Total No. of Questions : 8]

SEAT No. :

P2056

[Total No. of Pages : 3

[4639] - 201

M. Tech.

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM- 201: Complex Analysis

(2013 Pattern) (Semester -II)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $f(z) = u(x, y) + iv(x, y)$ is differentiable at $z_0 = x_0 + iy_0$ then prove that u and v must satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$. **[5]**

b) Give an example to show that $\log(z_1 z_2) \neq \log z_1 + \log z_2$ where $\log z$ denotes the principal branch of logarithm. **[3]**

c) Sketch the following set and determine whether it is a domain in the complex plane. **[2]**

$$S = \{ z \in \mathbb{C} \mid |z+3| > 4 \}.$$

Q2) a) Let z_0 and w_0 be points in the z and w planes, respectively, prove that **[4]**

$$\lim_{z\to\infty} f(z) = w_0 \text{ if and only if } \lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0.$$

b) Find the value of the integral $\int_C \bar{z}\, dz$ where C is the right-hand half **[4]**

$z = 2e^{i\theta}$ $\left(\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$ of the circle $|z| = 2$, from $z = -2i$ to $z = 2i$.

c) Show that the function $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in any domain. **[2]**

P.T.O.

Q3) a) Find the analytic function $f(z)$ whose real part is $u(x, y) = \frac{y}{x^2 + y^2}$ in some domain . **[4]**

b) Show that $|\cos z|^2 = \cos^2 x + \sin h^2 y$. **[4]**

c) State Cauchy-Goursat theorem. **[2]**

Q4) a) State and prove Cauchy's residue theorem. **[5]**

b) Prove that if f is entire and bounded in the complex plane, then $f(z)$ is constant throughout the plane. **[5]**

Q5) a) State Taylor's theorem. Also derive the Taylor series representation. **[5]**

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} \frac{(z-i)^n}{(1-i)^{n+1}} \qquad \left(|z-i| < \sqrt{2}\right).$$

b) Let f be analytic everywhere inside and on a simple closed contour C, taken in the positive sense. If z_0 is any point interior to C, then prove that

$$f(z_o) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0}\, dz$$ **[5]**

Q6) a) Use residues to evaluate the improper integral $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$. **[5]**

b) Show that if $f(z) = \left(\frac{z}{\overline{z}}\right)^2$ then the limit $\lim_{z \to 0} f(z)$ does not exist. **[3]**

c) Evaluate $\int_c f(z)\, dz$ where the contour C is the positively oriented circle $|z| = 1$, and $f(z) = z^2 + 3$. **[2]**

Q7) a) Prove that if a function f that is analytic at a point z_0 has a zero of order m there, then there is a function g_i which is analytic and non zero at z_0, such that $f(z) = (z - z_0)^m\, g(z)$. **[5]**

b) Let $f(z)=\dfrac{z^2-i}{(z^2+4)(z-3)}$, determine the singular points of the function $f(z)$ and state why the function is analytic everywhere except at those points. **[3]**

c) Find the series expansion of the function $f(z)=\dfrac{e^z}{z^2}$ in powers of z. **[2]**

Q8) a) Define the terms: **[3]**

i) simple pole

ii) singular point

iii) isolated singular point

b) Let C denote the polygonal line from 0 to i and then from i to $1+i$. Evaluate $\int_C f(z)\,dz$ where $f(z)=y-x-3x^2i$. **[5]**

c) Write the Laurent series in powers of z that represent the function $f(z)=\dfrac{z+1}{z-1}$, when $1<|z|<\infty$. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2057

[Total No. of Pages : 2

[4639] - 202

M.Tech.(Mathematics)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM- 202: Algebra - I

(2013 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that the order of every element in a finite group is finite. **[4]**

b) Let (G, $*$) be an finite abelian group. $f: G \to G$ defined by $f(x) = x^{-1}\ \forall\ x \in G$. Show that *f* is onto isomorphism. **[4]**

c) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 3 & 7 & 6 \end{pmatrix}$ find order of σ. **[2]**

Q2) a) Prove that every finite integral domain is a field. **[4]**

b) Let F be the ring of all functions mapping $\mathbb{R}$ into $\mathbb{R}$. Let H be the subring of F consisting of all the constant functions in F. Is H an ideal in F? Why? **[4]**

c) Define: Cyclic subgroup. **[2]**

Q3) a) Prove that a subgroup H of a group G is normal if and only if $xHx^{-1} = H\ \ \forall\ x \in G$. **[5]**

b) Prove that no group of order 36 is simple. **[5]**

Q4) a) Prove that every group is isomorphic to a group of permutations. **[5]**

P.T.O.

b) If $f : G \to G'$ is a group homomorphism with $N = \text{Ker} f$. Prove that N is normal subgroup of G. **[5]**

Q5) a) Show that $(\mathbb{Q}^+, *)$ is a group where $a*b = \dfrac{ab}{2}$ $a, b \in \mathbb{Q}^+$. **[5]**

b) Find all solutions of $x^2 + 2x + 4 = 0$ in $\mathbb{Z}_6$. **[3]**

c) Is union of two subgroups of a group a subgroup? Justify. **[2]**

Q6) a) Prove that if F is a field every ideal in F(x) is principal. **[4]**

b) Find all subgroups of a cyclic group of order 10. **[4]**

c) Define: Even permutation. **[2]**

Q7) a) Define: Class equation. What are conjugate classes of S_3? Verify class equation for S_3. **[4]**

b) Prove that in any group (G, $*$) left and right cancellation laws hold. **[4]**

c) Define: Quotient group. **[2]**

Q8) a) Show that the groups $G = \{1, -1, i, -i\}$ and $(\mathbb{Z}_4, +4)$ are isomorphic. **[4]**

b) Define: Polynomial ring F[x] over a field F. Let $f(x) \in F[x]$. Let $f(x)$ be of degree 2 or 3. Prove that $f(x)$ is reducible over F if and only if it has a zero in F. **[4]**

c) Define: Homomorphism of rings. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2058

[Total No. of Pages : 4

[4639] - 203
M.Tech. (Mathematics)
INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS
MIM- 203: Numerical Analysis
(2013 Pattern) (Semester -II)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions out of Eight.*
2) *Figures to the right indicate full marks.*

Q1) Attempt each of the following.

a) Assume that 'g' is a continuous function and that $\{p_n\}_{n=0}^{\infty}$ is a sequence generated by fixed point iterations. If $\lim_{n\to\infty} p_n = p$, then prove that '$p$' is a fixed point of $g(x)$. **[4]**

b) Solve the following system of linear equations by Gaussian Elimination method **[4]**

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

c) Define the following: **[2]**

i) Truncation Error

ii) Round-off error.

