Total No. of Questions : 4]

SEAT No. :

PC4059

[Total No. of Pages : 2

[6334]-1001

First Year M.A./M.Sc.

MATHEMATICS

MTS-501-MJ : Linear Algebra

(2023 Credit Pattern) (Semester-I)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicates full marks.*

Q1) Attempt any three of the following: **[6]**

a) Show that any two bases of a finite dimensional vector space have the same number of elements.

b) Let T be a invertible linear operator on finite dimensional vector space V over field F, then show that 0 is not eigenvalue of T.

c) Prove that if ϕ is bilinear form on V then $\phi(\alpha, 0) = 0 = \phi(0, \alpha)$ for all $\alpha \in V$.

d) Find the matrix of an identity linear operator on $\mathbb{R}^3$ with respect to basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$.

Q2) Attempt any two of the following: **[10]**

a) Let V be finite dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalizable if and only if minimal polynomial for T has the form $(x - C_1)(x - C_2) \ldots\ldots (x - C_k)$ where $C_1, C_2, \ldots C_k$ are distinct elements of F.

b) Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

c) Write all possibe Jordan canonical forms if the characteristic polynomial is $(x + 2)^3(x + 3)^2$.

Q3) Attempt any two of the following: **[10]**

a) Let T be a linear transformation from V into W, then prove that T is non-singular if and only if T carries each linearly independent subset of V onto a linearly independent subset of W.

b) Let V and W be inner product space over the same field and let T is linear transformation from V to W. Then show that T preserves inner product if and only if $\|T\alpha\| = \|\alpha\|$ for every $\alpha \in V$.

c) Let $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. Which are the coordinates of the vector (a, b, c)?

P.T.O.

Q4) A) Attempt any one of the following : **[4]**

a) Let V be an inner product space and T a self adjoint linear operator on V. Then show that each characteristic value of T is real and characteristic vectors of T associated with distinct characteristic value are orthogonal.

b) Prove that a normal and nilpotent operator is zero operator.

B) Attempt any one of the following : **[5]**

a) Describe explicitely all bilinear forms f on $\mathbb{R}^3$ with the property that $f(\alpha, \beta) = f(\beta, \alpha)$ for all α, β.

b) Let V be the vector space of polynomial functions from $\mathbb{R}$ into $\mathbb{R}$ which have degree less than or equal to 3 and let D is differential operator on V. Find the matrix of D with respect to standard basis.

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Total No. of Questions : 7]

SEAT No. :

PC4060

[Total No. of Pages : 3

[6334]-1002
First Year M.A./M.Sc.
MATHEMATICS
MTS-503 MJ : Group Theory
(2023 Credit Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicates full marks.*

Q1) Attempt any five of the following : **[10]**

a) Determine the operation * on $\mathbb{R}$ defined by $a*b = a + b + ab$, is associative.

b) How many elements of order 6 in S_5.

c) Find all generators for $\mathbb{Z}/48\mathbb{Z}$.

d) Let G and H be groups and $\phi : \mathrm{G} \rightarrow \mathrm{H}$ be a homomorphism. Show that $\phi(\mathrm{g}^{-1}) = [\phi(\mathrm{g})]^{-1}, \forall g \in \mathrm{G}$.

e) Write a class equation of the group Q_8.

f) State fundamental theorem of finitely generated abelian groups.

g) Prove that $\mathbb{Z}_{30} \cong \mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5$.

Q2) Attempt any two of the following : **[10]**

a) Prove that if H and K are subgroups of G then so is their intersection $\mathrm{H} \cap \mathrm{K}$.

b) Let G and H be a groups and $\phi : \mathrm{G} \rightarrow \mathrm{H}$ be a homomorphism. prove that $\phi(x^n)=[\phi(x)]^n$ for all $n \in \mathbb{Z}^+$.

c) Let $\mathrm{G} = \{z \in \mathbb{C} \mid z^n = 1, \text{for some } n \in \mathbb{Z}^+\}$. Prove that G is a group under multiplication.

P.T.O.

Q3) Attempt any two of the following : **[10]**

a) Let Δ and Ω are nonempty sets. If $|\Delta| = |\Omega|$ then prove that $S_\Delta \cong S_\Omega$.

b) Draw the lattice of subgroups of the group D_8.

c) Prove that any two cyclic groups of the same order are isomorphic.

Q4) Attempt any two of the following : **[10]**

a) Let G be a group of order $p^\alpha m$, where p is a prime not dividing m. Prove that sylow p-subgroups of G exists.

b) Let $H = \langle(1\ 2\ 3)\rangle$. Prove that H is normal subgroup of S_3.

c) State and prove third isomorphism theorem.

Q5) Attempt any two of the following : **[10]**

a) Let N be any subgroup of the group G. Prove that the set of left cosets of N in G form a partition of G.

b) Let G be a group and let A be nonempty set and G act on the set A. If $a, b \in A$ and $b = g.a$, for some $g \in G$ then prove that $G_b = gG_a g^{-1}$.

c) Let G be a finite group and $g_1, g_2, \ldots, g_r$ be representatives of the distinct conjugacy classes of G not contained in the centre Z(G) of G.

Prove that $|G| = |Z(G)| + \sum_{i=1}^{r} |G : C_G(g_i)|$.

Q6) Attempt any two of the following : **[10]**

a) Let $m, n \in \mathbb{Z}^+$. Prove that $\mathbb{Z}_m \times \mathbb{Z}_n \cong \mathbb{Z}_{mn}$ if and only if $(m, n) = 1$.

b) Let $a, b, c \in G$. Prove that $[a, bc] = [a, c]\,(c^{-1}[a, b]c)$.

c) Show that the order of S_n is $n!$.

Q7) Attempt any two of the following : **[10]**

a) Let $H = \langle x \rangle$ be a cyclic group. Prove that every subgroup of H is cyclic.

b) List the abelian groups of order 243.

c) Let G be a group, $x, y \in G$ and $H \leq G$. Prove that H is normal subgroup of G if and only if $[H, G] \leq H$.

Total No. of Questions : 7]

SEAT No. :

PC4061

[Total No. of Pages : 3

[6334]-1003

M.A./M.Sc.-I

MATHEMATICS

MTS - 504 : MJ : Ordinary Differential Equations

(2023 Credit Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***All question are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[10]**

a) Solve the differential equation $y' - 2y = 1$.

b) Find the solution of differential equation $3y'' + 2y' = 0$

c) Determine whether the functions $\phi_1(x) = \cos x$ and $\phi_2(x) = \sin x$ are linearly dependent or independent on $-\infty < x < \infty$.

d) Write the general form of

i) Legendre's equation

ii) Bessel's equation

e) Find the singular point of Euler's equation.

f) Determine whether the differential equation $(x^2 + xy)\,dx + xy\,dy = 0$ is exact.

g) Define Lipschitz condition.

P.T.O.

Q2) Attempt any two of the following. **[10]**

a) Explain the method of solving the differential equation $y' + ay = b(x)$ where a is constant and $b(x)$ is continuous function on some interval I.

OR

b) Consider the differential equation $x^2y' + 2xy = 1$ on $0 < x < \infty$.

i) Show that every solution tends to zero as $x \to \infty$

ii) Find a solution ϕ which satisfy $\phi(2) = 2\,\phi(1)$.

c) Solve the differential equation $Ly' + Ry = E$ where L, R and E are constants.

Q3) Attempt any two of the following. **[10]**

a) Prove that the two solutions ϕ_1 and ϕ_2 of the differential equation $L(y) = y'' + a_1y' + a_2y = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2) \neq 0$ for all x in I.

b) Solve the initial value problem.

$y'' - 2y' - 3y = 0,\ y(0) = 0,\ y'(0) = 1$

c) Find all solutions of the equation $y'' + 9y = \sin 3x$.

