1 \ , & \text{otherwise} \end{cases}$$

Q6) a) If x and y are any real numbers, then prove that, **[8]**

i) $[x]+[y]\leq[x+y]\leq[x]+[y]+1$ and

ii) $\left[\frac{[x]}{m}\right]=\left[\frac{x}{m}\right]$ if m is a positive integer.

b) Define a divisor function d. and if n is positive integer then prove that, **[6]**

$$d(n)=\prod_{p^{\alpha}\|n}(\alpha+1)$$

Q7) a) Let $f(n)$ be a multiplicative function and let F (n) = $\sum_{d|n} f(d)$. Then prove that F (n) is multiplicative. **[6]**

b) Any polynomials $f(x)$ and $g(x)$, not both identically zero, have a common divisor $h(x)$ that is a linear combination of $f(x)$ and $g(x)$. Then prove that $h(x) \mid f(x)$, $h(x) \mid g(x)$ and $h(x) = f(x)$. F(x) + $g(x)$ G (x) for some polynomials F (x) and G (x). **[5]**

c) Prove that 19 is not a divisor of $4n^2+4$ for any integer n. **[3]**

Q8) a) If ξ is an algebraic number of degree n, then prove that, every number in Q (ξ) can be written uniquely in the form $a_0 + a_1 \xi + ... + a_{n-1} \xi^{n-1}$ where the a_i are rational numbers. **[7]**

b) If (a,p) = 1 then prove that, **[5]**

$$\left(\frac{a^2}{p}\right) = 1, \quad \left(\frac{a^2 b}{p}\right) = \left(\frac{b}{p}\right)$$

c) List all integer $1 \le x \le 100$ which satisfy $x \equiv 7 \pmod{17}$ **[2]**

✤✤✤✤

Total No. of Questions : 8]

SEAT No. :

PC-3717

[Total No. of Pages : 2

[6334]-405
M.Sc.
MATHEMATICS
MTUTO-145 : Algebraic Topology
(2019 Pattern) (CBCS) (Semester - IV)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that if $h, h' : x \to y$ are homotopic and $k, k' : y \to z$ then koh and $k'oh'$ are homotopic. **[5]**

b) Define the term. **[4]**

i) simply connected

ii) star convex

c) Find the star convex set that is not convex. **[5]**

Q2) a) Give an example of a non identity covering map from s' onto s'. **[5]**

b) Define a covering map. Show that a covering map is a local homomorphism. **[4]**

c) If G = $G_1 * G_2$ show that $\frac{G}{[G,G]} \cong \left(\frac{G_1}{[G_1,G_1]}\right) \oplus \left(\frac{G_2}{[G_2,G_3]}\right)$. **[5]**

Q3) a) Find spaces whose fundamental group is isomorphic to the following groups. **[8]**

i) $\mathbb{Z}_n \times \mathbb{Z}_m$

ii) $\mathbb{Z}_n \otimes \mathbb{Z}_m$

b) Define : **[6]**

i) A deformation retract

ii) A strong deformation retract

P.T.O.

Q4) a) Let $A \subset X$ and $\{A_i \mid i = 1, 2...\}$ is a collection of connected subset of X each of which intersects A. Show that $A \cup \left\{\bigcup_{i=1}^{\infty} A_i\right\}$ is connected. **[5]**

b) Prove that the fundamental group of real projective plane is isomorphic to a cyclic group of order two. **[5]**

c) Prove that a covering map $P : \tilde{X} \to X$ is open. **[4]**

Q5) a) Prove that the closed ball $B^n (n \geq 1)$ has the fixed point property. **[6]**

b) Prove that $\mathbb{R}^n$ is homeomorphic to $\mathbb{R}^m$ if and only if $n = m$. **[4]**

c) Prove that the map $P : R \to S'$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map. **[4]**