Q2) Attempt each of the following:

a) Derive the recursive formula $P_k = \dfrac{(N-1)P_{k-1} + \dfrac{A}{P_{k-1}^{N-1}}}{N}$ for $k = 1, 2, \ldots$ for finding N^{th} root of A, where $f(x) = x^N - A$ and N is positive integer. **[4]**

P.T.O.

b) The table below gives the values of tan x for $0.10 \leq x \leq 0.30$ **[4]**

x	0.10	0.15	0.20	0.25	0.30
$y = \tan x$	0.1003	0.1511	0.2027	0.2553	0.3093

Find the value of tan (0.12)

c) Explain the term ill conditioning with the suitable example. **[2]**

Q3) Attempt each of the following:

a) Assume that $f \in C[a, b]$ and that there exists a number $r \in [a, b]$ such that $f(r) = 0$. If $f(a)$ and $f(b)$ have opposite signs and $\{c_n\}_{n=0}^{\infty}$ represents the sequence of midpoints generated by bisection process, then prove that $|r - c_n| \leq \frac{b-a}{2^{n+1}}$, for n = 0, 1, 2,... and therefore the sequence $\{c_n\}_{n=0}^{\infty}$ converges to the zero $x = r$, that is $\lim_{n\to\infty} c_n = r$. **[4]**

b) Find the parabola $y = A + Bx + Cx^2$ that passes through the three points (1, 1), (2, -1) and (3, 1) **[4]**

c) Find the absolute error and the relative error, if the true value $x = 0.000012$ and calculated value $\hat{x} = 0.000009$. **[2]**

Q4) Attempt each of the following:

a) Assume that $f \in C^3[a, b]$ and that $x-h, x, x+h \in [a, b]$ then show that $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$. Furthermore, there exists a number $c = c(x)$ in $[a, b]$ such that $f'(x) = \frac{f(x+h) - f(x-h)}{2h} + E_{trunc}(f, h)$ where $E_{trunc}(f, h) = \frac{-h^2 f^{(3)}(c)}{6} = O(h^2)$. The term $E(f, h)$ is called the truncation error. **[4]**

b) Solve the following system of linear equations by Gauss-Elimination Method. **[4]**

$3x+2y+4z= 7$
$2x+y+ z =7$
$x+3y +5z=2$

c) Find the interval $[a, b]$ where the real root of $f(x) = x \sin(x)$ lie. **[2]**

Q5) Attempt each of the following:

a) Show that the Lagrangian polynomial passing through the points (x_0, y_0) and (x_1, y_1) is given by $y = L_1(x) = y_0 \dfrac{(x - x_1)}{(x_0 - x_1)} + y_1 \dfrac{(x - x_0)}{(x_1 - x_0)}$. **[4]**

b) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ using the false position method. **[4]**

c) Prove the relation $\Delta - \nabla = \delta^2$ where

Δ = Forward difference operator.

∇ = Backward difference operator.

δ = Central difference operator. **[2]**

Q6) Attempt each of the following:

a) Show that the iterative formula to evaluate roots of $f(x) = 0$ is a sequence of intervals $\left[x_{n-1}, x_n\right]$ given by: **[4]**

$$c_n = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

b) Find, from the following table, the area bounded by the curve and the x-axis from $x = 7.47$ to $x = 7.52$ using trapezoidal rule. **[4]**

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

c) Consider the graph $y = f(x) = \cos x$ over the interval [0.0, 1.2]. Use nodes $x_0 = 0.0$ and $x_1 = 1.2$ to construct a linear interpolating polynomial $p_1(x)$. **[2]**

Q7) Attempt each of the following:

a) Factorize the following matrix $A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$ into the form LU where

L = Unit lower triangular matrix

U = Upper triangular matrix **[5]**

b) State and prove composite Simpson's $\frac{1}{3}^{rd}$ rule of numerical integration. **[5]**

Q8) Attempt each of the following:

a) Use Runge-Kutta second order formula for solving the differential equation $\frac{dy}{dx} = y - x$ where $y(0) = 2$. Find $y(0.1)$ considering $h = 0.1$ **[5]**

b) Solve the differential equation $y' = -y$ with the initial condition $y(0) = 1$. Take $h = 0.01$ and find $y(0.01)$, $y(0.02)$ and $y(0.03)$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2060

[Total No. of Pages : 2

[4639] - 205
M.Tech.
COMPUTER SCIENCE
INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS
MIM - 205 : Data Structures Using C
(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions out of eight.***
2) ***All questions carry equal marks.***
3) ***Figures to the right indicate full marks.***

Q1) Answer following questions:

a) Define queue. Explain its static implementation. **[4]**

b) Differentiate between stack and array. **[4]**

c) Show the contents of recursive stack for evaluating fact (3). **[2]**

Q2) Answer following questions:

a) Write a 'C' program to implement circular queue using array. **[5]**

b) Write a menu driven program to insert, delete and display elements from a doubly linked list. **[5]**

Q3) Answer following questions:

a) Write a short note on : Generalized linked list. **[4]**

b) Write a 'C' function for bubble sort. **[4]**

c) Define the terms **[2]**

i) Complete binary Tree

ii) Leaf node.

