Total No. of Questions : 8]

SEAT No. :

PA-3108

[Total No. of Pages : 4

[5907]-11
M.A./M.Sc.
MATHEMATICS
MTUT-111 : Linear Algebra
(2019 Pattern) (Semester - I) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $T \in L(V, V)$ for some finite dimensional vector space V over F. Then prove that the following statements are equivalents : **[7]**

i) T is invertible.

ii) T is one to one.

iii) T is onto.

b) Let $T \in L(V, V)$ be given by

$T(u_1) = u_2$

$T(u_2) = u_1$

Where $\{u_1 = (1, 0), u_2 = (0, 1)\}$ is a basis for V. Let A be matrix of T with respect to the basis $\{u_1, u_2\}$ and B be the matrix of T with respect to the new basis $\{w_1, w_2\}$ given by

$W_1 = 3u_1 - u_2$

$W_2 = u_1 + u_2$

Find A, B and an invertible matrix X such that $X^{-1}AX = B$. **[7]**

P.T.O.

Q2) a) Let S be a subspace of a vector space V over a field F, such that S is generated by n vectors a_1, ... a_n. Suppose b_1, ..., b_m are vectors in S, with $m > n$ then prove that the vectors b_1, ..., b_m are linearly dependent. **[7]**

b) Determine whether the following set of points are vertices of parallelogram or not. **[4]**

$$\langle 0, 0\rangle, \langle 1, 1\rangle, \langle 4, 2\rangle, \langle 3, 1\rangle$$

c) Test the linear dependence of the following set of vectors in R_3. **[3]**

$$\langle 1, 1, 2\rangle, \langle 3, 1, 2\rangle, \langle -1, 0, 0\rangle$$

Q3) a) Show that, a linear transformation $T \in L(V, V)$ is diagonable if and only if the minimal polynomial of T has the form **[7]**

$M(x) = (x - \xi_1)$ .... $(x - \xi_s)$ with distinct ξ_1, ξ_2, ..., ξ_3 in F.

b) Let T be a linear transformation on a vector space over the complex number such that **[7]**

$T(V_1) = -V_1 - V_2$

$T(V_2) = V_1 - 3V_2$

Where $\{V_1 = (1, 0), V_2 = (0, 1)\}$ is a basis for the vector space then find

i) Characteristic Polynomial of T.

ii) Minimal Polynomial of T.

iii) Characteristic vector of T.

iv) Characteristic roots of T.

Q4) a) Let $T \in L(V, V)$ be a linear transformation where V is a finite dimensional. Show that the following statements are equivalents : **[7]**

i) T is orthogonal transformation.

ii) Inner product $(T(u), T(v))$ = Inner Product (u, v) for all $u, v \in V$.

iii) For some orthonormal basis $\{u_1, \ldots\ldots\ldots, u_n\}$ of v the vectors $\{T(u), \ldots\ldots\ldots\ldots, T(u_n)\}$ also form an orthonormal set.

b) Show that, the functions $f_n(x) = \sin nx$, $n = 1, 2, \ldots.$ form an orthonormal set in the vector space $C([-\pi, \pi])$ of continuous real valued functions on the closed interval $[-\pi, \pi]$ with respect to the inner product

$(f, g) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cdot g(x) dx$ for continuous functions $f.\ g \in C([-\pi, \pi])$.

[7]

Q5) a) Let V be a vector space over a field F and let Y be a subspace of V. Then prove that, the relation R on the set V defined by vRv' if $v-v' \in Y$ is an equivalence relation. **[6]**

b) Let T be a normal transformation and let v and v' be eigenvectors for T and its adjoint T' simultaneously such that v and v' belongs to distinct eigenvalues for T. Then prove that $(v, v') = 0$. **[6]**

c) Find the companion matrix of $(x^2 + 1)^2$ over the rational field. **[2]**

Q6) a) Find the perpendicular distance from the Point (1, 5) to the line passing through the Points (1, 1) and (–2, 0) by using Gram schmidt process. **[6]**

b) If V be a vector space over an algebraically closed field F, then prove that, every irreducible invariant subspace W relative to $T \in L(V, V)$ has dimension 1. **[5]**

c) Compute $A_1 \dot{x} B_1$ where $A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. **[3]**

Q7) a) let U and V be the finite dimensional vector space with bases $\{u_1, \ldots\ldots, u_k\}$, $\{v_1, \ldots\ldots, v_k\}$ respectively then prove that $U_1 = \{(u, 0) \mid u \in U\}$ and $V_1 = \{(0.v) \mid v \in V\}$ are subspaces of U + V and U + V is the direct sum $U_1 \oplus V_1$. **[7]**

b) Let T be an invertible linear transformation on a vector space V over C with Hermitian scalar product then prove that T can be expressed in the form T = US, where S is positive & U is unitary. **[7]**

Q8) a) Let $T \in L(V, V)$, let $\{v_1, \ldots\ldots\ldots, v_n\}$ be a basis of V and $\{f_1, \ldots\ldots\ldots, f_n\}$ the dual basis of V*. Let A be the matrix of T with respect to the basis $\{v_1, \ldots\ldots\ldots, v_n\}$. Then prove that, the matrix of T* with respect to the basis $\{f_1, \ldots\ldots\ldots, f_n\}$ is the transpose matrix ${}^tA$. **[6]**

b) Define an Unitary transformation. Show that, $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ is unitary matrix. **[4]**

c) Find the rational canonical form over the field of rational numbers of matrix A, where $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ **[4]**

□□□

Total No. of Questions : 8]

SEAT No. :

PA-3109

[Total No. of Pages : 3

[5907]-12
M.A./M.Sc.
MATHEMATICS
MTUT - 112 : Real Analysis
(2019 Pattern) (CBCS) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) i) Let E_1 and E_2 are measurable sets then prove that, **[4]**

$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$

ii) Is closed interval $G\delta$ – set? Justify. **[3]**

b) i) Let f is measurable function over a measurable set E then prove that $|f|$ is measurable. **[4]**

ii) A function f defined on $\mathbb{R}$ as **[3]**

$$f(x) = \begin{cases} x\sin\dfrac{1}{x} & , \quad if \ x \neq 0 \\ 0 & , \quad if \ x = 0 \end{cases}$$

Find upper and lower derivatives of f at $x = 0$.