Q6) a) Prove that a non-empty open connectd subset of $\mathbb{R}^2$ is path connected. **[6]**

b) Determine the fundamental groups of the following spaces : **[8]**

i) $\mathbb{R}^2 - (\mathbb{R}^+ \times 0)$

ii) $\{x \in \mathbb{R}^2 \mid \| x \| < 1\}$

iii) The solid sphere

iv) Torus T with one point removed

Q7) a) Prove that the fundamental group of the torus is a free abelian group of rank 2. **[4]**

b) Show that if X is an infinite wedge of circles, then X do not satisfy the first countability axiom. **[5]**

c) Given spaces X and Y. Let [X, Y] denote the set of homotopy classes of maps of X into Y. Let I = [0, 1] show that $\alpha : \pi_1(X, x) \to \pi_1(X, x_1)$ by $\hat{\alpha}([F]) = [\overline{\alpha}] * [F] * [\alpha]$ then show that $\hat{\alpha}$ is group homomorphism. **[5]**

Q8) a) Show that the unit closed solid n-sphere B^n is a contractible space. **[4]**

b) Show that fundamental group of circle is isomorphic to additive group of integers. **[6]**

c) Show that $f : s' \to s'$ given by $f(z) = z^3$ is a covering projection. **[4]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

PC-3718

[Total No. of Pages : 3

[6334]-406
M.A./M.Sc.
MATHEMATICS
MTUTO-146 : Representation Theory of Finite Groups
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of scientific calculator is allowed.***

Q1) a) State and prove Cayley-Hamilton theorem. **[5]**

b) If $q(A) = 0$ then prove that $m_A(x) \mid q(x)$. **[4]**

c) Prove that $(AB)^* = B^*A^*$. **[5]**

Q2) a) Define following terms : **[4]**

i) G-invariant subspace

ii) Irreducible representation

b) Prove that $\phi : \frac{z}{4z} \to C^*$ given by $\phi(m) = i^m$ is a representation. **[5]**

c) Define representation $\phi^{(1)} : \frac{z}{nz} \to C^*$ by $\phi^{(1)}[m] = e^{\frac{2\pi i m}{n}}$ and $\phi^{(2)} : \frac{z}{nz} \to C^*$ by $\phi^{(2)}[m] = e^{\frac{-2\pi i m}{n}}$. Find $(\phi^{(1)} \oplus \phi^{(2)})[m]$. **[5]**

P.T.O.

Q3) a) Let $\phi : G \to GL(v)$ be equivalent to a decomposable representation prove that ϕ is decomposable. **[7]**

b) Show that every representation of a finite group G is equivalent to a unitary representation. **[7]**

Q4) a) State and prove Schur's Lemma. **[6]**

b) Let $T : v \to w$ be in $Hom_G(\phi, \rho)$. Show that ker T is G-variant subspace of V and $T(v) = ImT$ is a G-variant subspace of w. **[4]**

c) Define following terms : **[4]**

i) Regular representation

ii) Character table

Q5) a) Let $\phi : G \to GL(v)$ and $\rho : G \to GL(w)$ be representations and suppose that $T : v \to w$ be a linear transformation. Prove the following statements. **[7]**

i) $T^{\#} = \frac{1}{|G|} \sum_{g \in G} \rho_g^{-1} \, T \, \phi_g \in Hom_G(\phi, \rho)$

ii) If $T \in Hom_G(\phi, \rho)$ then $T^{\#} = T$

iii) The map $P : Hom(v, w) \to Hom(\phi, \rho)$ defined by $P(T) = T^{\#}$ is an onto linear map.

b) Let ϕ be a representation of G. Prove that for all $g, h \in G$ the equality $\chi_\phi(g) = \chi_\phi(hgh^{-1})$ holds. **[3]**

c) Prove that there are at most $|C_1(G)|$ equivalence classes of irreducible representations of G. **[4]**