Q4) Attempt any two of the following. **[10]**

a) Solve the differential equation $y''' - y' = x$.

b) compute the Wronskian $WC\phi_1, \phi_2, \phi_3)(x)$ at $x = 0$ for the functions $\phi_1(x) = e^x$, $\phi_2(x) = xe^x$ and $\phi_3(x) = x^2e^x$.

c) Find two linearly independent power series solutions in powers of x for the equation $y'' - xy = 0$

Q5) Attempt any two of the following. **[10]**

a) Let ϕ_1 be the solutions of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I. Then show that the second solution ϕ_2 on I of L(y) = 0 is given by $\phi_2(x) = \phi_1(x)\int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2}.\exp\left[-\int_{x_0}^{s} a_1(t)dt\right]ds$.

b) Compute the indicial polynomial and it's root for the differential equation $x^2y'' + xy' + (x^2 - ¼) = 0$.

c) Show that infinity is not a regular singular point for the Bessel's equation.

Q6) Attempt any two of the following. **[10]**

a) Explain the method of successive approximations for finding solutions of the equation $y' = f(x,y)$.

b) Solve the differential equation $2xy\,dx + (x^2 + 3y^2)dy = 0$.

c) Show that the function $f(x,y) = 4x^2 + y^2$

i) satisfies the Lipschitz condition on the set $s : |x| \leq 1, |y| \leq 1$.

ii) does not satisfy Lipschitz condition on the strip $s : |x| \leq 1, |y| < \infty$

Q7) Attempt any two of the following. **[10]**

a) Explain the variable separable method for the first order differential equations.

b) If $\overline{v}_1, \overline{v}_2, ... \overline{v}_n \in \mathbb{R}^n$ are n-eigen vectors of the n × n matrix A and the corresponding eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ are real and distinct then prove that $\overline{x}(t) = c_1\overline{v}_1e^{\lambda_1 t} + c_2\overline{v}_2e^{\lambda_2 t} + ... + c_n\overline{v}_ne^{\lambda_n t}$ is the general solution of the homogeneous system $\overline{x}' = A\overline{x}$.

c) Find the general solution of $\overline{x}' = A\overline{x}$ where $A = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

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Total No. of Questions : 4]

SEAT No. :

PC4062

[Total No. of Pages :3

[6334]-1004

First Year M.A./M.Sc.

MATHEMATICS

MTS - 506(A) MJ : Advanced Numerical Analysis

(2023 Credit Pattern) (Semester- I)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Use of single, non-programmable scientific calculator is allowed.***
3) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Show that when Newton's method is applied to the equation $\frac{1}{x} - a = 0$ the resulting iteration function is $g(x) = x(2 - ax)$.

b) Determine the corresponding rate of convergence for the function

$$f(x) = \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}.$$

c) Compute the condition number k_∞ for the matrix $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$.

d) Show that the matrix $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$ has no LU decomposition.

P.T.O.

Q2) Attempt any two of the following. **[10]**

a) Derive the closed Newton cotes formula with $n = 3$

$$\int_a^b f(x)dx = \left(\frac{b-a}{8}\right)\left[f(a) + 3f(a+\Delta x) + 3f(a+2\Delta x) + f(b)\right].$$

b) Find the solution of initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 6,\ x(1) = 1$.

Using second order Runge-Kutta method using two steps.

c) Construct the Householder matrix H for $w = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}^T$.

Q3) Attempt any two of the following. **[10]**

a) Verify that the equation $x^4 - 18x^2 + 45 = 0$ has a root on the interval (1, 2). Perform four iterations of the secant method using $p_0 = 1$, $p_1 = 2$.

b) For the matrix $A = \begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$ with initial vector $x^{(0)} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

perform two interations of the power method to find the dominant eigen value and corresponding eigen vector.

c) Construct the Neville's table for the following data set take $\bar{x} = 3.7$

x	2	4	5
y	–1	4	8

Q4) a) Attempt any one of the following. **[4]**

i) Derive the following backward difference approximation for the second derivative, $f''(x_0) = \frac{f(x_0 - 2h) - 2f(x_0 - h) + f(x_0)}{h^2}$.

ii) Approximate the value of definite integral $\int_1^2 \frac{1}{x} dx$ by composite Simpson's $\frac{1}{3}^{rd}$ rule, taking $n = 6$.

b) Attempt any one of the following. **[5]**

i) Solve the following system of linear equations by SOR method. Start with $x^{(0)} = [0 \quad 0 \quad 0]^T$ & $w = 0.9$. Perform two iterations $4x_1 - x_2 = 2, -x_1 + 4x_2 - x_3 = 4, -x_2 + 4x_3 = 10$.

ii) Apply Euler's method to find the solution of the initial value problem $\frac{dx}{dt} = \frac{t}{x}, 0 \le t \le 5, x(0) = 1$. Using four steps.

Total No. of Questions : 4]

SEAT No. :

PC4063

[Total No. of Pages :2

[6334]-1005

First Year M.A./M.Sc.

MATHEMATICS

MTS - 506(B) - MJ : Number Theory

(2023 Credit Pattern) (Semester- I)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Symbols and Notations have their usual meanings.***

Q1) Attempt any three of the following. **[6]**

a) For any positive integer n, prove that $2/n(n-1)$.

b) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then prove that $ac \equiv bd \pmod{m}$.

c) Evaluate the Legender's symbol $\left(\frac{10}{11}\right)$.

d) If $x \in \mathbb{R}$ and $m \in \mathbb{Z}$ then prove that $[x+m] = [x] + m$.

Q2) Attempt any two of the following. **[10]**

a) If x, y, z is a pythagorean triple then prove that at least one of the x, y or z is divisible by 5.

b) Prove that the product of two primitive polynomial is primitive.

c) Let p be a prime. Prove that $x^2 \equiv -1 \pmod{p}$ has solution if and only if $p = 2$ or $p \equiv 1 \pmod{4}$.

Q3) Attempt any two of the following. **[10]**

a) If $d = g.c.d\,(b, c)$ then prove that there exists a integer's x_0 and y_0 such that $d = bx_0 + cy_0$.

b) Evaluate the Legender's symbol $\left(\frac{-42}{61}\right)$.

c) State and prove De - Poliganic formula.

P.T.O.

Q4) a) Attempt any one of the following. **[4]**

i) Prove that 19 is not a divisor of $4n^2 + 4$ for any integer n.

ii) Prove that 3 is quadratic residue of 13 but quadratic non residue of 7.

b) Attempt any one of the following. **[5]**

i) Define algebric integer and prove that among all the rationals the 0, ±1, ±2, ... are only algebric integer's.

ii) If ξ is algebric number of degree n then prove that every number in $\mathbb{Q}(\xi)$ can be uniquely express as $a_0 + a_1.\xi + a_2.\xi^2 + ... + a_{n-1}.\xi^{n-1}$ where $a_i \in \mathbb{Q}$.

Total No. of Questions : 4]

SEAT No. :

PC4064

[Total No. of Pages :2

[6334]-1006

M.A./M.Sc.-I

MATHEMATICS

MTS - 506(C) - MJ : COMBINATORICS

(2023 Credit Pattern) (Semester- I)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Find the rook polynomial for a full 4 × 4 board.

b) Find a recurrence relation for the ways to distribute *n* identical balls into *k* distinct boxes with between two and four balls in each box.

c) How many even six - digit numbers are there?

d) Find a generating function for the number of ways to make '*r*' cents change in pennies, nickels, dimes and quarters.

Q2) Attempt any two of the following. **[10]**

a) Show that by a combinatorial argument

$$\binom{n}{0}^2+\binom{n}{1}^2+\binom{n}{2}^2+\ldots+\binom{n}{n}^2=\binom{2n}{n}$$

b) Using generating function, solve the recurrence relation $a_n=a_{n-1}+n(n-1),\ a_0=1$.

c) State pigeonhole principle. Show that any collection of 8 positive integers whose sum is 20 has a subset summing to 4.

P.T.O.

Q3) Attempt any two of the following. **[10]**

a) How many ways are there to paint the 10 identical rooms in a hotel with five colors if at most three rooms can be painted green, at most three painted blue, at most three red, and no constraint on the other two colors, black and white?

b) Suppose a school with 120 students offers Yoga and Karate. If the number of students taking Yoga alone is twice the number taking Karate (possibly, Karate and Yoga), if 25 more students study neither skill than study both skills, and if 75 students take at least one skill, How many students study Yoga?

c) How many r - digit ternary sequence in which:

i) No digit occurs exactly twice?

ii) 0 and 1 each appear a positive even number of times?