Q2) a) i) Let $\{A_k\}_{k=1}^{\infty}$ be an ascending collection of measurable sets. Prove that $m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k\to\infty} m(A_k)$

ii) Let $\{B_k\}_{k=1}^{\infty}$ be descending collection of measurable sets and $m(B_1) < \infty$ then prove that $m\left(\bigcap_{k=1}^{\infty} B_k\right) = \lim_{k\to\infty} m(B_k)$.

[7]

b) Prove that the union of countable collection of a measurable set is measurable. **[7]**

P.T.O.

Q3) a) State and prove Egoroff's theorem. **[7]**

b) Let $\{fn\}_{n=1}^{\infty}$ be a sequence of measurable functions on E that converges pointwise almost everywhere on E to the function *f* then prove that *f* is measurable. **[7]**

Q4) a) Let *f* be a continuous on closed, bounded interval $[a, b]$ and absolutely continuous on [a, *b*]. Prove that the family of divided difference functions $\{\mathrm{Diff}_h^{\,f}\}_{0<h\leq 1}$ is uniformly integrable over $[a, b]$. **[7]**

b) Let function $f(x)$ defined on [0, 1] by **[7]**

$$f(x)=\begin{cases} x^2\sin\left(\dfrac{1}{x^2}\right) & , \quad for\ 0<x\leq 1 \\ 0 & , \quad for\ x=0 \end{cases}$$

Show that f' is not integrable over [a, b].

Q5) a) Prove that every interval is measurable. **[7]**

b) Let *f* be an extended real valued function on E, also *f* is measurable on E and $f = g$ almost everywhere on E then prove that *g* is measurable on E. **[7]**

Q6) a) Let *f* and *g* are measurable functions then prove that, **[7]**

i) the integer powers f^k, $k \geq 1$ are measurable.

ii) for finite valued *f* and *g*, the $f + g$ and $f.g$ are measurable.

b) Let *f* be a absolutely continuous function on closed, bounded interval $[a, b]$. Prove that *f* is differentiable almost everywhere on (a, b) and

$$\int_a^b f' = f(b) - f(a).$$ **[7]**

Q7) a) Let *f* be integrable over the closed bounded interval $[a, b]$. Then prove that $f(x) = 0$ for almost all $x \in [a,b]$ if and only if $\int_{x_1}^{x_2} f = 0$ for all $(x_1, x_2) \subseteq [a, b]$. **[7]**

b) State and prove Borel-Cantelli Lemma. **[7]**

Q8) a) Let E be a set of finite outer measure and $\mathcal{F}$ a collection of closed, bounded intervals that covers E in the sense of Vitali. Prove that for each $\in > 0$ there is a finite disjoint sub collection $\{I_k\}_{k=1}^{n}$ of $\mathcal{F}$ for which

$$m^*\left[E \sim \bigcup_{k=1}^{n} I_k\right] < \in.$$ **[7]**

b) Prove that outer measure is translation invariant. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3110

[Total No. of Pages : 3

[5907]-13

M.A./M.Sc.

MATHEMATICS

MTUT - 115: Ordinary Differential Equations

(2019 Pattern) (Semester - I) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Figures to the right indicate full marks***
2) ***Attempt any Five questions.***

Q1) a) Show that $\phi(x) = e^{-ax}\int_{x_0}^{x} e^{at}b(t)dt + ce^{-ax}$ is a solution of the equations $y' + ay = b(x)$, a is constant and $b(x)$ is a continuous function on an interval I. **[7]**

b) Solve the equation Ly' + Ry = E where L, R, E all are positive constants. Also show that every solution tends to E/R as $x \to \infty$. **[7]**

Q2) a) Explain the method of solving the equation $y' + ay = b(x)$. **[7]**

b) i) Show that the function $\phi(x) = 2 + e^{-x}$ is the solution of the equation $y' + y = 2$. **[2]**

ii) Show that every solution of the equation $x^2 y' + 2xy = 1$ on $(0, \infty)$ tends to zero as $x \to \infty$. **[5]**

Q3) a) i) If $\phi(x)$ is a function with a continuous derivative on I = [0, 1] satisfying $\phi'(x) - 2\phi(x) \le 1$ and $\phi(o) = 1$ on I. Then show that $\phi(x) \le \frac{3}{2}e^{2x} - \frac{1}{2}$. **[4]**

ii) If $\phi(x)$ is the solution of the equation $y' + iy = x$ such that $\phi(o) = 2$ then find $\phi(\pi)$. **[3]**

b) Show that by formal substitution $z = y^{1-k}$ transforms the equation $y' + \alpha(x)\, y = \beta(x)\, y^k$ into $z' + (1-k)\alpha(x)\, z = (1-k)\, \beta(x)$ Hence find all the solutions of $y' - 2xy = xy^2$. **[7]**

P.T.O.