Q4) Answer following questions:

a) Write a short note on :Implementation of linked list using array. **[4]**

b) Convert the following postfix expression to infix. **[4]**

i) ABCDE –+$EF*-/

c) Give the best case and worst case complexity of insertion sort. **[2]**

P.T.O.

Q5) Answer following questions:

a) Explain Binary tree representations using array. **[4]**

b) Write an algorithm to multiply two polynomials. **[4]**

c) What is circular queue? **[2]**

Q6) Answer following questions:

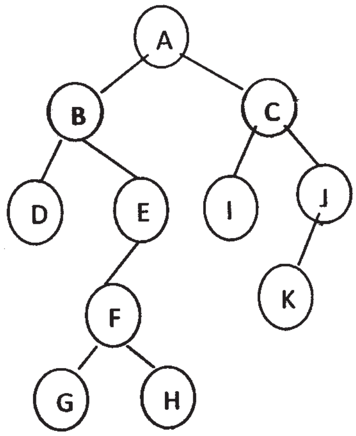
a) Write a function to reverse a singly linked list. **[4]**

b) Write a short note on applications of stack. **[4]**

c) List the types of linked list. **[2]**

Q7) Answer following questions:

a) Define Tree: Give preorder, postorder, inorder traversals for the following. **[4]**



b) Differentiate between static and dynamic implementation of a binary search tree. **[4]**

c) What is a difference between complete and strictly binary tree. **[2]**

Q8) Answer following questions:

a) Write a recursive function to search for an element in a binary search tree. **[4]**

b) Explain BFS method of traversal. **[4]**

c) Give maximum number of nodes in a complete binary tree of height h. **[2]**

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Total No. of Questions :5]

SEAT No. :

P2033

[Total No. of Pages :3

[4639]-21

M.Tech. -I (Mathematics)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 201: Real and Complex Analysis

(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any EIGHT of the following: **[16]**

a) Define: A measurable set.

b) If A = {1, 2, 3, ..., 100} then show that the outer measure of A is zero.

c) Prove that if $m * A = 0$, then $m*(A \cup B) = m*B$.

d) Show that if E is measurable set, then each translate $E + y$ of E is also measurable.

e) If A and B are disjoint measurable sets contained in $E \subseteq \mathbb{R}$ then

$$\int_{A\cup B} f = \int_A f + \int_B f,$$ where f is integrable over E.

f) Show that $\{z^n\}$ is a null sequence for $|z| < 1$.

g) Define: Entire function.

h) Show that the function $u(x, y) = 4xy - x^3 + 3xy^2$ is harmonic in $\mathbb{C}$.

i) State Liouville's theorem.

j) Discuss the singularities of the function $f(z) = \frac{1}{\cos(1/z)}$

P.T.O.

Q2) a) Attempt any ONE of the following: **[6]**

i) Let <E_n> be an infinite decreasing sequence of measurable sets, that is a sequence with $E_{n+1} \subset E_n$ for each *n*. Let m E_1 be finite ; then show that $m\left[\bigcap_{i=1}^{\infty} E_i\right] = \lim_{n\to\infty} m\, E_n$

ii) Prove that the interval (a,∞) is measurable subset of $\mathbb{R}$.

b) Attempt any TWO of the following: **[10]**

i) If *f* is a measurable function and if *f* = *g* almost everywhere then show that g is measurable.

ii) Let ϕ and ψ be simple functions which vanish outside a set of finite measure, then $\int(a\phi+b\psi) = a\int\phi+b\int\psi$, and if $\phi \geq \psi$ almost everywhere, then $\int\phi \geq \int\psi$.

iii) Show that, if m^* E = 0, then E is measurable.

Q3) a) Attempt any ONE of the following: **[6]**

i) State and prove Lebesgue convergence theorem.

ii) Let *f* be a non negative function which is integrable over a set E. Then prove that given $\in > 0$ there is a $\delta > 0$ such that for every set $A \subset E$ with m A < δ, $\int_A f < \in$.

b) Attempt any TWO of the following: **[10]**

i) Let *f* be a bounded function defined on [*a*, *b*]. If *f* is Riemann integrable on [*a*, *b*], then it is measurable and

$$R\int_a^b f(x)\,dx = \int_a^b f(x)\,dx$$

ii) Let *f* be a non - negative measurable function. If *f* = 0 a.e. then show that if $\int_E f = 0$ then *f* = 0 a.e in E, where E is measurable set.

iii) Show that the sum and product of two simple functions are simple.

Q4) a) Attempt any ONE of the following: **[6]**

i) If $\lim_{z \to z_0} f(z) = w_0$ and g is a function which is continuous at w_0 then show that $\lim_{z \to z_0} [gof(z)] = g(w_0)$

ii) State and prove Morera's theorem.

b) Attempt any TWO of the following: **[10]**

i) Show that every Möbius transformation is a combination of translation, rotation, magnification and inversion transformations.

ii) Check differentiability of the function

$$f(x+iy) = u + iv = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} + i\left[\dfrac{x^3 + y^3}{x^2 + y^2}\right], & \text{if } \begin{array}{l} x \neq 0, \\ y \neq 0 \end{array} \\ 0 & , \text{ if } \begin{array}{l} x = 0, \\ y = 0 \end{array} \end{cases}$$

iii) Evaluate the integral $\mathrm{I} = \int_{\gamma} \frac{dz}{1+z}$, where γ is any curve in $D = \{z \mid \operatorname{Im} z > 0\}$, which joins $-1 + i$ to $1 + 2i$.