Q4) a) Attempt any one of the following. **[5]**

i) State and prove Inclusion - Exclusion formula.

ii) How many 10 letter words are there in which each of the letters e, n, r, s occur

1) at most once?

2) at least once?

b) Attempt any one of the following. **[4]**

i) How many non negative integer solutions are there to the equalities $x_1 + x_2 + ... + x_6 \leq 20$ and $x_1 + x_2 + x_3 \leq 7$?

ii) How many distributions of 24 different object into three different boxes are there with twice as many objects in one box as in the other two combined?

Total No. of Questions : 4]

SEAT No. :

PC4065

[Total No. of Pages :2

[6334]-1007

M.A./M.Sc. - I

MATHEMATICS

MTS - 506(D) - MJ : Lattice Theory

(2023 Credit Pattern) (Semester- I)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Symbols and notations have their meanings.***

Q1) Attempt any three of the following. **[6]**

a) Give an example of poset with maximal element in which not every element is included in a maximal element.

b) Does one-one join homomorphism implies meet homomorphism? Justify your answer.

c) Show that, $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$ for all elements x, y, z in lattice L.

d) Define, modular lattice.

Q2) Attempt any two of the following. **[10]**

a) Let $\phi : L \rightarrow K$ be an onto homomorphism, I be an ideal of L and J be an ideal of K then show that, $\phi(I)$ is an ideal of K and $\phi^{-1}(J) = \{a \in L / \phi(a) \in J\}$ is an ideal of L.

b) Show that in a lattice, maximal element is unique if exists but it is not true for poset.

c) Let I is an ideal and D is dual ideal. If $I \cap D \neq \phi$ then show that $I \cap D$ is convex sublattice and every convex sublattice can be express in this

P.T.O.

form.

Q3) Attempt any two of the following. **[10]**

a) Let L be a lattice. show that L is modular if and only if L does not contain pentagon as a sublattice.

b) Show that, a lattice is distributive if and only if it is isomorphic to a ring of sets.

c) Show that every maximal chain C of the finite distributive lattice L is of length |J(L)|.

Q4) a) Attempt any one of the following. **[4]**

i) Any complete lattice that satisfies the join Infinite Distributive Identity (JID) then prove that it is a pseudo complemented distributive lattice.

ii) In a distributive lattice with pseudo complementation L, if L is a stone algebra then show that $(a \wedge b)^* = a^* \vee b^*$ for $a, b \in L$.

b) Attempt any one of the following. **[5]**

i) Prove that every distributive lattice is modular but not conversly. Also find the smallest modular lattice which is not distributive.

ii) Let L be a modular lattice and $a, b \in$ L. Show that $\phi_b : x \mapsto x \wedge b,\ x \in [a, a \vee b]$ is an isomorphism of $[a, a \vee b]$ and $[a \wedge b, b]$

Total No. of Questions : 4]

SEAT No. :

PC4066

[Total No. of Pages : 1

[6334]-1008

First Year M.A./M.Sc.

MATHEMATICS

MTS-508 MJ: Research Methodology

(2023 Credit Pattern) (Semester - I)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) What is mean by Research?

b) What are mathematical tools used in Research methodology?

c) What do you mean by presentation?

d) Explain the term patent.

e) What is mean by Thesis?

Q2) Attempt any two of the following. **[10]**

a) Explain h-index.

b) Write a short note on scopus.

c) Explain significance of Research Report writing.

Q3) Attempt any two of the following. **[10]**

a) Write a short note on ZMATH.

b) Write a short note on Research Proposal.

c) Write a difference between survey and Experiment.

Q4) a) Attempt any one of the following. **[5]**

i) Distinguish between Research methods and methodology.

ii) Explain the concept of ISSN and ISBN and their importance.

b) Attempt any one of the following. **[4]**

i) Describe in detail tools of data collection.

ii) Explain the characteristics of Research Methods.

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Total No. of Questions : 4]

SEAT No. :

PC4067

[Total No. of Pages : 1

[6334]-2001

M.A./M.Sc. - I

MATHEMATICS

MTS - 551 MJ : Topology

(2023 Credit Pattern) (Semester - II)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Show that a subset of a countable set is countable.

b) Show that a set $\mathbb{Z}$ of all integers in countably infinite.

c) Show that the set $F = \{xxy|x \geq 0 \text{ and } y \geq 0\}$ is closed in $\mathbb{R}^2$.

d) Define : Lindelöf space.

Q2) Atempt any two of the following. **[10]**

a) State and prove pasting lemma.

b) Prove that every finite point set in a Hausdorff space X is closed.

c) Show that every order topology is Hausdorff.

Q3) Attempt any two of the following. **[10]**

a) Prove that every compact subspace of a Hausdorff space is closed.

b) Prove that image of a compact space under a continuous map is compact.

c) Show that [0, 1] and (0, 1) are not homeomorphic.

Q4) Attempt any one of the following. **[9]**

a) Show that $\mathbb{R}_R$ and I_0^2 are not metrizable.

b) Prove that every regular space with a countable basis is normal.

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Total No. of Questions : 7]

SEAT No. :

PC4068

[Total No. of Pages : 2

[6334]-2002

First Year M.A./M.Sc.

MATHEMATICS

MTS - 553 MJ : Ring Theory

(2023 Credit Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicates full marks.***

Q1) Attempt any five of the following : **[10]**

a) Prove that every Boolean ring is commutative.

b) Decide the ideal (x) in $\mathbb{Z}[x]$ is maximal ideal.

c) Let R be a commutative ring and let $a, b \in$ R with $b \neq 0$. Then prove that $a \in$ (b) if and only if $(a) \subseteq (b)$.

d) Define a Dedekind - Hass norm.

e) Give an example of a Unique Factorization Domain but not principal Ideal Domain.

f) Determine whether the polynomial $x^3 + x + 1$ in $\mathbb{Z}_3[x]$ is irreducible.

g) Let $p(x, y) = 2x^3 + xy - y^2$ and $q(x, y) = -3xy + 2y^2 + x^2y^3$ be polynomials in $\mathbb{Z}[x, y]$. Find the degree of p and q in the variable y with coefficients in $\mathbb{Z}[x]$.

Q2) Attempt any two of the following : **[10]**

a) Prove that any finite integral domain is field.

b) Show that the ideal $(2, x)$ in $\mathbb{Z}$[x] is not a maximal ideal.

c) Prove that every ideal in a Euclidean domain is principal.

Q3) Attempt any two of the following : **[10]**

a) Let R be an integral domain. If two elements d and d' of R generate the same principal ideal. Then prove that $d' = ud$ for some unit u in R.

b) Prove that $\mathbb{Z}\left[\sqrt{-5}\right]$ is not a Principal Ideal Domain.

c) Prove that every prime element in an integral domain is irreducible but converse is not true.

P.T.O.

Q4) Attempt any two of the following : **[10]**

a) If R is commutative ring such that the polynomial ring R[x] is a Principal Ideal Domain. Then prove that R is necessarily a field.

b) Determine all the representations of the integer 493 as a sum of two squares.

c) Let F be a field. Then prove that the polynomial ring F[x] is a Euclidean domain.

Q5) Attempt any two of the following : **[10]**

a) State and prove Eisenstein's Criterion.

b) Let I be an ideal of the ring R. Then prove that $R[x]/I[x] \cong (R/I)[x]$.

c) Let R be a commutative ring. Then prove that the ideal P is a prime ideal in R if and only if the quotient ring R/P is an integral domain.

Q6) Attempt any two of the following : **[10]**

a) Let R be a ring and A be a subring and B be an ideal of R. Then prove that $(A+B)/B \cong A/(A \cap B)$.

b) Determine whether the map $\phi : Q[x] \rightarrow Q$ define as $\phi(p(x)) = p(o)$, $p(x) \in Q[x]$, is ring homomorphism.

c) Prove that the quotient ring $\mathbb{Z}[i]/(1+i)$ is a field of order 2.