Q4) a) i) Show that the functions $\phi_1(x) = x^2$ and $\phi_2(x) = x|x|$ are linearly independent on $(-\infty,\infty)$ **[3]**

ii) Find the solution of the initial value problem

$y'' + (4i + 1)\, y' + y = 0$ with $y(0) = 0 = y'(0)$ **[4]**

b) i) If $\phi(x)$ is a function having continuous derivative on $(0, \infty)$ such that $\phi'(x) + 2\,\phi(x) \le 1$, $\forall x \varepsilon [0,\infty]$ and $\phi(0) = 0$. then show that $\phi(x) < \frac{1}{2}$ For $x \ge 0$. **[5]**

ii) Explain the difference between order and degree of the differential equation. **[2]**

Q5) a) Show that every solution $\psi(x)$ of $L(y) = b(x)$ on I is of the form $\psi(x) = \psi_p + c_1\phi_1 + c_2\phi_2$. Where, $b(x)$ is continuous on I, ψ_p is particular solution and ϕ_1, ϕ_2 are linearly independent solutions of $L(y) = 0$. **[7]**

b) Find the solution of the equation $y'' - y' - 2y = e^{-x}$. **[7]**

Q6) a) Explain the method for solving non-homogeneous equation of order n. **[7]**

b) Compute the solution $\phi(x)$ of the equation $y''' + y'' + y' + y = 1$ which satisfies $\phi(0) = 0$, $\phi'(0) = 1$ and $\phi''(0) = 0$. **[7]**

Q7) a) i) Find two linearly independent solutions of the equation

$(3x - 1)^2\, y'' + (9x - 3)\, y' - 9y = 0$ for $x > 1/3$ **[4]**

ii) Show that there is a solution of the form x^r of the equation $x^2 y'' + xy' - y = 0$, $x > 0$ and r is constant . **[3]**

b) Explain the method of reduction of order for solving n^{th} order homogeneous equation. **[7]**

Q8) a) Show that if ϕ_1 is a solution of the equation

$y'' + a_1(x)\, y' + a_2(x)\, y = 0$ on I with $\phi_1(x) \neq 0$ on I then

$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi_1]^2} \exp\left[-\int_{x_0}^{x} a_1(t)\, dt \right] ds$ is the second solution on I.

[7]

b) Explain the variable separable method for first order differential equation $y' = f(x, y)$. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3111

[Total No. of Pages : 3

[5907]-14
M.A./M.Sc.
MATHEMATICS
MTUT - 114 : Advanced Calculus
(2019 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $(x, y) \neq (0, 0)$. Let $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ **[6]**

Find the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along the line $y = mx$. Is it possible to define $f(0, 0)$, so as to make f continuous at $(0, 0)$?

b) A scaler field f is defined on $\mathbb{R}^n$, by the equation $f(x) = a.x$, where a is constant vector compute $f'(x; y)$, for arbitrary x and y. **[4]**

c) If $f(x,y,z) = x\bar{i} + y\bar{j} + z\bar{k}$. Prove that the Jacobian matrix D $f(x, y, z)$ is the identity matrix of order 3. **[4]**

Q2) a) Prove that, if f is differentiable at 'a' with total derivative Ta. Then the derivative $f'(a\,;y)$ exist for every y in $\mathbb{R}^n$ and we have

$$\text{Ta}\,(y) = f'(a\,;y)$$

Moreover, $f'(a\,;y)$ is a linear combination of the component of y. In fact, if $y = (y_1, y_2 \ldots y_n)$ we have,

$$f'(a\,;y) = \sum_{k=1}^{n} \mathrm{D}_k f(a) y_k$$ **[6]**

b) Calculate the line integral of the vector Field f along the path described by $f(x,y) = (x^2 - 2xy)\bar{i} + (y^2 - 2xy)\bar{j}$, from $(-1, 1)$ to $(1, 1)$ along the parabola $y = x^2$. **[4]**

c) Verify that the mixed partial derivatives $\mathrm{D}_1(\mathrm{D}_2 f)$ and $\mathrm{D}_2(\mathrm{D}_1 f)$ are equal for function $f(x, y) = x^4 + y^4 - 4x^2 y^2$. **[4]**

P.T.O.

Q3) a) Prove that, if ϕ be a differentiable scalar field with a continuous gradient $\Delta\phi$ on an open connected set S in $\mathbb{R}^n$. Then for any two points *a* and *b* joined by piecewise smooth path α is $\int_a^b \overline{\nabla}\phi \cdot d\overline{\alpha} = \phi(\overline{b}) - \phi(\overline{a})$. **[6]**

b) Calculate the line integral of vector field $\overline{f}(x,y,z) = x\overline{i} + y\overline{j} + (xz - y)\overline{k}$, from (0, 0, 0) to (1, 2, 4) along a line segment. **[4]**

c) Show that, given vector field is not gradient $f(x,y) = x\overline{i} + xy\overline{j}$. **[4]**

Q4) a) Prove that, if α and β be equivalent piecewise Smooth path. Then

$\int_c fd\alpha = \int_c fd\beta$, if α & β trace out C in same direction and

$\int_c fd\alpha = -\int_c fd\beta$, if α & β trace out C in opposite direction. **[6]**

b) Give any two basic properties of line integral. **[4]**

c) Prove that, if $f : S \to \mathbb{R}^m$, then *f* is continuous at a point if and only if each component *f k* is continuous at that point. **[4]**

Q5) a) State and prove Green's theorem. **[6]**

b) Evaluate the double integral,

$\iint_\varphi (x^3 + 3x^2y + y^3)dxdy$, where φ = [0,1] × [0, 1]. **[4]**

c) Find the Jacobian for cylindrical transformation. **[4]**

Q6) a) Use a suitable linear transformation to evaluate the double integral $\iint_S e^{\frac{y-x}{y+x}}dxdy$, where S is the triangle bounded by $x + y = 2$ & two coordinate axes. **[6]**

b) Define Bounded set of content zero. Give any two consequences to this definition. **[4]**

c) Evaluate $\iiint_S \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) dx\, dy\, dz$, where S is the solid bounded by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. **[4]**

Q7) a) Prove that, if V is solid in 3-space bounded by an orientable closed surface S and let n be the unit outer normal to S. If f is continuously differentiable vector field on V, then $\iiint_V (div\, \overline{F}) dx\, dy\, dz = \iint_S \overline{F} \cdot \overline{n}\, ds$. **[6]**

b) If $\overline{F}(x, y, z) = x\overline{i} + y\overline{j} + z\overline{k}$. Then find div $\overline{F}$, curl $\overline{F}$. **[4]**

c) Define: **[4]**

i) Smooth Surface

ii) Tangent Plane to Surface.