Q5) a) Attempt any ONE of the following: **[6]**

i) If f has a pole of order n at z_0, then

$$\text{Res } [f(z); z_0] = \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} [(z - z_0)^n f(z)] \Big|_{z = z_0}.$$

ii) For any closed curve γ and $a \notin \gamma$, the index or a winding number $\eta(\gamma; a)$ is an integer.

b) Attempt any TWO of the following: **[10]**

i) Evaluate $\int_{|z - \frac{\pi}{2}| = 1} \tan z \; dz$.

ii) Let $f(z) = \log\left[\dfrac{z^n}{z^{n-1}}\right]$, $|z| > 1$, & 'n' is positive integer. Find the Laurent series expansion for f.

iii) Prove that the four distinct points z, z_1, z_2, z_3 all lie on a circle or on a line if and only if, their cross ratio (z, z_1, z_2, z_3) is a real number.

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Total No. of Questions :5]

SEAT No. :

P2034

[Total No. of Pages :3

[4639]-22

M.Tech. (Mathematics)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 202: Algebra - II

(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Eight of the following: **[16]**

a) Show that the set $S = \{1, 1 + x, 1 + x^2\}$ is a basis for $P_2(x)$, the vector space of polynomial of degree ≤ 2.

b) Let W_1 and W_2 be subspaces of a vector space V and $W_1 \cup W_2$ is a subspace of V. Show that either $W_1 \subset W_2$ or $W_2 \subset W_1$.

c) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be given as $T(x, y) = (x + y + 1, x - y)$. Is T a linear transformation? Justify.

d) Find the eigen values of the matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

e) Let V be a finite dimensional vector space and $T: V \to V$ be a linear transformation. Show that T is one - one linear transformation if and only if T is onto.

f) Construct a field of 4 elements.

g) Show that $\mathbb{Q}\left(\sqrt{2}, i\right) = \mathbb{Q}\left(\sqrt{2} + i\right)$.

h) Give an example of an algebraic extension of $\mathbb{Q}$ which is not a finite extension.

i) Show that if $[E : F] = 2$, then E / F is a normal extension.

j) Let E be a finite extension of a field F. If $[E : F] = 7$ and $\alpha \in E$, $\alpha \notin F$, then show that $E = F(\alpha)$.

P.T.O.

Q2) a) Attempt any ONE of the following: **[6]**

i) Let V be a vector space over a field F spanned by $V_1, V_2, ---, V_m$. Prove that any linearly independent subset of V is finite and contains almost m elements.

ii) Let V be an inner product space. For any two vectors $\alpha, \beta \in V$, prove that $\|\alpha+\beta\|^2 + \|\alpha-\beta\|^2 = 2\|\alpha\|^2 + 2\|\beta\|^2$.

b) Attempt any TWO of the following: **[10]**

i) Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}.$$

Determine whether T is diagonalizable.

ii) Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ be a Linear operator defined by $T(x, y) = (x-y, x+y)$. Is T invertible ? If so, find T^{-1}.

iii) Show that $W=\{(x, x, x) \mid x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

Q3) a) Attempt any ONE of the following: **[6]**

i) Let $T:V \to W$ be a linear transformation and V be a finite dimensional vector space. Show that rank (T) + nullity (T) = dim (V).

ii) Let $S = \{\alpha_1, \alpha_2, ---, \alpha_n\}$ be a set of non - zero vectors in an inner product space V. If every pair of distinct vectors in S are orthogonal, then prove that S is linearly independent.

b) Attempt any TWO of the following: **[10]**

i) If W is a subspace of a vector space V, then show that

dim (W) + dim(W^o) = dim(V).

ii) Use Gram-Schmidt process to transform the basis B = {(1, 0, 1), (1, 0, –1), (0, 3, 4)} into an orthonormal basis of $\mathbb{R}^3$ with Euclidean inner product on $\mathbb{R}^3$.

iii) Let $B = \{1, x-1, x^2-2x+1\}$ be a basis for the vector space P_2. Determine the coordinate vector of $v = 2x^2 - 5x + 5$ relative to basis B.

Q4) a) Attempt any ONE of the following: **[6]**

i) If E is a finite extension of a field F of degree m and K is a finite extension of E of degree n, then prove that $[K : F] = m \cdot n$

ii) Let F be a field and $f(x) \in F[x]$ be a non-constant polynomial. Prove that there exists an extension E of F which contains a root of $f(x)$.

b) Attempt any TWO of the following: **[10]**

i) Let F be a finite field of characteristic p. Define $\phi : F \to F$ by $\phi(a) = a^p, \ \forall a \in F$. Show that ϕ is an automorphism of F.

ii) Prove that if $\alpha \in E$ is algebraic over F of an odd degree, then show that $F(\alpha) = F(\alpha^2)$.

iii) Find the degree of the extension field $Q(\sqrt{3}, \sqrt{5})$ over the field of rationals and over $Q(\sqrt{3})$.

Q5) a) Attempt any ONE of the following: **[6]**

i) If E is a finite field of characteristic p, then prove that E contains exactly p^n elements for some positive integer n.

ii) Prove that any algebraic extension of a field F of characteristic o is a separable extension.

b) Attempt any TWO of the following: **[10]**

i) Find the splitting field of $x^4 - 2$ over Q.

ii) Show that an algebraically closed field has no proper algebraic extension.

iii) Show that $f(x) = x^3 - 3x^2 + 3x - 3$ is an irreducible polynomial over Q.

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Total No. of Questions :5]

SEAT No. :

P2035

[Total No. of Pages :4

[4639]-23

M.Tech. (Mathematics)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 203: Discrete Mathematical Structures - II

(2008 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks :80***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any eight of the following: **[16]**

a) Draw all simple non isomorphic unlabelled graphs on 3 vertices.

b) Draw a simple four regular graph on six vertices.

c) Define complete symmetric diagraph.

d) Define bipartite graph.

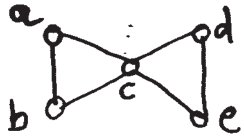
e) Does every disconnected graph G have an isolated vertex? Justify.

f) Find the edge connectivity of K_4.

g) When is a diagraph G said to be an arborescence?

h) Define chromatic number of a graph G.

i) Is the following graph Eulerian? Justify.



j) Give an example of a self complementary graph on five vertices.