Q7) Attempt any two of the following : **[10]**

a) Prove that a polynomial of degree two or three over a field F is reducible if and only if it has a root in F.

b) Let R and S be rings and $\phi : R \rightarrow S$ be an ring homomorphism. Prove that kernel of ϕ is ideal of R.

c) Prove that set of integers $\mathbb{Z}$ is a Euclidean domain with norm $N(a) = |a|$.

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Total No. of Questions : 7]

SEAT No. :

PC4069

[Total No. of Pages : 3

[6334]-2003
First Year M.A./M.Sc.
MATHEMATICS
MTS 554MJ : Advanced Calculus
(2023 Pattern) (Semester-II) (Credit Pattern)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[10]**

a) Find the gradient of the function $f(x,y,z)=x^2y^3z^4$.

b) Show that, for the function $f(x,\mathrm{y})=\dfrac{x-y}{x+y}$, if $x+y\neq 0$ repeated limit exist but not equal.

c) Give an example of path which is not smooth path.

d) State the general formula for the change of variables in double integral.

e) Compute the fundamental vector product of

$\overline{r}(u,v)=(u+v)\overline{i}+(u-v)\overline{j}+4v^2\overline{k}$

f) Find Jacobian determinant for polar co-ordinates.

g) Define surface integral.

Q2) Attempt any two of the following: **[10]**

a) Let $g(t)=f(a+ty)$. If one of the derivatives $g'(t)$ or $\mathrm{F}'(a+ty;y)$ exists then show that the other also exists and the two are equal,

$$g'(t)=\mathrm{F}'(a+ty;y)$$

In particular, when $t=0$ we have $g'(0)=\mathrm{F}'(a\,;y)$.

P.T.O.

b) Evaluate the directional derivative of the scalar field $f(x,y,z)=\left(\frac{x}{y}\right)^z$ at point (1, 1, 1) in the direction of $2\bar{i}+\bar{j}-\bar{k}.$

c) Calculate the line integral of the vector field $\bar{F}$;

$\bar{F}(x,y)=(2a-y)\bar{i}+x\bar{j}$ along the path described by

$\bar{\alpha}(t)=a(t-\sin t)\bar{i}+a(1-\cos t)\bar{j},\ 0\le t\le 2\pi$

Q3) Attempt any two of the following: **[10]**

a) Prove that, if ϕ be a differentiable scalar field with continuous gradient ∇ ϕ on an open connected set S in $\mathbb{R}^n$. Then for any two points a and b joined by piecewise smooth path α in S we have

$$\int_a^b \nabla\phi\cdot d\bar{\alpha}=\phi(\bar{b})-\phi(\bar{a})$$

b) Evaluate $\iint_Q \sin^2 x\cdot\sin^2 y\ dxdy$, where $Q=[0,\pi]\times[0,\pi]$

c) A force field $\bar{F}$ in 3-space is given by the formula

$\bar{F}(x,y,z)=yz\bar{i}+xz\bar{j}+x(y+1)\bar{k}.$

Calculate the work done by $\bar{F}$ in moving a particle once around the triangle with vertices (0, 0, 0), (1,1,1), (–1,1,–1) in that order.

Q4) Attempt any two of the following: **[10]**

a) Prove that if ϕ be real valued function that is continuous on an interval [a,b] then graph of ϕ has content zero.

b) Evaluate the line integral $\int_c (5-xy-y^2)dx-(2xy-x^2)dy$ where C is the square with vertices (0,0), (1,0), (1,1), (0,1) traversed counter clockwise.

c) State and prove Gauss divergence theorem.

Q5) Attempt any two of the following: **[10]**

a) Prove that, curl (curl $\overline{\mathrm{F}}$) = grad (div $\overline{\mathrm{F}}$) – $\overline{\nabla}^2\overline{\mathrm{F}}$, if the components of $\overline{\mathrm{F}}$ have continuous mixed partial derivatives of second order.

b) Find the fundamental vector product for the surface

$\overline{r}(x,\mathrm{y}) = x\overline{i} + u\overline{j} + \mathrm{F}(x,\mathrm{y})\overline{k}.$

What are the singular points of this surface?

c) If a scalar field F is fifferentiable at a, then show that, F is continous at a.

Q6) Attempt any two of the following. **[10]**

a) Find all differentiable vector field $\overline{\mathrm{F}}:\mathbb{R}^3 \to \mathbb{R}^3$ for which Jacobian matrix DF is identity matrix of order 3.

b) State and prove chain rule for derivative of vector field.

c) State and prove Green's Theorem.

Q7) Attempt any two of the following. **[10]**

a) Find the gradient vector of the function,

$\mathrm{F}(x,y,z) = \log\left(x^2 + 2y^2 - 3z^2\right).$

b) State and prove Stokes theorem.

c) Determine whether or not the vector field

$\overline{\mathrm{F}}(x,y) = 3x^2y\ \overline{i} + x^3y\overline{j}$ is a gradient on any open subset of $\mathbb{R}^2$.

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Total No. of Questions : 4]

PC4070

SEAT No. :

[Total No. of Pages : 3

[6334]-2004

First Year M.A./M.Sc.

MATHEMATICS

MTS-556(A)-MJ : Graph Theory

(2023 Credit Pattern) (Semester - II)

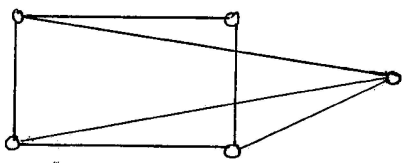
Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

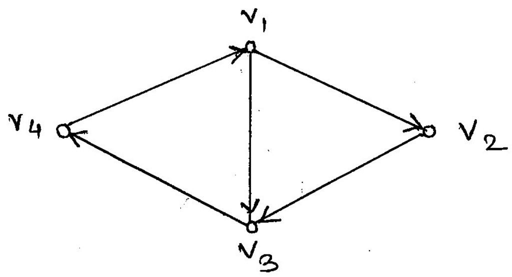
Q1) Attempt any three of the following : **[6]**

a) Find the complement of the following graph.

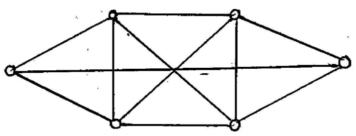


b) Prove that a connected graph G with n vertices has at least (n–1) edges.

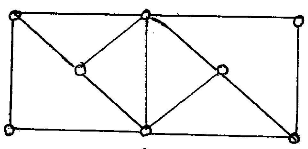
c) Find the indegree and outdegree of each vertex in the following graph.



d) Find the Clique number for the following graphs.



G_1



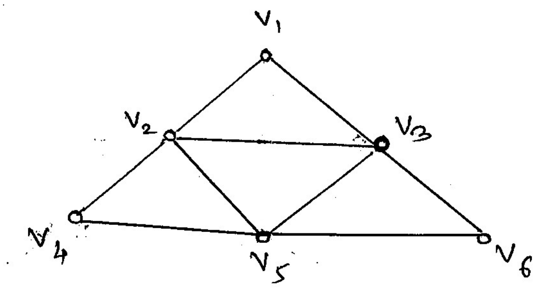
G_2

P.T.O.

Q2) Attempt any two of the following : **[10]**

a) Prove that an edge '*e*' of a graph G is a bridge if and only if *e* is not a part of any cycle in G.

b) Find an Eulerian trial in the following graph using Fluery's algorithm.

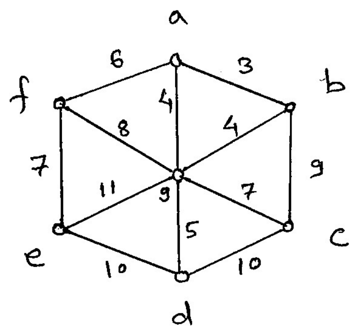


c) Let G be a non-empty graph. Prove that $\chi(G) = 2$ if and only if G is bipartite.