Q6) a) Prove that the set $\chi_1, \chi_2, \ldots, \chi_s$ is an orthonormal basis for $z(L(G))$. **[4]**

b) Let G_1, G_2 be abelian groups and suppose that $\chi_1, \chi_2, \ldots, \chi_m$ and $\varphi_1, \varphi_2, \ldots, \varphi_n$ are irreducible representations of G_1 and G_2 respectively. Show that the functions $\alpha_{ij} : G_1 \times G_2 \to C^*$ with $1 \le i \le m$, $1 \le j \le n$ given by $\alpha_{ij}(g_1, g_2) = \chi_i(g_1) \cdot \varphi_j(g_2)$ forms complete set of irreducible representations. **[7]**

c) The class functions form the center of L(G). **[3]**

Q7) a) Define following terms : **[4]**

i) Fourier transform

ii) Periodic functions

b) Prove that $f : G \to C$ is a class function if and only if $a * f = f * a$ for all $a \in L(G)$. **[7]**

c) Prove that the map $T : L(G) \to L(\hat{G})$ given by $Tf = \hat{f}$ is an invertible linear transformation. **[3]**

Q8) a) Prove that the linear map $T : L(G) \to L(\hat{G})$ given by $Tf = \hat{f}$ provides a ring isomorphism between $(L(G), +, *)$ and $(L(\hat{G}), +, \cdot)$. **[7]**

b) Let L be the regular representation of G prove that the decomposition $L \sim d_1\, \varphi^{(1)} \oplus d_2\, \varphi^{(2)} \oplus \ldots\ldots\ldots\ldots \oplus d_s\, \varphi^{(s)}$ holds. **[5]**

c) State spectral theorem. **[2]**

⌘⌘⌘

Total No. of Questions : 8]

SEAT No. :

PC-3719

[Total No. of Pages : 3

[6334]-407
M.A./M.Sc.
MATHEMATICS
MTUTO - 147 : Coding Theory
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let C be an $[n, k, d]$ - linear code over finite field F_q then prove that, **[5]**

i) For all $u \in F_q^n$, $|C + u| = |C| = q^k$

ii) There are q^{n-k} different cosets of C.

b) For $S = \{101, 111, 010\} \subseteq F_2^3$, find F_2-linear span <S> and its compliment $S^{\perp}$. **[5]**

c) Show that binary hamming codes are perfect codes. **[4]**

Q2) a) Let V be a vector space over F_q. If dim (V) = k then prove that **[7]**

i) V has q^k elements.

ii) V has $\frac{1}{K!}\prod_{i=0}^{k-1}(q^k - q^i)$ different bases.

b) Construct the incomplete maximum likelihood decoding table for binary code C = {101, 011, 111}. **[4]**

c) Let C be a binary linear code with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Find d(C). **[3]**

P.T.O.

Q3) a) For an integer $q > 1$ and integers n, d such that $1 \le d \le n$. Prove that

$$A_q(n,d) \le \frac{q^n}{\sum_{i=0}^{\left[\frac{d-1}{2}\right]} \binom{n}{i}(q-1)^i}$$

where $[x]$ denotes greatest integer less than or equal to x. **[5]**

b) If $g(x) = (1 + x)(1 + x^2 + x^3) \in F_2[x] \mid (x^7 - 1)$ is a generator polynomial of cyclic code C then find C and dim (C). **[5]**

c) If C and D are two linear codes over F_q of same length then prove that $C \cap D$ is also a linear code over F_q. **[4]**

Q4) a) Let $g(x)$ be the generator polynomial of an ideal of $F_q[x] \mid (x^n - 1)$. If the degree of $g(x)$ is n-k then prove that the dimension of cyclic code corresponding to the ideal is k. **[5]**

b) Let $x^6 - 1 = (1+x)^2(1+x+x^2)^2 \in F_2[x]$. Then find number of cyclic codes in F_2^6. Also find cyclic code generated by $(1 + x + x^2)^2$. **[5]**

c) If C = {000, 111, 011, 100} then find extended code $\overline{C}$ and its distance $d(\overline{C})$, where $C \subseteq F_2^3$. **[4]**