Q8) a) Prove that, if a scalar field F is differentiable at a, then f is continuous at a. **[6]**

b) Define **[4]**

i) Rectifiable curve

ii) Jorden curve

c) Evaluate, $\int_C \frac{(x+y)dx - (x-y)dy}{x^2 + y^2}$, where C is the circle $x^2 + y^2 = a^2$, traversed once in a counter clockwise direction. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3112

[Total No. of Pages : 3

[5907]-15

F.Y. M.A./M.Sc.

MATHEMATICS

MTUT113 : Group Theory

(2019 Pattern) (Semester-I) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let G be a group and $a \in G$, C(a) denote the centralizer of a in G. prove that C(a) is a subgroup of G. **[5]**

b) Let G be a group and $a \in G$, prove that if a has infinite order then $a^i = a^j$ if and only if $i = j$. **[5]**

Also prove that if a is of finite order n then $a^i = a^j$ if and only if n divides $i - j$.

c) Give an example of a non - Abelian group all of whose subgroups are Abelian. **[4]**

Q2) a) Let G be a group of finite order. Prove that the number of elements of order d is a multiple of $\varphi(d)$. **[5]**

b) Let S be a finite group and σ denotes a permutation of S. Prove that $\sigma = \alpha_1\alpha_2....\alpha_n$ where $\alpha_1, \alpha_2, ...\alpha_n$ are disjoint cycles. **[5]**

c) Let $\alpha = (1\ 2)\ (4\ 5)$, $\beta = (1\ 6\ 5\ 3\ 2)$ be permutations in S_6. Compute each of the following. **[4]**

i) α^{-1}

ii) $\beta\alpha$

iii) $\alpha\beta$

P.T.O.

Q3) a) Prove that Aut $(\mathbb{Z}_n)$ is isomorphic to u(n), forevery positive integer n. **[5]**

b) Let A_n denote the set of even permutations of S_n then prove that A_n is a subgroup of S_n of order $\frac{n!}{2}$. **[5]**

c) Let $G = SL_2(\mathbb{R})$, the group of 2×2 matrices over $\mathbb{R}$ with determinant 1. Prove that $\varphi_m(A) = MAM^{-1}$ for all A in G & *fix* M in G is an isomorphism of $SL_2(\mathbb{R})$. **[4]**

Q4) a) State and prove Lagrange's theorem. **[5]**

b) Let H and K be two subgroups of a group G define $HK=\{hk/h\in H,k\in K\}$, prove that $|HK| = \frac{|H||K|}{|HnK|}$. **[5]**

c) Let G = {(1), (1 3 2) (4 6 5) (7 8), (1 3 2) (4 6 5), (4, 2 3) (4 5 6), (1 2 3) (4 5 6) (7 8), (7 8)}. **[4]**

Find the following

i) $orb_G(1)$, $stab_G(1)$

ii) $orb_G(2)$, $stab_G(2)$

Q5) a) Prove that the group of complex numbers under addition is isomorphic to $\mathbb{R} \oplus \mathbb{R}$. **[5]**

b) Determine the number of cyclic subgroup of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$. **[5]**

c) Express U(165) as an external direct product of U-groups in four different ways. **[4]**

Q6) a) Let G be a group and $Z(\sigma)$ denote the center of G, prove that $G/z(G) \approx$ Inn (G) where Inn (G) – inner automorphism of G. **[5]**

b) Prove that if H is a subgroup of G having index 2 in G then H is normal in G. **[5]**

c) Prove that an Abelian group of order 33 is cyclic. **[4]**

Q7) a) Let φ be a homomorphism from a group G to a group $\overline{G}$ and let H be a subgroup of G . Then prove that. **[5]**

i) If $\overline{K}$ is a subgroup of $\overline{G}$ then $\varphi^{-1}(\overline{K})$ is a subgroup of G.

ii) If $\overline{K}$ is a normal subgroup of $\overline{G}$ then $\varphi^{-1}(\overline{K}) = \{K \in G / \varphi(K) \in \overline{K}\}$ is a normal subgroup of G.

iii) If φ is onto and kerφ=$\{e\}$ then φ is an isomorphism from G to $\overline{G}$.

b) Let H be a subgroup of G, N(H) denote the normalizer of G and C(H) denote the centralizer of H. Then prove that N(H)/C(H) is isomorphic to a subgroup of Aut(H). **[5]**

c) Suppose that φ is a homomorphism from U(30) to U(30) with ker φ=$\{1, 11\}$. If $\varphi(7) = 7$ find all elements of U(30) that map to 7. **[4]**

Q8) a) If G is a group of order pq's where p, q are primes $p < q$ and p does not divide q–1, then prove that G is cyclic and $G \approx \mathbb{Z}_{pq}$. **[5]**

b) Let G be a group of order 99. Then prove that $G \approx \mathbb{Z}_{99}$ or $G \approx \mathbb{Z}_3 \oplus \mathbb{Z}_{33}$. **[5]**

c) Show that Cl(a) = {a} if and only if a$\in$z(G), where Z(G) is a center of G and Cl(a) denote conjugacy class of a$\in$G. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3113