P.T.O.

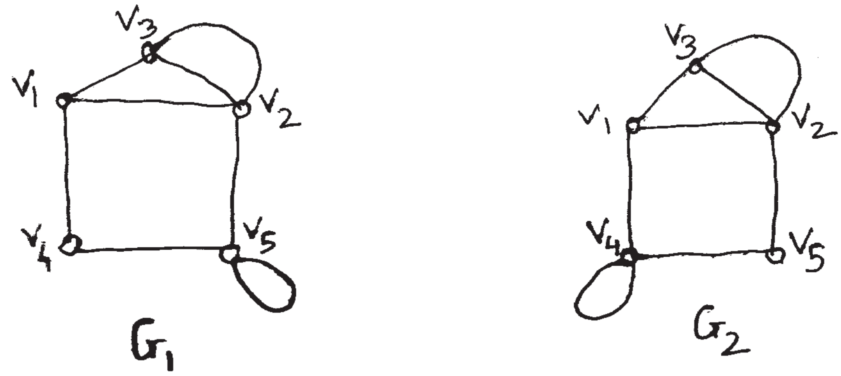
Q2) a) Attempt any one of the following: **[6]**

i) Prove that the number of vertices of odd degree in a graph is always even.

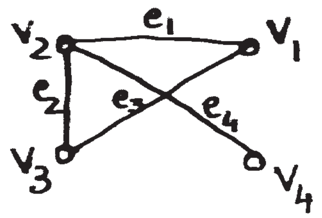
ii) Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

b) Attempt any two of the following: **[10]**

i) Examine whether the following pairs of graphs are isomorphic or not:



ii) Find the adjacency matrix of the following graph.



iii) Write a short note on the Chinese Postman Problem.

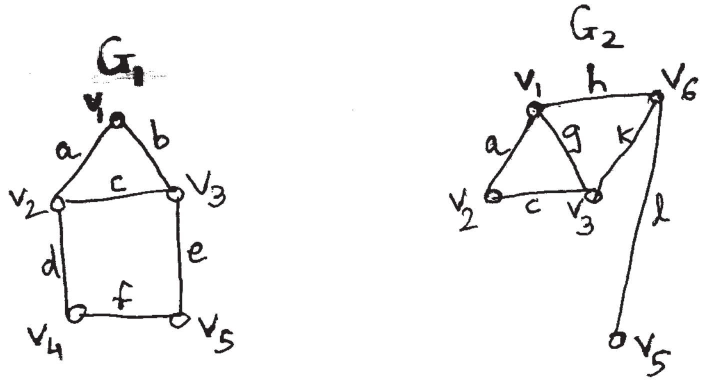
Q3) a) Attempt any one of the following: **[6]**

i) Prove that a graph G with *n* vertices, *n*–1 edges and no circuits is connected.

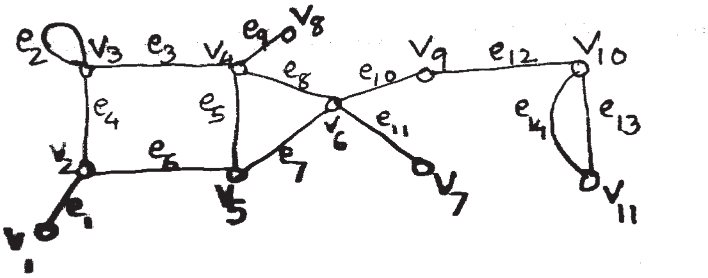
ii) Prove that a tree with *n* vertices has n–1 edges.

b) Attempt any two of the following: **[10]**

i) Find $G_1 \cap G_2$ where



ii) Find all the bridges in the following graph G.



iii) Explain depth first search algorithm for a graph.

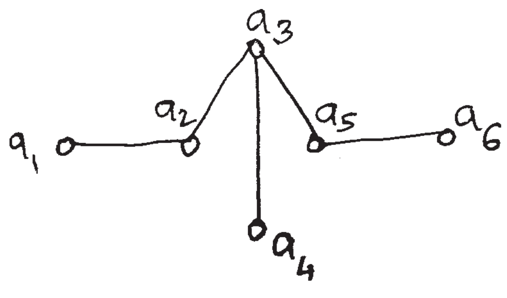
Q4) a) Attempt any one of the following: **[6]**

i) Prove that every tree has either one or two centers.

ii) Prove that if a graph has exactly two vertices of odd degree, there must be a path joining these two vertices.

b) Attempt any two of the following: **[10]**

i) Find eccentricity of each vertex in the following tree. Hence find centre of the tree.



ii) Let T be a binary tree with n vertices. Show that T has $\frac{n+1}{2}$ pendant vertices.

iii) Explain sequential colouring algorithm for colouring a graph G.

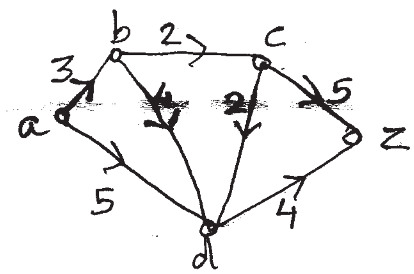
Q5) a) Attempt any one of the following: **[6]**

i) Prove that the complete graph on five vertices is non planar.

ii) Prove that a connected graph G is an Euler graph if and only if it can be decomposed into circuits.

b) Attempt any two of the following: **[10]**

i) Determine the maximal flow from *a* to *z* in the network given below. The numbers assigned to the edges represent their capacities.



ii) Prove that in any simple connected planar graph with f regions, *n* vertices and *e* edges ($e > z$) the following inequalities hold: $e \geq \frac{3}{2} f$ and $e \leq 3n - 6$.

iii) Find the smallest integer *n* such that K_n has atleast 600 edges.