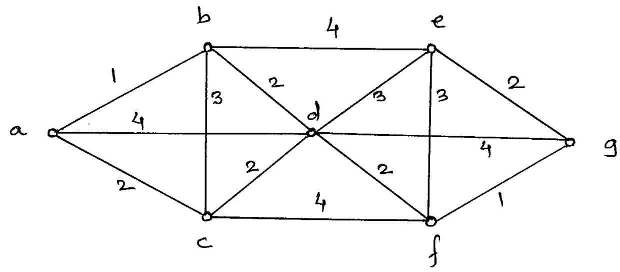
Q3) Attempt any two of the following : **[10]**

a) Prove that for each pair of positive integers *n* and *k*, both greater than one, the *de* Bruijn diagram $D_{n,k}$ has a directed Euler tour.

b) Find a minimal spanning tree for the following weighted graph using Prim's algorithm. Also find weight of the minimal spanning tree.



c) Solve the chinese postman problem for the following graph.



Q4) a) Attempt any one of the following : **[4]**

i) Prove that in any graph G there is an even number of odd vertices.

ii) 1) Define Hamiltonian graph.

2) Define Eulerian graph.

3) Give an example of a graph which is Hamiltonian but not Eulerian.

4) Give an example of a graph which is Eulerian but not Hamiltonian.

b) Attempt any one of the following : **[5]**

i) Prove that a connected graph G is Euler if and only if degree of every vertex is even.

ii) Check whether the graph G is connected or not whose adjacency matrix is $A(G)=\begin{bmatrix}0&0&1&0&0\\0&0&0&1&0\\1&0&0&0&1\\0&1&0&0&1\\0&0&1&1&0\end{bmatrix}$

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Total No. of Questions : 4]

SEAT No. :

PC4071

[Total No. of Pages : 3

[6334]-2005

First Year M.A. /M.Sc.

MATHEMATICS

MTS-556 (B) MJ : Dynamical Systems

(2023 Credit Pattern) (Semester-II)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[3×2=6]**

a) Find e^{At} if $A=\begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$.

b) Assume that M (t) is fundamental matrix solution of $x = A(t)\,x$ and c is a constant matrix with $\det(c) \neq 0$. Show that $M(t)\,c$ is a fundamental matrix solution.

c) Show that $\dot{x} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} x$ has saddle at (:).

d) Find polar form of the differential system

$$\dot{x} = y + x\left(1 - x^2 - y^2\right)$$

$$\dot{y} = -x + y\left(1 - x^2 - y^2\right)$$

Hence find limit cycles of the system.

P.T.O.

Q2) Attempt any two of the following: **[2×5=10]**

a) Let x^* be an equilibrium point of $\dot{x} = F(x)$, where F is twice continuously differentiable function. Assume all the eigenvalues of $DF(x^*)$ has negative real part. Then show that x^* is asymptotically stable.

b) Sketch the phase portrait of $\dot{x} = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} x.$

c) Show that any periodic orbit of the system

$\dot{x} = y, \quad \dot{y} = -x + \left(1 - x^2\right) y$ is stable.

Hence show that it has only one periodic orbit.

Q3) Attempt any two of the following: **[2×5=10]**

a) Assume that a forward orbit $\{\theta(t,q) \ : \ t \geq 0\}$ of the differential system $\dot{x} = F(x), \ \ F \in C^2\left(\mathbb{R}^2\right)$ is bounded. Then show that limit set $w(q)$ either contains a fixed point or a periodic orbit.

b) Consider system $\dot{x} = x(a - by - fx), \dot{y} = y(-c + ex - hy)$ where all the parameters *a, b, c, e, f* and *h* are positive. Using Dulac criterion show that this system does not have periodic orbit. Also find expression of poin care map p and using p′ show that again system has no periodic orbit.

c) Find polar form of the system

$$\dot{x} = y + x\left(1 - x^2 - y^2\right)\left(4 - x^2 - y^2\right)$$

$$\dot{y} = -x + y\left(1 - x^2 - y^2\right)\left(4 - x^2 - y^2\right)$$

Hence show that the system has one attracting limit cycle and one repelling limit cycle.

Q4) a) Attempt any one of the following: **[1×4=4]**

i) Let C be a diagonal matrix, T be invertible symmetric matrix and

$$f(x)=\left(f_1(x_1), f_2(x_2), \ldots\ldots f_n(x_n)\right)^t$$

$$b=\left(b_1, b_2, \ldots . b_n\right)^t$$

Then show that the system

$$\dot{x}=-cx+\mathrm{T}f(x)+b$$

has Lyapunov function

$$\mathrm{V}(x)=\mathrm{F}(x)-\frac{1}{2}\left(f(x)+\mathrm{B}\right)^t\ \mathrm{T}\left(f(x)+\mathrm{B}\right)$$

Where $\mathrm{F}(x)=\sum_{i=1}^{n} c_i \int_0^{f_i(x_i)} f_i^{+}(s)ds$

$c=\left(c_1, c_2, \ldots . c_n\right)^t, \quad \mathrm{B}=\mathrm{T}^{-1}b$.

ii) Assume that the system $\dot{x}_i=x_i\left(r_i+\sum_{j=1}^{n} a_{ij}\ x_j\right)$ for $i=1,2,3,\ldots\ldots n$

has no fixed point in $\mathbb{R}_{+}^{n}$. Then show that for any

$x_0 \in \mathbb{R}_{+}^{n},\ w(x_0)\cap \mathbb{R}_{+}^{n}=\phi$ and $\alpha(x_0)\cap \mathbb{R}_{+}^{n}=\phi$

Where $\mathbb{R}_{+}^{n}=\{x=(x_1, x_2, \ldots x_n) \mid x_i>0\}$

$w(x_0)$ is forward limit set of x_0

$\alpha(x_0)$ is backward limit set of x_0

b) Attempt any one of the following: **[1×5=5]**

i) Let $\dot{x}=-\nabla \mathrm{G}(x)$ be a gradient system.

Let z be an α limit point or w limit point.

Then show that z is a fixed point of the system $\dot{x}=-\nabla \mathrm{G}(x)$

ii) Consider system $\dot{x}=-\nabla \mathrm{G}(x)$, where $\mathrm{G}(x)=\mathrm{G}(x,y)=\dfrac{-x^2+x^4+y^2}{2}$

for all $x=(x,y)\in \mathbb{R}^2$ show that (0,0) is saddle and $(\neq 1,0)$ are nodes of the system. Also sketch the phase portrait of the system.

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Total No. of Questions : 4]

SEAT No. :

PC4072

[Total No. of Pages : 2

[6334]-2006
First Year M.A./M.Sc.
MATHEMATICS
MTS-556(C)-MJ : Coding Theory
(Credit 2023 Pattern) (Semester - II)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following : **[3×2=6]**

a) Define cyclic code and give example on it.

b) Is C = {(0, 1, 1, 2), (2, 0, 1, 1), (1, 2, 0, 1), (1, 1, 2, 0)} $\leq F_3^{\,4}$ cyclic code? Justify.

c) Let C be a binary linear code with parits check matrix.

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find d(c)

d) Define the following terms

i) Generator Matrix

ii) Optimal code

Q2) Attempt any two of the following : **[2×5=10]**

a) If $9(x) = (1 + x)(1 + x^2 + x^3) \in F_2(x)/(x^7 - 1)$ is a generator polynomial of cyclic code C then find C and dim(c).

b) Show that binary Hamming codes are perfect codes.

c) Construct the incomplete maximum likelihood decoding table for the binary code

C = {000, 001, 010, 011}.

P.T.O.

Q3) Attempt any two of the following : **[2×5=10]**

a) Let $q = 2$ and let $S = \{0100, 0101\}$. Find $S^{\perp}$ and verify $\dim(\langle S\rangle) + \dim(S^{\perp}) = n$.

b) For the ternary code $c = \{00122, 12201, 20110, 22000\}$, use the nearest neighbour decoding rule to decode the word 01122.

c) If q is a prime power then show that $B_q(n,n) = A_q(n, n) = q$.