[Total No. of Pages : 4

[5907]-21
F.Y.M.A./M.Sc.
MATHEMATICS
MTUT - 121 : Complex Analysis
(2019 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions out of eight quetions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $P:\mathbb{C}\to\mathbb{C}$ be a polynomial function then prove that $|P|:\mathbb{C}\to\mathbb{R}$ attains its infimum. **[5]**

b) Let $f:U\to V$ and $g:v\to\mathbb{R}$ be such that *f* is differentiable at $z_0\in U$ and *g* is differentiable at $w_0=f(z_0)\in v$. then prove that *gof* is differentiable at z_0 and $D(gof)_{z_0}=D(g)_{w_0}\ 0\ D(f)_{z_0}$. **[5]**

c) If $z=x+iy$ then show that $\frac{|x|+|y|}{\sqrt{2}}\le|z|\le|x|+|y|$. **[4]**

Q2) a) Let U be a convex open subset of $\mathbb{R}^2$ and $f:U\to\mathbb{R}$ be a differentiable function such that $D(f)_z=0$ for all $z\in U$. Then prove that $f(z)=c$, a constant on U. **[5]**

b) Show that the directional derivatives of the function

$$f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)\neq(0,0)\end{cases}$$

exists at (0, 0) in any direction $u=(u_1, u_2)$ **[4]**

P.T.O.

c) Show that the function f(z) = z Re (z) is complex differentiable at $z = 0$ only. **[3]**

d) Find the length of the are c where C is the circle of radius r and centre at origin. **[2]**

Q3) a) Let Ω be a simply connected domain in $\mathbb{C}$ and f be a holomorphic function on it. Then show that for any simple closed contour γ in $\Omega, \int_{\gamma} f(z)dz = 0.$ **[5]**

b) For all points w such that $|w-a| < r,$ show that $\int_c \frac{dz}{z-w} = 2\pi i$. **[5]**

c) Evaluate $\int_c \frac{z^3}{2z-1}dz$ where c is unit circle traversed in counterclockwise direction. **[4]**

Q4) a) Prove that for a continuous complex valued function f defined in a region Ω, the integral $\int_w fdz = 0$ for all closed contours w if and only if f is the derivative of a holomorphic function on Ω. **[5]**

b) Given a complex differentiable function f such that $|f(z)| \leq 1, \forall |z| < 1$ find a variable upper bound for $\left|f^{(n)}(z)\right|$ in the disc $|z| \triangleleft 1$. Conclude that $\left|f^{(n)}(0)\right| \leq n!$ **[4]**

c) Determine the nature of singularity at $z = 0$ for the function $f(z) = \frac{\sin z}{z^2}$. If it is removable, find the value of f at $z = 0$. If it is pole, find the singular part. **[3]**

d) Determine the value of the integration $\int_c e^{-z^2} dz$ where c is the circle $|z-5| = 2$. **[2]**

Q5) a) Let $f : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$ be a holomorphic function then prove that f is a rational function. **[5]**

b) Let f and g be holomorphic functions on a domain Ω. Suppose $k \subset \Omega$ is such that for every $z \in k$, $f(z) = g(z)$ and k has a limit point Ω. Then show that $f \equiv g$ on Ω. **[5]**

c) Compute the residues at all singular points of the function $f(z) = \frac{e^z}{z^2 - 1}, z \neq \pm 1$ **[4]**

Q6) a) Let a be a pole of order n of f and let $g(z) = (z-a)^n . f(z)$ with g holomorphic and $g(a) \neq 0$. Then show that the residue of f at is a is $\mathrm{R}_a(f) = \frac{g^{(n-1)}(a)}{(n-1)!}$ **[5]**

b) Prove the Jordan's inequality $\mathrm{J} := \int_0^{\pi} e^{-R\sin\theta} d\theta < \frac{\pi}{R}, R > 0$ **[4]**

c) Obtain the Laurent series expansion for the function $f(z) = \frac{1}{1-z}$ for the region $\mathrm{A} = \{z/|z-2| > 1\}$ **[5]**

Q7) a) Show that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1 = a^2}}, \ -1 < a < 1.$ **[7]**

b) Show taht $\int_{\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}.$ **[7]**

Q8) a) Let $f : \mathrm{D} \to \overline{\mathrm{D}}$ be a holomorplic function such that $f(0) = 0$. Then show that $|f(z)| \leq |z|$ and $|f'(0)| \leq 1$. Further show the following conditions are equivalent.

i) There exists $z_0 \neq 0$ with $|z_0| < 1$ and $|f(z_0)| = |z_0|$.

ii) $|f'(0)| = 1$.

ii) $f(z) = cz$ for some $|c| = 1$. **[6]**

b) Let f be holomorphic on $\overline{D}$ and $|f(z)| < 1$. for $|z| = 1$. Then show that f has exactly one fixed point inside D. **[5]**

c) Find the cauchy's principal value of $\int_{-\infty}^{\infty} \frac{e^{iax}}{1-x} dx$ for $H{:}|z| < 1.$. **[3]**

Total No. of Questions : 8]

SEAT No. :

PA-3114

[Total No. of Pages : 3

[5907]-22
M.A./M.Sc. - I
MATHEMATICS
MTUT - 122 : General Topology
(2019 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right, indicate full marks.***

Q1) a) If C is an infinite subset of $\mathbb{Z}_+$, then show that C is countably infinite. **[6]**

b) Let A be a set. Prove that there is no injective map $f : p(\mathrm{A}) \to \mathrm{A}$, and there is no surjective map $g : \mathrm{A} \to p(\mathrm{A})$. **[4]**

c) Prove that there exists a well ordered set A having largest element Ω, such that the section S_Ω of A by Ω is uncountable but every other section of A is countable. **[4]**