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Total No. of Questions :5]

SEAT No. :

P2037

[Total No. of Pages :3

[4639]-25

M.Tech.

Computer Science

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM-205: Data Structures using C

(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any eight of the following: **[16]**

a) Differentiate between stack and array.

b) What is queue? Explain primitive operations on queue.

c) Write a short note on insertion sort.

d) Differentiate between static and dynamic memory allocation.

e) Define data structure stack with its applications.

f) Define strictly binary tree.

g) List any two graph traversal techniques and the data structure used in them.

h) Write a short note on quick sort.

i) What is graph? Explain the term isolated vertex.

Q2) Attempt any four of the following: **[16]**

a) What are the different types of linked list? Explain in brief.

b) Write a 'C' program to implement circular queue. Using array.

c) Write following'C' functions for static queue representation.

i) DeleteQ( )

ii) Isempty Queue( )

P.T.O.

d) Sort the following numbers using bubble sort

3, 97, 65, 71, 23, 57, 93, 100.

e) Write a short note on doubly linked list.

Q3) Attempt any four of the following: **[16]**

a) Describe array and linked list representation of binary trees.

b) Write a function in 'C' to insert a node in a BST.

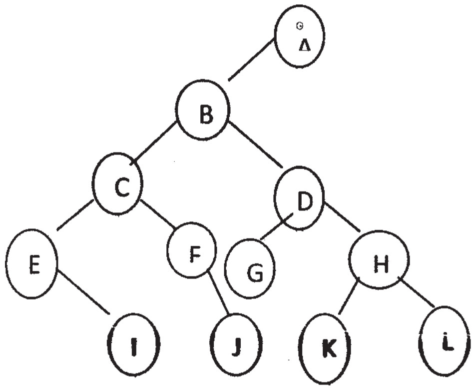
c) Explain need of dynamic implementation of stack.

d) Write the algorithm to convert prefix expression to infix expression.

e) Write a short note on array as ADT.

Q4) Attempt any four of the following: **[16]**

a) Find preorder, postorder and inorder tree traversal for following binary tree,



b) Convert following infix expression to postfix expression.

(A/((B $ (C+(D-E))))-E*F)))

c) Explain the terms:

i) Degree

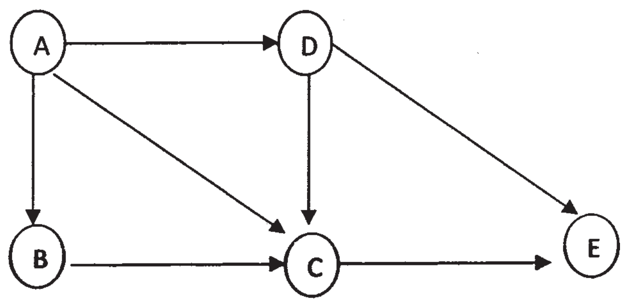
ii) Forest

iii) Siblings

iv) Depth of tree

d) Write a program to implement "push" and "pop" operation of stack using linked list.

e) Construct adjacency list for the following graph.



Q5) Attempt any four of the following: **[16]**

a) Write time complexity of merge sort for best case, average case and worst case.

b) Write an algorithm to implement insertion sort.

c) Write a short note on priority queue.

d) Write a 'C' program to insert and delete and display an element in doubly linked list.

e) What are athe differences between singly linked list and doubly linked list.

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Total No. of Questions : 5]

SEAT No. :

P2043

[Total No. of Pages : 3

[4639]-41

M.Tech. (Mathematics)

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 401 : Topology

(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any EIGHT of the following: **[16]**

a) Define a basis for a topology.

b) Show that (0, 1) and [0, 1] with standard topology are not homeomorphic.

c) Show that $\mathbb{Z}$, the set of all integers is $\mathbb{R}$ is not a connected set.

d) Is the collection $\tau_{\infty} = \{U \mid X\text{-}U \text{ is infinite or all of } X\}$ a topology on X? Justify.

e) Define First countable space.

f) If X = $\mathbb{R}$, Y = [0, 1] $\cup \{2\}$, then show that $\{2\}$ is an open subset of Y.

g) Define Lindelöf space.

h) State Urysohn's Metrization theorem.

i) Give an example of a non Hausdorff space.

j) Show that $\mathbb{R}$ is Locally compact.

Q2) a) Attempt any ONE of the following: **[6]**

i) If $\{\tau_\alpha\}$ is a family of topologies on a set X, then prove that $\bigcap_{\alpha} \tau_\alpha$ is a topology on X.

ii) Let X be a topological space and Y be a subspace of X. If $\mathcal{B}$ is a basis for the topology on X, then prove that $\mathcal{B}_Y = \{B \cap Y \mid B \in \mathcal{B}\}$ is a basis for the subspace topology on X.

b) Attempt any Two of the following: **[10]**

i) If X is a Hausdorff space, then a sequence of points of X converges to atmost one point of X.

ii) Let X and Y be topological spaces. Let A be closed in X and B be closed in Y. Prove that $A \times B$ is closed in $X \times Y$.

iii) Prove that (0, 1) is homomorphic to $\mathbb{R}$.

Q3) a) Attempt any ONE of the following: **[6]**

i) Show that a subspace of a regular space is regular.

ii) Let X and Y be topological spaces, and let $f: X \rightarrow Y$ be a given map. Prove that following are equivalent.

1) f is continuous

2) For every subset A of X, $f\left(\overline{A}\right) \subset \overline{f(A)}$

3) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

b) Attempt any two of the following: **[10]**

i) Show that a compact subset of a Hausdorff space is closed.

ii) Prove that every map on a discrete space is continuous.

iii) Give an example of a topological space which is first countable but not second.

Q4) a) Attempt any ONE of the following: **[6]**

i) Prove that product of two connected spaces is connected.

ii) Prove that a closed subspace of normal space is normal.

b) Attempt any TWO of the following: **[10]**

i) Prove by an example that the product of Lindelöf spaces need not be Lindelöf.

ii) Show that every second countable space is separable.

iii) Show that every completely regular space is regular.