Q4) a) Attempt any one of the following : **[1×4=4]**

i) Show that each monic divisior of $x^n - 1$ is the generator polynomial of some cyclic code in F_q^n.

ii) Find a generator matrix and a parity - check matrix for the linear code $C = \langle S\rangle$ where $S = \{110000, 011000, 001100, 000110, 000011\} \subseteq F_3^6$.

b) Attempt any one of the following : **[1×5=5]**

i) Find number of distinct bases for vector space $V = \langle S\rangle$ over F_2, where $S = \{0001, 0010, 0100\}$.

ii) Suppose that codewords from the binary code {000, 100, 111} are being sent over binary symmetric channel with crossover probability P = 0.03. Use the Maximum likehood decoding rule to decode word W = 011.

Total No. of Questions : 4]

SEAT No. :

PC4073

[Total No. of Pages : 4

[6334]-2007

First Year M.A./M.Sc.

MATHEMATICS

MTS 556 (D) MJ : Operations Research

(2023 Credit Pattern) (Semester-II)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figure to the right indicates full marks.***

Q1) Attempt any three of the following. **[6]**

a) Write the dual of following LP problem.

Minimize $Z = x_1 + 2x_2$

subject to $2x_1 + 4x_2 \le 160$

$x_1 - x_2 = 30$

$x_1 \ge 10$

and $x_1, x_2 \ge 0$

b) Write the following LPP in equation form

maximize $Z = 3x_1 + 5x_2 + 4x_3$

subject to $2x_1 + 3x_2 \le 8$

$2x_2 + 5x_3 \le 10$

$3x_1 + 2x_2 + 4x_3 \le 15$

and $x_1, x_2, x_3 \ge 0$

c) Define slack and surplus variable in LPP.

d) Define general non-linear programming problem

P.T.O.

Q2) Attempt any two of the following: **[10]**

a) Solve the following LP problem using simplex method

Max $Z = 4x_1 + 3x_2$

Subject to constraints $2x_1 + x_2 \le 1000$

$x_1 + x_2 \le 800$

$x_1 \le 400$

$x_2 \le 700$

and $x_1, x_2 \ge 0$

b) A firm manufacturing two product A and B on machine I and II as shown below.

Machine	Product A	Product B	Available hours.
I	30	20	300
II	5	10	110
Profit per units (Rs.)	6	8	

The total time available is 300 hrs and 110 hrs on machine I and II respectively. Product A and B contribute a profit of Rs. 6 and Rs. 8 per unit respectively. Formulate this as an LP problem also write it's dual.

c) Solve the following LP problem.

max $Z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \le 18$

$x_1 \le 4$

$x_2 \le 6$

and $x_1, x_2 \ge 0$

Q3) Attempt any two of the following: **[10]**

a) Solve the following LP problem using Gomory's cutting plane method.

minimize $Z = x_1 + x_2$

subject to $3x_1 + 2x_2 \le 5$

$x_2 \le 2$

$x_1, x_2 \ge 0$ and are integers.

b) Solve the following Non-Linear programming problem using graphical method.

maximize $Z = x_1 + x_2$

subject to $x_1 x_2 - 2x_2 \geq 3$

$3x_1 + 2x_2 \leq 24$

and $x_1, x_2 \geq 0$

c) Use Beale's method to solve following quadratic programming problem:

Minimize $Z = -4x_1 + x_1^2 - 2x_1x_2 + 2x_2^2$

Subject to $2x_1 + x_2 \geq 6$

$x_1 - 4x_2 \geq 0$

and $x_1, x_2 \geq 0$

Q4) a) Attempt any one of the following: **[4]**

i) Solve the following LP problem to show that these have an unbounded solution.

Maximize $Z = 3x_1 + 5x_2$

Subject to $x_1 - 2x_2 \leq 6$

$x_1 \leq 10$

$x_2 \geq 1$

and $x_1, x_2 \geq 0$

ii) Solve the following LP problem to show that these have an Infeasible solution.

Maximize $Z = 6x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 5$

$x_2 \geq 8$

and $x_1, x_2, \geq 0$

b) Attempt any one of the following: **[5]**

i) Find the effect of the following changes on the optimal solution of the following LP problem.

Max $Z = 45x_1 + 100x_2 + 30x_3 + 50x_4$

Subject to $7x_1 + 10x_2 + 4x_3 + 9x_4 \geq 1200$

$3x_1 + 40x_2 + x_3 + x_4 \leq 800$

and $x_1, x_2, x_3, x_4 \geq 0$

		$C_j \rightarrow$	45	100	30	50	0	0
Cost per unit C_B	Variables in Basis B	Solution value b $(=X_B)$	x_1	x_2	x_3	x_4	s_1	s_2
30	x_3	$800/3$	$5/3$	0	1	$7/3$	$4/5$	$-1/15$
100	x_2	$40/3$	$1/30$	1	0	$-1/30$	$-1/150$	$2/75$
$z = \frac{28,000}{3}$		z_j	$1600/30$	100	30	$-11/3$	$110/15$	$50/75$
		$c_j - z_j$	$-25/3$	0	0	$\frac{-55}{3}$	$\frac{-22}{3}$	$\frac{-2}{3}$

1) 'x_1' - column in the problem changes from $[7,3]^T$ to $[7,5]^T$.

2) 'x_1' column changes from $[7,3]^T$ to $[5,8]^T$.

ii) Solve the following LP problem

maximize $Z = 3x_1 + 9x_2$

subject to $x_1 + 4x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

and $x_1, x_2 \geq 0$

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Total No. of Questions : 4]

SEAT No. :

PC4074

[Total No. of Pages : 2

[6334]-3001

S.Y.M.A./M.Sc.

MATHEMATICS

MTS-601-MJ : Complex Analysis

(2023 Credit Pattern) (Semester-III)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate marks.*

Q1) Attempt any three of the following. **[6]**

a) Find real and imaginary parts of $\frac{1}{z}$

b) Find the radius of converges of $\sum_{n=0}^{\infty} k^n z^n$ where k is integer and $k \neq 0$

c) Evaluate $\int_{v} \frac{dz}{z-a}$, $v(t) = a + re^{it}$, $0 \leq t \leq 2\Pi$

d) Evaluate cross ratio (2, 1–i, 1, 1+i)

Q2) Attempt any two of the following: **[10]**

a) Let u & v be real valued functions defined on a region G and suppose that u and v have continuous partial derivatives. Then show that F:G$\to\mathbb{C}$ defined by $f(z) = u(z) + iv$ is analytic if and only if u and v satisfy the Cauchy-Riemann equation.

b) Show that for $a > 1$ $\int_{0}^{\Pi} \frac{d\theta}{a+\cos\theta} = \frac{\Pi}{\sqrt{a^2-1}}$

c) Let G be a region and suppose that f is a non-constant analytic function on G then show that for any open set U in G, f(U) is open.

P.T.O.

Q3) Attempt any two of the following: **[10]**

a) If G is a region and $F:G \to \mathbb{C}$ is an analytic function such that there is a point ä in G with $|f(a)| \geq |f(z)|$ for all Z in G then show that f is constant.

b) Evaluate the integral

$$\int_{v} \frac{\sin z}{z^3}\, dz, \quad v(t) = e^{it}, \quad 0 \leq t \leq 2\Pi$$

c) Calculate the square roots of i

Q4) a) Attempt any one of the following: **[5]**

i) Show that $u(x,y) = \log\left(x^2 + y^2\right)^{½}$ is harmonic function on $G = \mathbb{C} - \{0\}$.

ii) Let G be an open set and let $F:G \to \mathbb{C}$ be a differentiable function then show that F is analytic on G

b) Attempt any one of the following: **[4]**

i) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\Pi}{\sqrt{2}}$

ii) Let f is bounded entire function then show that f is constant.

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Total No. of Questions : 7]

SEAT No. :

PC4075

[Total No. of Pages :3

[6334]-3002

Second Year M.A./M.Sc.

MATHEMATICS

MTS - 603 MJ : Field Theory

(2023 Credit Pattern) (Semester- III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any five of the following. **[10]**

a) If the characteristics of a field F is a Prime P then prove that $P.\alpha = 0 \ \forall \ \alpha \in F$.

b) Find the smallest extension of $\mathbb{Q}$ having a root of $x^2 - 2 \in \mathbb{Q}[x]$.

c) Find the minimal polynomial for $1+\sqrt{2}$ over $\mathbb{Q}$.

d) Define

i) Algebraic closure of a field F.

ii) Separable polynomial.

e) Find fixed field of Aut $\left(\mathbb{Q}\left(\sqrt{2}\right)/\mathbb{Q}\right)$.

f) Define

i) A character of a group.

ii) Frobenius automorphism.

g) Give an example of a Galois extension of $\mathbb{Q}$ of degree *n* with a cyclic Galois group.