Q2) a) Let X and Y be two topological spaces and $\mathrm{A} \subset \mathrm{X}, \mathrm{B} \subset \mathrm{Y}$. Prove that product topology on A × B is the same as the subspace topology of A × B as subspace of X × Y. **[6]**

b) Find closure of $\mathrm{K} = \left\{ \frac{1}{n} \middle| n \in \mathbb{N} \right\}$ with usual topology on $\mathbb{R}$ and with finite complement topology on $\mathbb{R}$. **[4]**

c) With usual notations prove or disprove: **[4]**

i) $(\mathrm{A} \cap \mathrm{B})^\circ = \mathrm{A}^\circ \cap \mathrm{B}^\circ$

ii) $(\mathrm{A} \cup \mathrm{B})^\circ = \mathrm{A}^\circ \cup \mathrm{B}^\circ$

P.T.O.

Q3) a) Prove that the collection **[6]**

$S\left\{\pi_1^{-1}(U) / U \text{ is open in } X\right\} U \left\{\pi_2^{-1}(V) / V \text{ open in } Y\right\}$ is a subbasis for the product topology on X × Y.

b) Let Y be a subspace of X, let A be a subset of Y, let $\overline{A}$ denote the closure of A in X. Then prove that closure of A in Y equals $\overline{A} \cap Y$. **[4]**

c) Prove that every finite point set in a Hausdorff space X is closed. **[4]**

Q4) a) Let $\rho : X \to Y$ be a quotient map; let A be a subspace of X that is saturated with respect to ρ; let $q : A \to P(A)$ be the map obtained by restriction? Then prove the following: **[7]**

i) If A is either open or closed in X then *q* is quotient map.

ii) If ρ is either an open map or a closed map, then *q* is quotient map.

b) Prove that an uncountable product of $\mathbb{R}$ with itself is not metrizable.**[7]**

Q5) a) Let $f_n : X \to Y$ be a sequence of continuous functions from the topological space X to the metric space Y. Prove that, if (f_n) converges uniformly to *f*, then *f* is continuous. **[6]**

b) A finite Cartesian product of connected spaces is connected. **[4]**

c) Prove that, the ordered square I_o^2 is connected but not path connected.**[4]**

Q6) a) Let X be a metrizable space. Then prove that the following are equivalent: **[7]**

i) X is compact.

ii) X is limit point compact.

iii) X is sequentially compact.

b) Let X be a topological space. Then prove that X is compact if and only if for every collection $\mathcal{C}$ of closed sets in X having the finite intersection property, the intersection $\bigcap_{C \in \mathcal{C}} C$ of all the elements of $\mathcal{C}$ is nonempty. **[7]**

Q7) a) State and prove Tychonoff theorem. **[7]**

b) Define First and Second countable spaces. Establish relation between them. **[7]**

Q8) a) Prove that, every metrizable space is normal. **[6]**

b) Prove that subspace of completely regular space is completely regular. **[4]**

c) Let X be locally compact Hausdorff, let A be a subspace of X. Show that if A is closed in X or open in X, then A is locally compact. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3115

[Total No. of Pages : 2

[5907]-23

M.A./M.Sc

MATHEMATICS

MTUT - 123 : Ring Theory

(2019 Pattern) (Semester - II) (Credit system)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figres to the right indicate full marks.***

Q1) a) Let R be a commutative ring with 1, a_0 is unit in R and a_1, $a_2 ... a_r$ are nilpotent elements in R. Prove htat $a(x) = a_0 + a_1x + a_2x^2 + ... + a_rx^r$ is unit in R [x]. **[6]**

b) Show that $R = z\left[\sqrt{-2}\right]$ has no units other than $\pm$ 1, where $S = z\left[\sqrt{2}\right]$ has infinity many units. **[5]**

c) Describe the units in the ring given by ZXQXZ. **[3]**

Q2) a) Show that (Q,+) has no maximal subgroup. **[6]**

b) Prove that the characteristic of a local ring is either zero or power of a prime. **[6]**

c) Give an example of degree one polynomial over z_6 which has three distinct roots in z_6. Justify. **[2]**

Q3) a) If R is ring with 1 and I is left ideal in R such that I $\neq$ R, then prove that there exist a maximal left ideal M of R such that I $\subseteq$ M. **[6]**

b) With usual notations prove that $\sqrt{(30)} = \sqrt{(60)} = \sqrt{(300)}$. **[4]**

c) Define local ring and give an example of local ring. **[4]**

Q4) a) Prove that R/I is a field if and only if I is a maximal ideal. **[6]**

b) Let f : R $\rightarrow$ S be a surjective ring homomorphism prove that inverse image of a maximal ideal. In S is a maximal ideal in Q. **[4]**

c) Show that, field of fraction of z [i] is Q [i]. **[4]**

P.T.O.