Q5) a) Attempt any ONE of the following: **[6]**

i) Prove that every metrizable space is normal.

ii) Prove that a continuous image of a path connected space is path-connected.

b) Attempt any TWO of the following: **[10]**

i) Show that $\mathbb{R}$ with lower limit topology is not connected.

ii) Prove that a finite union of compact set is compact.

iii) Give an example of a regular space which is not normal.

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Total No. of Questions : 5]

SEAT No. :

P2044

[Total No. of Pages : 3

[4639]-42
M.Tech.
INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS
MIM - 402 : Computer Networks
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***All the questions are compulsory.***
2) ***Neat diagrams must be drawn wherever necessary.***
3) ***Figures to the right indicate full marks.***
4) ***Assume suitable data if necessary.***

Q1) Attempt any eight of the following: **[8 × 2 = 16]**

a) What is the difference between N/W layer delivery and transport layer delivery?

b) What are peer-to-peer processes?

c) List out five components of data communication system.

d) What is the difference between half-duplex & full-duplex transmission mode? Give example of each.

e) Name three types of transmission impairment.

f) Distinguish between data rate and signal rate.

g) Find the error, if any, in the following IPV4 addresses.

i) 75.45.301.14

ii) 221.34.7.8.20

h) Define multiplexing.

i) What is the significance of the twisting in twisted-pair cable?

j) What is the hamming distance?

Q2) Attempt any <u>four</u> of the following: **[4 × 4 = 16]**

a) Briefly describe the services provided by Data Link Layer.

b) Differentiate between FDMA and TDMA.

c) Define virtual LAN and explain in brief with suitable diagram.

d) Explain in detail following connecting devices along with suitable diagram.

i) Repeaters.

ii) Bridges.

e) Describe "flooding" which is used by multicast distance routing algorithm in detail.

Q3) Attempt any <u>four</u> of the following: **[4 × 4 = 16]**

a) Write a note on classful addressing.

b) What are the different characteristics of line coding?

c) What is a congestion control? List out two broad categories of it. Explain in brief any prevention policy for congestion control.

d) Differentiate between static and dynamic routing table.

e) Explain with suitable example : - physical, logical and port addresses.

Q4) Attempt any <u>four</u> of the following: **[4 × 4 = 16]**

a) Define Piggybacking & its usefulness.

b) Write a note on CSMA.

c) Compare and contrast the Go-Back-N ARQ and Selective-Repeat ARQ.

d) Write a note on PCM. Draw a diagram.

e) Describe Flow Control and Error Control.

Q5) Attempt any <u>four</u> of the following: **[4 × 4 = 16]**

a) Differentiate between ISO-OSI and TCP/IP reference model.

b) Draw UDP datagram format diagram and explain its fields.

c) Write a note on Bluetooth. Describe its architecture with suitable diagram.

d) Describe Nyquist theorem for noiseless channel. Using Nyquist formula solve the following:

Consider a noiseless channel with a bandwidth of 3000Hz transmitting a signal with two signal levels. Calculate the maximum bit rate.

e) Define the following terms:-

i) Checksum.

ii) Collision.

iii) Frequency.

iv) Hub.

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Total No. of Questions : 5]

SEAT No. :

P2045

[Total No. of Pages : 2

[4639]-43

M.Tech.

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 403 : Web Technologies

(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***

Q1) Attempt any Eight of the following: **[16]**

a) What is the task of a DNS name server?

b) What is the purpose of Accept field in an HTTP request?

c) Give any four primitive types in javascript.

d) What is MIME?

e) What is the form of an HTML comment?

f) Give any four predefined character classes in javascript.

g) What are three categories of perl variable.

h) What potential advantage do servlet have over CGI programming?

i) What is the purpose of DTD?

j) What is PHP? What are two modes of PHP processor?

Q2) Attempt any Four of the following: **[16]**

a) Explain XML Name space.

b) Explain concept of constructor in javascript with example.

c) Write PHP script to print average & median of an array of numbers.

d) Write note on CGI. pm module. What is the purpose of shortcuts in CGI.pm?

e) Give two ways in which hashes differ from arrays. What statement adds the element (joe, 42) to the hash % guys?

Q3) Attempt any 4 (four) of the following: **[16]**

a) Write a perl script which reads a file specified on command line & print its contents in upper case.

b) Explain any one control statement with example in PHP.

c) Write note on XML-Document structure.

d) What are cookies? Where are they stored. How is a cookie added to a response by a servlet?

e) Explain form handling in PHP with example.

Q4) Attempt any Four of the following: **[16]**

a) Explain rsort & ksort with example in PHP.

b) What is an IP Address? How IPV_6 is different from IPV_4.

c) Explain functions to implement stack in perl. With example.

d) What is main deficiency of HTML? What is the goal of XML? Give two primary task of a validating XML parser.

e) Write a javascript for reading three numbers using a prompt & print the largest of them.

Q5) Attempt any Four of the following: **[16]**

a) Explain use of implode & explode function in PHP with example.

b) What is SAX parser? Give advantages of DOM parser over SAX parser.

c) Explain HTTP servlet request handling.

d) Write short note on XML processor.

e) What is difference between "=="and "===". What is output of following script.

Var X=5

document - write (X = = = "5")

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Total No. of Questions : 5]

SEAT No. :

P2046

[Total No. of Pages : 3

[4639]-44

M.Tech.

INDUSTRIAL MATHEMATICS WITH COMPUTER APPLICATIONS

MIM - 404 : Design and Analysis of Algorithms

(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*

Q1) Attempt any eight of the following: **[16]**

a) Define θ notation. Show that $2 + 5n = \theta(n)$.

b) Define time & space complexity of an algorithm.

c) Define P & NP class.

d) What is a Heap? Is (40, 10, 2, 1) a heap?

e) Define forward edge and backward edge.

f) What is amortised analysis? How amortised cost is defined in potential method of amortized analysis.

g) What is the idea behind Merge sort?

h) Explain divide and conquer strategy.

i) What is negative weight cycle. How it affects shortest path calculation.

j) Explain string editing problem.