Q2) Attempt any two of the following. **[10]**

a) Show that $Q\left(\sqrt[3]{2}\right)$ over Q is not Galois over Q.

b) Show that Galois group of $x^3 - 2 \in \mathbb{Q}[x]$ is a group of symmetric of triangle.

c) State fundamental theorem of Galois theory.

P.T.O.

Q3) Attempt any two of the following. **[10]**

a) Let G be a finite subgroup of automorphism of a field K and F be a fixed then prove that Aut $(K/\mathbb{F}) = G$ and the extension K/F is Galois with G as Galois group.

b) Show that the Irreducible polynomial $x^4 + 1 \in \mathbb{Z}/[x]$ is reducible over $\mathbb{Z}_p$.

c) Show that $Q\left(\sqrt{2}+\sqrt{3}\right) = Q\left(\sqrt{2},\sqrt{3}\right)$.

Q4) Attempt any two of the following. **[10]**

a) Let $P(x) \in F[x]$ be an Irreducible polynomial of degree n over the field F, $K = F[x]/\langle P(x)\rangle$ and $Q = x \bmod \langle P(x)\rangle \in K$ then prove that $\{1, Q, Q^2, ..., Q^{n-1}\}$ from a basis for vector space K over F.

b) If α is an algebraic over F then prove that there is unique monic Irreducible polynomial $M_{\alpha F}(x) \in F[x]$ which has α as a root.

c) Show that $\left[Q\left(\sqrt{2}\right);Q\right] = 6$ and hence show that $x^3 - \sqrt{2}$ is Irreducible polynomial over $Q\left(\sqrt{2}\right)$.

Q5) Attempt any two of the following. **[10]**

a) Define composite Field. Show that composite of two fields $Q\left(\sqrt{2}\right)$ and $Q\left(\sqrt[3]{2}\right)$ is $Q\left(\sqrt[6]{2}\right)$.

b) Show that the degree of a Spliting field of n^{th} degree polynomial over a field F is atmost $n!$ over F.

c) Show that every Irreducible polynomial over a finite field $\mathbb{F}^p$ is seperable.

Q6) Attempt any two of the following. **[10]**

a) Show that the spliting field of $F(x) = x^4 - 2 \in Q[x]$ over $\mathbb{Q}$ is $\mathbb{Q}\left(2^{1/4}, i\right)$.

b) Let F be any field and $\overline{F}$ be a algebraic closure of F then prove that $\overline{F}$ is algebraically closed.

c) Define Cyclotomic polynomial. Find $\Phi_n(x)$ for $n = 1, 2, 3, 4$.

Q7) Attempt any two of the following. **[10]**

a) Prove that squaring the circle is impossible using straight edges and compass.

b) If $[K_1 : F] = n$ and $[K_2 : F] = m$ where K_1, K_2 finite extension of F contained in K and m & n are relatively prime then prove that

$[K_1K_2 : F] = [K_1 : F]\,[K_2 : F]$.

c) If F is field and $[F(\alpha) : F]$ is odd then prove that $F(\alpha) = F(\alpha^2)$.

Total No. of Questions : 7]

SEAT No. :

PC4076

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[6334]-3003
S.Y.M.A./M.Sc.
MATHEMATICS
MTS - 604 - MJ : Differential Geometry
(2023 Credit Pattern) (Semester - III)

Time : 3 Hours] **[Max. Marks : 70**

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any Five of the following. **[10]**

a) Define maximal integral curve with an example.

b) Explain the difference between a plane and a hyperplane.

c) What is the range of the gauss map in $\mathbb{R}^3$.

d) If α(t) = (t^2, t^3) Find $\dot{\alpha}$(t), $\ddot{\alpha}$(t).

e) Give an example of an orientable n-surface in 1 $\mathbb{R}^{n+1}$. Justify.

f) Define Weingarten map.

g) What is a tangent space? Explain.

Q2) Attempt any two of the following. **[10]**

a) Define complete vector field and check wheather the vector field x (x_1, x_2) = $(x_1, x_{2,}$ 2.0) is complete or not.

b) Explain why an integral curve cannot cross itself as does the parametrized Curve.

c) If S is connected n-surface in $\mathbb{R}^{n+1}$ Then show that there exist on S exactly two smooth unit normal vector fields N_1 and N_2 with $N_2(p) = - N_1(p)$ for all p$\in$ S.

P.T.O.

Q3) Attempt any two of the following. **[10]**

a) Show that the set S of all unit vectors at all points of $\mathbb{R}^2$ Forms a 3-surface in $\mathbb{R}^4$.

b) Define geodesics. Hence, show that geodesics have constant speed.

c) Show that the Weingarten map is self-adjoint.

Q4) Attempt any two of the following. **[10]**

a) Let $g(t) : I \rightarrow \mathbb{R}$ is a smooth function and C be the graph of g(t) Then show that the curvature of C at a point (t, g(t)) is $g''(t)/ (1+(g^1 (t)^2)^{3/2}$

b) If C is an oriented plane Curve. Then show that there exist a global parametrization of C if and only if C is connected.

c) State and prove lagrange's Multiplier Theorem.

Q5) Attempt any two of the following. **[10]**

a) If C is a Connected plane Curve and $\beta : I \rightarrow C$ is a unit speed global parametrization of C. Then show that, β is eithe one to one or periodic.

b) Find the Global parametrization of the curve $(x_1 - a)^2 + (x_2 - b)^2 = r^2$.

c) If S is an oriented 2–Surface in $\mathbb{R}^3$ and $p \in S$. Then show that for each

$u, v \in Sp$, $Lp (u) \times Lp (v) = k (p) u \times v$.

Q6) Attempt any two of the following. **[10]**

a) State and prove any two properties of Levi-Civita parallel.

b) Show that the gradient of f at $p \in f^{-1}$ (c) is orthogonal to f^{-1} (c) at 'p'.

c) Let S be an n-surface in $\mathbb{R}^{n+1}$ let p, q$\in$ S and α(t) is a piccewise smooth parametrized Curve from p, to q. Then show that the parallel transport $p_{\alpha}: S_p \to S_q$ along α(t) is a vector space isomorphism which preservesdot products.

Q7) Attempt any two of the following. **[10]**

a) Let a,b,c$\in \mathbb{R}$ be such that ac–b^2 >0. show that the maximum and minimum values of the function g(x_1, x_2) = $x_1^2+x_2^2$ on the ellipse $ax_1^2+2bx_1x_2+Cx_2^2=1$ are of the form $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$.Where λ_1, λ_2 are the eigenvalues of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$.

b) Show that M$\ddot{o}$bius band is an unorientable 2-surface in $\mathbb{R}^3$.

c) Show that the unit n-sphere $x_1^2+x_2^2+ \ldots +x_{n+1}^2=1$ is connected for n>1.

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Total No. of Questions : 4]

SEAT No. :

PC4077

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[6334]-3004

Second Year M.A./M.Sc.

MATHEMATICS

MTS-611(A) MJ : Mathematical Statistics

(2023 Credit Pattern) (Semester- III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Define conditional probability of an event.

b) State Bayes Theorem.

c) Define mean and variance of chi-squared distribution.

d) Define simple linear regression model.

Q2) Attempt any two of the following. **[10]**

a) Prove that, two random variables x and y with joint p.d.f $f(x, y)$ are stochastically independent if and only if $f_{xy}(x, y)$ can be expressed as the product of a non-negative function of x - alone and a non-negative function of y - alone i.e. if $F_{XY}(x, y) = h_X(x).K_Y(y)$, where $h(\cdot) \geq 0$ and $k(\cdot) \geq 0$.

b) Ten is the average number of oil tankers arriving each day at a certain port. The facilities at port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

c) Given the frequency function:

$$f(x, \theta) = \frac{1}{\theta} \quad , 0 \leq x \leq \theta$$

$$= 0 \quad , \text{elsewhere}$$

and that we are testing the null hypothesis. $H_0 : \theta = 1$ against $H_1 : \theta = 2$, by means of a single observed value of x. What would be the sizes of the type I and type II errors. If you choose the interval

i) $0.5 \leq x$

ii) $1 \leq x \leq 1.5$ as the critical regions.