Q5) a) Prove that every euclidean domain is principal ideal domain. **[7]**

b) Prove that homomorphic image of a PIR is a PIR C (Principal ideal ring). **[7]**

Q6) a) Prove that every Principal ideal domain is a factorisation domain. **[6]**

b) Show that $1+x+x^2+......+x^{p-1}$ is irreducible in z [x] for any prime P. **[5]**

c) Show that in the ring z[i] the elements 3+4i and 4–3i are associates. **[3]**

Q7) a) Prove that Q = K [x], the polynomial ring in one variable over a field K is euclidean. **[7]**

b) Let R be a PID with 1 and a, b $\in$ R*. Then prove that d = gcd (a, b) if and only if (d) = (a) + (b). **[7]**

Q8) a) If P, N are submodules of module M. Then prove that there exist natural isomorphism $\frac{P+N}{P} \cong \frac{N}{N \cap P}$. **[7]**

b) Prove that an R-module M is simple if and only if M $\cong$ R\I for some maximal left ideal in R. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3116

[Total No. of Pages : 4

[5907]-24
M.A./M.Sc. - I
MATHEMATICS
MTUT - 124 : Advanced Numerical Analysis
(CBCS 2019 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Determine the corresponding rate of convergence for the function. $f(x)=\dfrac{2^n+3}{2^n+7}$ **[4]**

b) Prove that if $g:[a,b]\to[a,b]$ is continuous function on [a, b], differentiable on (a,b) and there exists a constant $k<1$ such that, $|g'(x)|\le k<1\ \forall x\in(a,b)$ then

i) The sequence $\{p_n\}$ generated by $p_n=g(p_{n-1})$ converges to the fixed point p for any $p_0\in(a,\ b)$

ii) $|p_n-p|\le\dfrac{k^n}{1-k}|p_1-p_0|$

[5]

c) Use the secant method to determine p_5, the fifth approximation to root of $f(x)=x^7-3$ in (1, 2) with $p_0=1$ and $p_1=2$. **[5]**

Q2) a) The sequence listed below was obtained from fixed point iteration applied to the function $g(x)=e^{-x}$, which has a unique fixed point. Applying Aitken's Δ^2 - method to the given sequence.

Find $\hat{p}_3$, $\hat{p}_4$ and $\hat{p}_5$. **[6]**

1	1,00000
2	0.3678794412
3	0.6922006276
4	0.5004735006
5	0.6062435351

b) Show that when Newton's method is applied to the equation $x^2 - a = 0$, the resulting iteration function is $g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$ **[4]**

c) Define: **[4]**

i) Orthogonal Matrix

ii) Round off Error

Q3) a) Solve the following system of equations by using Gaussian elimination with scaled partial pivoting

$$
\begin{aligned}
3x_1 + x_2 + 4x_3 - x_4 &= 7 \\
2x_1 - 2x_2 - x_3 + 2x_4 &= 1 \\
5x_1 + 7x_2 + 14x_3 - 8x_4 &= 20 \\
x_1 + 3x_2 + 2x_3 + 4x_4 &= -4
\end{aligned}
$$

[5]

b) Find the LU decomposition of the matrix

$$
A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}
$$

by using crout decomposition method. **[5]**

c) Solve the following system by Jacobi method starting with vector $x^{(0)} = [0\ 0\ 0]^T$ performance iteration: **[4]**

$$2x_1 - x_2 = -1$$
$$-x_1 + 4x_2 + 2x_3 = 3$$
$$2x_2 + 6x_3 = 5$$

Q4) a) Solve the following system of linear equations by SOR method, start with $x^{(0)} = [0\ 0\ 0]^T$ and w = 0.9 (perform 3 iteration)

$$5x_1 + x_2 + 2x_3 = 10$$
$$-3x_1 + 9x_2 + 4x_3 = -14$$
$$x_1 + 2x_2 - 7x_3 = -33$$

[5]

b) Solve the following system of non-linear algebric equations by using Broyden's method start with $x^{(0)} = [111]^T$ (perform 3 iteration)

$$x_1^3 - 2x_2 - 2 = 0$$
$$x_1^3 - 5x_3^2 + 7 = 0$$
$$x_2 x_3^2 - 1 = 0$$

[5]

c) Let A be an n×n matrix with eigenvalues $\lambda_1, \lambda_2 ...., \lambda_n$ and associated eigenvectors $V_1, V_2, ..., V_n$. Then prove that,

If B = a_0I + a_1A + a_2A^2 + ... +a_mA^m = P$^{(A)}$

where p is the polynomial $p(x) = a_0 + a_1x + a_2x^2 + ... + a_mx^m$, then the eigen values of B are $p(\lambda_1)p(\lambda_2), ..., p(\lambda_n)$ with associated eigenvector $V_1, V_2, ..., V_n$. **[4]**

Q5) a) Explain the QR algorithm for finding the eigenvalues of symmetric Tridiagonal matrix. **[7]**

b) Construct the house holder matrix H for $W = \left[\frac{2}{3}\ \frac{1}{3}\ \frac{2}{3}\right]^T$ **[4]**

c) If $f(x) = \ln x$, find $f'(2)$ for h = 0.1, 0.01, 0.001 **[3]**

Q6) a) Derive the open Newton - Cotes formula with $n = 3$: $\mathrm{I}(f) \approx \mathrm{I}_3$, open

$$(f) = \frac{b-a}{24}\left[11F(a+\Delta x) + f(a+2\Delta x) + f(a+3\Delta x) + 11f(a+4\Delta x)\right]$$ **[5]**

b) Derive the following forward difference approximation for the second derivative:

$$\frac{f''(x_0) \approx f(x_0) - 2f(x_0+h) + f(x_0+2h)}{h^2}$$ **[4]**

c) Prove that : If f is continuous on [a, b], g is integrable on [a, b] and $g(x)$ does not change sign on [a, b], then there exist a number $\xi \in [a,b]$ such that, $\int_a^b f(x).g(x)\,dx = f(\xi)\int_a^b g(x)\,dx$ **[5]**

Q7) a) Use Euler's method to solve initial value problem $\frac{dx}{dt} = \frac{t}{x}, 0 \le t \le 5,\ x(0) = 1$ **[7]**

b) Use the Runge Kutta method of order N = 4 to solve initial value problem. $\frac{dx}{dt} = 1 + \frac{x}{t},\ 1 \le t \le 6,\ x(1) = 1.$ **[7]**