Q5) a) Let S be a subset of F_q^n. Prove that dim (<S>) + dim $(S^{\perp}) = n$. **[5]**

b) Let S = {0100, 0101} $\subseteq F_2^4$. Verify that dim (<S>) + dim $(S^{\perp}) = n$. **[5]**

c) If C = <S> with S = {12101, 20110, 01122, 11010} $\subseteq F_3^5$ then find basis of C. **[4]**

Q6) a) Prove that **[8]**

i) The dimension of a q-ary BCH code of length q^m-1 generated by $g(x) = lcm\ \{M^{(a)}(x), M^{(a+1)}(x), \ldots, M^{(a+\delta-2)}(x)\}$ is independent of the choice of the primitive element α.

ii) A q-ary BCH code of length q^m-1 with designed distance δ has dimension at least $q^m-1-m(\delta-1)$.

b) Is C = {(0,1,1,2), (2,0,1,1), (1,2,0,1), (1,1,2,0)} cyclic code over F_3? Justify. **[3]**

c) Show that distance of binary hamming codes is 3. **[3]**

Q7) a) Let C be an $[n, k]$ - linear code over F_q with generator matrix G then prove that $V \in C^{\perp}$ if and only if $VG^T = 0$. **[5]**

b) If $C = \{0000, 1011, 0101, 1110\} \subseteq F_2^4$ is a linear code then decode

i) $w = 1101$

ii) $w = 0111$

by using nearest neighbour decoding for linear codes. **[6]**

c) Find information rate and relative minimum distance of repetition code $C = \{\lambda (1, 1, 1, \ldots, 1) \mid \lambda \in F_q\}$ of length n. **[3]**

Q8) a) Prove that a code C is u-error-detecting if and only if $d(C) \geq u + 1$. **[5]**

b) Suppose that codewords from the binary code {000, 100, 111} are being sent over a Binary Symmetric channel with crossover probability $P = 0.03$. Use the maximum likelihood decoding rule to decode $w = 010$. **[5]**

c) Find number of distinct bases for vector space V = <S> over F_2 where S = {0001, 0010, 0100} **[4]**

▽▽▽▽

Total No. of Questions : 8]

SEAT No. :

PC3720

[Total No. of Pages : 5

[6334]-408

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO - 148 : Probability and Statistics

(2019 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*
3) *Use of scientific calculator is allowed.*

Q1) a) Attempt following : **[4]**

i) Define Probability of an event

ii) Define Sample space

b) If a car agency sells 50% of its inventory of a certain foreign car equipped with side airbags, find a formula for the probability distribution of the number of cars with side airbags among the next 4 cars sold by the agency. **[5]**

c) Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens, and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, find **[5]**

i) The joint probability function $f(x, y)$,

ii) $P[(x, y) \in A]$, where A is the region $\{(x, y) / x + y \leq 1\}$

Q2) a) Show that covarience of two random variables X and Y with means μ_x and μ_y, respectively, is given by $\sigma_{XY} = E(XY) - \mu_X \mu_Y$. **[4]**

b) A random variable X has a mean $\mu = 8$, a variance $\sigma^2 = 9$ and an unknown probability distribution. Find **[5]**

$P(-4 < X < 20)$

$P(1\ X - 81 \geq 6)$

c) Let X and Y denote the amounts of two different types of impurities in a batch of a certain chemical product. Suppose that X and Y are independant random variables with variences $\sigma_X^2 = 2$ and $\sigma_Y^2 = 3$. Find the varience of the random variable $Z = 3X - 2Y + 5$. **[5]**

P.T.O.