P.T.O.

Q3) Attempt any two of the following: **[10]**

a) State and prove law of addition of probability and prove that, $P(\bar{A}) = 1 - P(A)$.

b) Suppose that the system contain a certain type of component whose time in years, to failure is given by T. The random variable T is modelled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different system, what is the probability that at least 2 are still functioning at the end of 8 years.

c) Explain the method of fitting of binomial distribution.

Q4) a) Attempt any one of the following: **[5]**

i) A card is drawn from a well-shuffled pack of playing cards. What is the probability that, it is either a spade or an Ace?

ii) If $x \geq 1$, is the critical region for testing $H_0 : \theta = 2$ against the alternative $\theta = 1$, on the basis of the single observation from the population.

$$f(x,\theta) = \theta \exp(-\theta x) \qquad ,0 \leq x < \infty$$

Obtain the value of type - I and type - II errors.

b) Attempt any one of the following: **[4]**

i) Show that the moment generating function of a normal random variable is $\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$ where μ and σ^2 are the parameters.

ii) Explain the concept of type - I and type - II errors with examples.

Total No. of Questions : 4]

SEAT No. :

PC4078

[Total No. of Pages : 2

[6334]-3005

M.A./M.Sc. - II

MATHEMATICS

MTS - 611 MJ(B) : Algebraic Topology

(2023 Credit Pattern) (Semester - III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Define Free group.

b) Define a covering map.

c) State the Borsuk - Ulam Theorem for S^2.

d) State the general lifting lemma.

Q2) Atempt any two of the following. **[10]**

a) A space X is said to be contractible if the identity map $i_X : X \to X$ is nulhomotopic. Show that if Y is contractible, then for any X, the set [X, Y] has a single element.

b) Show that if A is star convex, then A is simply connected.

c) Let Y have the discreate topology. Show that if $p : X \times Y \to X$ is projection on the first coordinate, then p is a covering map.

Q3) Attempt any two of the following. **[10]**

a) Show that $\mathbb{R}^1$ and $\mathbb{R}^n$ are not homeomorphic if n > 1.

b) Find a continous map of the torus into S^1 that is not nulhomotopic.

c) Let X be the wedge of circles S_α for $\alpha \in J$. Prove that X is normal.

P.T.O.

Q4) A) Attempt any one of the following. **[4]**

a) Let A be a 3 by 3 matrix of positive real numbers. Then prove that A has a positive real eigenvalue.

b) State the Seifert - van Kampen theorem.

B) Attempt any one of the following. **[5]**

a) Let $p : E \to B$ and $p' : E' \to B$ be covering maps. Let, $p(e_0) = p'(e'_0) = b_0$. Prove that, there is an equivalence $h : E \to E'$ such that $h(e_0) = e'_0$ if and only if the groups $H_0 = p_*(\pi_1(E, e_0))$ and $H'_0 = p'_*(\pi_1(E', e'_0))$ are equal.

b) Show that if A is a retract of B^2, then every continous map $f : A \to A$ has a fixed point.

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Total No. of Questions : 4]

SEAT No. :

PC4079

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[6334]-3006

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 611(C) MJ : Integral Transforms and Special Functions

(2023 Credit Pattern) (Semester - III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) As $t \to \infty$ test $f(t)$ for O. (e^{kt}) if $f(t) = t^2$.

b) Obtain the Laplace transform of $f(t) = U(t-2)\, e^{t-2}$.

c) Evaluate $\Gamma\left(\frac{-7}{2}\right)$.

d) Show that $B(x, y) = B(y, x)$.

Q2) Attempt any two of the following. **[10]**

a) Using partial fractions, find

$$L^{-1}\left\{\frac{s^2-7s+4}{(s-1)(s^2+4)}\right\}$$

b) Solve the following initial value problem :

$y''(t) + y(t) = U(t-\pi)\sin(t-\pi)$, $y(0) = 0$, $y'(0) = 0$.

c) Prove the following Recurrence Relations :

i) $\frac{d}{dx}\left\{x^n J_n(x)\right\} = x^n J_{n-1}(x)$

ii) $\frac{d}{dx}\left\{x^{-n} J_n(x)\right\} = -x^{-n} J_{n+1}(x)$

P.T.O.

Q3) Attempt any two of the following. **[10]**

a) Solve for $y(t)$ and also check the solution if $y(t) = t + \int_0^t y(\tau) \cdot e^{t-\tau} d\tau$.

b) Express the following integral in terms of gamma or beta functions and simplify. $\int_0^1 \frac{1}{\sqrt[3]{1-x^3}} dx$

c) When n is an integer (positive or negative), then prove that $J_{-n}(x) = (-1)^n J_n(x)$.

Q4) a) Attempt any one of the following. **[4]**

i) If $F_d(\omega)$ is the DFT of $f(t)$, then prove that :

1) $F_d(\omega + 2\pi) = F_d(\omega)$

2) $F_d(2\pi - \omega) = F_d(-\omega)$

ii) Prove that : $\int_0^{\pi/2} \cos^{2x-1}\theta \sin^{2y-1}\theta d\theta = \frac{\Gamma(x)\Gamma(y)}{2\Gamma(x+y)}$.

b) Attempt any one of the following. **[5]**

i) State and prove convolution theorem for Fourier transforms.

ii) Prove that, the general solution of

$$x^2 \frac{d^2 y}{dx^2} + (1-2\alpha) x \frac{dy}{dx} + \left\{\beta^2\gamma^2 x^{2\gamma} + \left(\alpha^2 - n^2\gamma^2\right)\right\} y = 0$$

is $Ax^\alpha J_n\left(\beta x^\gamma\right) + Bx^\alpha Y_n\left(\beta x^\gamma\right)$.

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Total No. of Questions : 4]

SEAT No. :

PC4080

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[6334]-3007

S.Y. M.A./M.Sc.

MATHEMATICS

MTS - 611(D) - MJ : Mechanics

(2023 Credit Pattern) (Semester- III)

Time : 2 Hours] ***[Max. Marks : 35***

Instructions to the candidates:

1) ***All questions are compulsory.***

2) ***Figures to the right indicate full marks.***

Q1) Attempt any three of the following. **[6]**

a) Find the Euler-Lagrange's equational of the function

$$I = \int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right) dx.$$

b) Explain principle of virtual work.

c) Find the potential energy of a particle of mass *m* moves in a plane under the action of force *F*with component $F_x = -k^2(2x + y)$, $F_y = -k^2(x + 2y)$ *k* is a constant.

d) State Hamilton's principle for conservative system.

Q2) Attempt any two of the following. **[10]**

a) A particle is constrained to move on the surface of a cylinder of fixed radius. obtain the Lagrange's equation of motion.

b) Set up the Lagrangian and the Lagrange's equation of motion for simple pendulum.

c) Show that the Euler-Lagrange equation of the functional $I(y(x)) = \int_{x_1}^{x_2} f\,(x, y, y' =) dx$ has first integral $f - y' \dfrac{\partial f}{\partial y'} =$ constant, if the integrand does not depend on *x*.

P.T.O.

Q3) Attempt any two of the following. **[10]**

a) Find the plane curve of fixed perimeter that encloses maximum area.

b) Find the shortest distance between $y^2 = 4x$ and $(x - 9)^2 + y^2 = 4$.

c) Explain the principle of Least action.

Q4) a) Attempt any one of the following. **[4]**

i) Show that the transformation $p = \frac{1}{Q}, q = pQ^2$ is canonical.

ii) Prove Hamilton's canonical equations of motion from Hamiltonian function.

b) Attempt any one of the following. **[5]**

i) Show that geodesic (shortest distance between two points) in a plane is straight line.

ii) Explain Atwood Machine and discuss its motion.