Q2) Attempt any TWO of the following: **[16]**

a) Write count sort algorithm and obtain its best case and worst case running time. Sort (5, 7, 22, 3, 1, 2) using count sort.

b) State Master's theorem. Solve following recurrence relations using Master's theorem.

i) $T(n) = 7T(n/3) + n^2$

ii) $T(n) = 2T(n/2) + \log n$

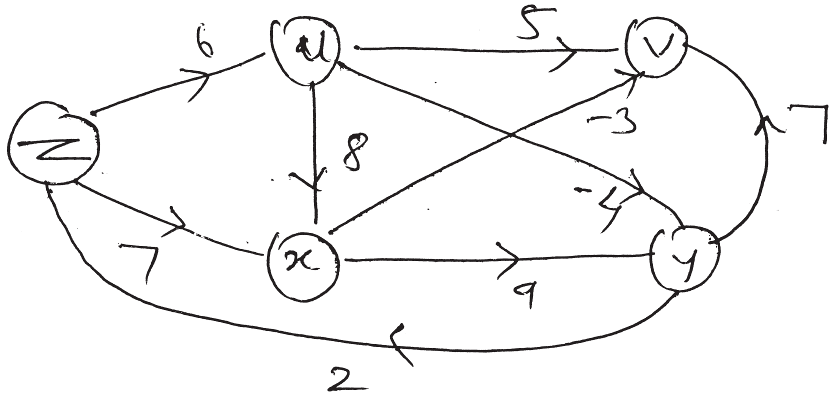
c) Explain Quick sort. What is its best case & worst case time complexity. Sort (65, 70, 75, 85, 82, 45) using quick sort.

P.T.O.

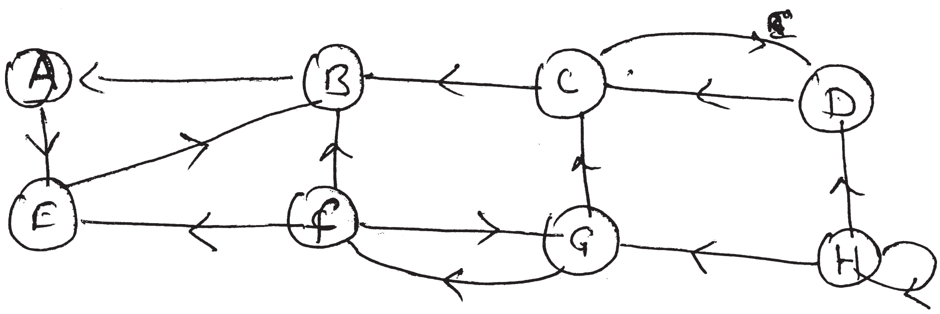
Q3) Attempt any two of the following: **[16]**

a) Explain matrix chain multiplication problem using dynamic programming. Illustrate it for chain of matrices A_1, A_2, A_3, & A_4 where $A_1 = 10 \times 5$, $A_2 = 5 \times 10$, $A_3 = 10 \times 20$, $A_4 = 20 \times 5$.

b) Explain Bellman Ford algorithm for calculating shortest path. What is it's time complexity. Apply Bellman ford algorithm for finding shortest path from source Z to all other vertices.



c) Explain algorithm based on DFS for finding strongly connected components of a directed graph G. Find strongly connected components of following graph.



Q4) Attempt any Four of the following: **[16]**

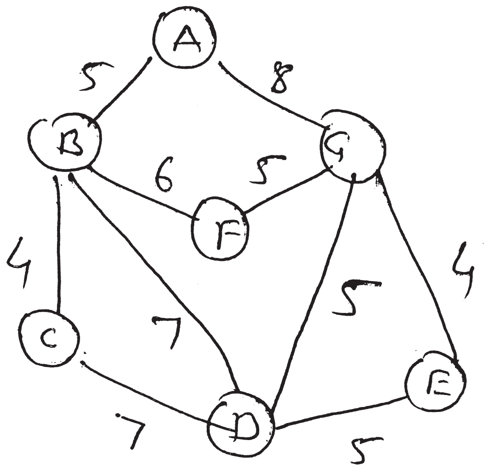
a) Illustrate LCS algorithm on the following sequence

$X = \langle A, B, C, B, D, A, B \rangle$

$Y = \langle B, D, C, A, B, A \rangle$

b) Write approximation algorithm for vertex-cover problem.

c) Using Kruskal's algorithm find spanning tree of following graph.



d) Explain Huffman coding algorithm.

e) If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$ prove that

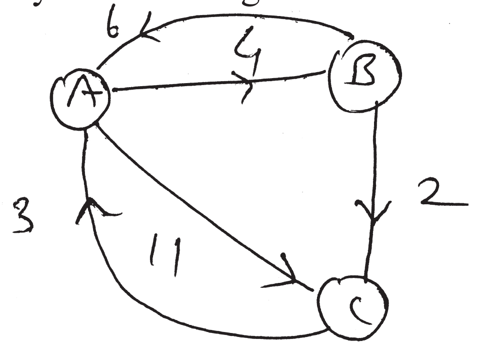
$f_1(n) + f_2(n) = O(\max(g_1(n), g_2(n)))$.

Q5) Attempt any Four of the following: **[16]**

a) Explain Prim's algorithm with example.

b) Show with an example that running time complexity of Ford-Fulkerson algorithm depends on choice of augmenting path.

c) Illustrate Floyd Warshall algorithm on following graph G.



d) Explain radix-sort algorithm. What is its time complexity.

e) Rank following functions in their increasing order of growth rates e^n, n^n, $n!$, $\log n^n$, n^2.