Q3) a) Show that mean and varience of the binomial distribution $b(x ; n, p)$ are $\mu = np$ and $\sigma^2 = npq$. **[4]**

b) A manufacturer of automobile tires reports that a shipment of 5000 sent to a local distributor, 1000 are slightly blemished. If one purchases 10 of these tires at random from the distributor, what is the probability that exactly 3 are blemished? **[5]**

c) For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found? **[5]**

Q4) a) Show that mean and varience of $n(x, \mu, \sigma)$ are μ and σ^2, respectively. Hence, the standerd deviation is σ. **[4]**

b) Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years? **[5]**

c) In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standerd deviation $\sigma = 0.0005$. On average, how many manufactured ball bearings will be scrapped? **[5]**

Q5) a) Explain the method of least square. **[4]**

b) In a certain type of metal test specimen, the normal stress on a specimen is known to be functionally related to the shear resistance. The following is a set of coded experimental data on the two variables : **[5]**

Normal stress, x	Shear Resistance, y
26.8	26.5
25.4	27.3
28.9	24.2
23.6	27.1
27.7	23.6
23.9	25.9
24.7	26.3
28.1	22.5
26.9	21.7
27.4	21.4
22.6	25.8
25.6	24.9

i) Estimate the regression line $\mu_{Y|x} = \beta_0 + \beta_1 x$

ii) Estimate the shear resistance for a normal stress of 24.5

c) Find a 95% confidence interval for β_1 in the regression line $\mu_{Y|x} = \beta_0 + \beta_1 x$, based on the pollution data of the following table **[5]**

Solid Reduction, x (%)	Oxygen demand Reduction, y (%)	Solid Reduction, x (%)	Oxygen demand Reduction, y (%)
3	5	36	34
7	11	37	36
11	21	38	38
15	16	39	37
18	16	39	36
27	28	39	45
29	27	40	39
30	25	41	41
30	35	42	40
31	30	42	44
31	40	43	37
32	32	44	44
33	34	45	46
33	32	46	46
34	34	47	49
36	37	50	51
36	38		

Q6) a) Prove that, $M_{X+a}(t) = e^{at} M_X(t)$. **[4]**

b) Compute and interpret the correlation coefficient for the following grades of 6 students selected at random : **[6]**

Mathematics grade	70	92	80	74	65	83
English grade	74	84	63	87	78	90

c) An experiment involves tossing a pair of dice, one green and one red and recording the numbers that come up. If x equals the outcome of the green die and y the outcome of red die then describe the sample space S. **[4]**

i) by listing the elements (x, y) :

ii) by using the rule method.

Q7) a) Show that mean and varience of the uniform distribution are $\mu = \frac{A+B}{2}$ and $\sigma^2 = \frac{(B-A)^2}{12}$. **[4]**

b) Three cards are drawn without replacement from the 12 face cards (jacks, queens and kings) of an ordinary deck of 52 playing cards. Let X be the number of kings selected and Y the number of jacks. **[5]**

Find,

i) The joint probability distribution of X and Y ;

ii) $P[(X, Y) \in A]$, where A is the region given by $\{(x, y) / x + y \geq 2\}$

c) The fraction X of male runners and the fraction Y of female runners who compete in marathon race are described by the joint destiny function,**[5]**

$$f(x, y) = \begin{cases} 8xy, & 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the covarience of X and Y.

Q8) a) The expected value of the sum or difference of two or more functions of the random variable X and Y is the sum or difference of the expected values of the functions. **[4]**

That is, prove that $E[g(X,Y) \pm h(X,Y)] = E[g(X,Y] \pm E[h(X,Y)]$

b) One prominent physician claims that 70% of those with lung cancer are chain smokers. If his assertion is correct, **[5]**

i) Find the probability that of 10 such patients recently admitted to a hospital, fewer than half are chain smokers.

ii) Find the probability that of 20 such patients recently admitted to a hospital, fewer than half are chain smokers.

c) Calculate karl pearsons coefficient of correlation for **[5]**

X	6	8	12	15	18	20	24	18	31
Y	10	12	15	15	18	25	22	26	28

x x x