Q8) a) Evaluate $\int_{-2}^{2} \frac{x}{5+2x}dx$ by using Trapezoidal rule by dividing the interval [–2, 2] into five equal sub intervals. **[3]**

b) Define: **[4]**

i) Relative error

ii) Triangular Matrix

c) Derive the difference equation for the four - step Adams - Bashforth method:

$$\frac{w_{i+1} - w_i}{h} = \frac{55}{24}f(t_i, w_i) - \frac{59}{24}f(t_{i-1}, w_{i-1}) + \frac{37}{24}f(t_{i-2}, w_{i-2}) - \frac{9}{24}f(t_{i-3}, w_{i-3})$$

Also, derive the associated truncation error: $\tau_i = \frac{251\lambda^4}{720}y^{(5)}(\xi)$ **[7]**

Total No. of Questions : 8]

SEAT No. :

PA-3117

[Total No. of Pages : 4

[5907]-25
M.A./M.Sc. - I
MATHEMATICS
MTUT - 125 : Partial Differential Equations
(CBCS 2019 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the charpits method for solution of Non-linear partial differential equations of first order. **[5]**

b) Attempt the following

i) Use charpits method to solve $P = (x + qy)^2$ **[5]**

ii) Solve $Z + 2u_x - (u_x + u_y)^2 = 0$ by Jacobi's Method. **[4]**

Q2) a) Explain the method of second order partial differential equation $Rr + Ss + Tt + f(x, y, z, p, q) = 0$ to a cannonical form if $S^2 - 4RT < 0$ **[5]**

b) Attempt the following **[5]**

i) Reduce the PDE

$$yu_{xx} + (x + y)u_{xy} + xu_{yy} = 0$$

to a cannonical form and hence solve it.

ii) Solve $\left(D^2 + DD^1 + D' - 1\right)Z = 0$ **[4]**

P.T.O.

Q3) a) Prove that if the PDE $F(D,D')Z = f(x,y)$ have repeated factor $(\alpha_r D + \beta_r D + \gamma_r)^2$ of $F(D,D')$ then **[6]**

$$Z = \exp\left(\frac{-\gamma_r x}{\alpha_r}\right)\left[\phi_r(\beta_r x - \alpha_r y) + x\phi_r(\beta_r x - \alpha_r y)\right]$$

b) Attempt the following

i) Write charpits auxillary equations of $z = px + qy + f(p, q)$ and Find complete integral of the equation $(p + q)(z - xp - yq) = 1$ **[5]**

ii) Solve $\left(D^2 - DD^1 + D' - 1\right)Z = 0$ **[3]**

Q4) a) Derive the Poisson equation of second order partial differential equation. **[5]**

b) Attempt the following. **[5]**

i) Prove that if the Dirichlet problem for a bounded region has a solution then it is unique. **[4]**

ii) Solve by the method of separation of variables

$$\frac{\partial^2 u}{\partial x^2} - \frac{2\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

Q5) a) Show that $u(x, t) = \left(c_1 \cos \lambda x + c_2 \sin \lambda x\right).e^{-\lambda^2 kt}$ is a solution of one dimensional heat equation $\frac{\partial u}{\partial t} = k.\frac{\partial^2 u}{\partial x^2}$ by method of separation variable method. **[5]**

b) Attempt the following

i) Solve $k\left(\frac{\partial^2 u}{\partial x^2}\right) = \frac{\partial u}{\partial t}$ for $0 < x < \pi,\ t > 0$ if $u_x(0,t) = u_x(\pi,t) = 0$ and $u(x, 0) = \sin x$ **[6]**

ii) Solve by $xu_x + yu_y = u_z^2$ by Jacobi's method. **[3]**

Q6) a) Solve

$$\begin{aligned}&\text{PDE}: u_{tt} - C^2 u_{xx} = 0;\ 0 \le x \le L,\ t > 0\\&\text{BC}_s : u(0,\ t) = 0;\ t > 0,\ u(L,\ t) = 0, t > 0\\&\text{IC}s : u(x,\ 0) = F(x),\ u_t(x,\ 0) = y(x)\end{aligned}$$ **[6]**

b) Attempt the following

i) A Square plate is bounded by the lines $x = 0$, $y = 0$, $x = 10$ and $y = 10$. Its faces are insulated. The temperature along and upper horizontal edge is given by $u(x, 10) = x(10 - x)$ while the other three faces are kept at 0°C. Find the steady state temperature in the plate. **[5]**

ii) Find the characteristics equations of differential equation $u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0$ **[3]**

Q7) a) Show that $T(x,t) = \frac{1}{\sqrt{4\pi\alpha t}} \exp\left[-(x-\xi)^2 / (4\alpha t)\right]$ is a solution of diffusion equation $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} - \infty < x < \infty, t > 0$ **[5]**

[7]

b) Attempt the following.

i) Show that $u(x,y) = \sum_n \text{An}\, e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$ is solution of $\nabla^2 u = 0$ by using separation variable method. **[6]**

ii) If $u = f(x+iy) + g(x-iy)$ where f and g are arbitrary functions, show that, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ **[3]**

Q8) a) Reduce the equation

$$(n-1)^2 \frac{\partial^2 z}{\partial x^2} - y^{2n} \frac{\partial^2 z}{\partial y^2} = n.y^{(2n-1)} \frac{\partial z}{\partial y}$$

to cannonical form and find its general solution. **[5]**

b) Attempt the following.

i) A string of length 'l' has its ends $x = 0$ and $x = l$ fixed , It is realised from rest in the position $y = \frac{[4\lambda x(1-x)]}{\lambda^2}$.

Find an expression for the displacement of the string at any subsequent time. **[6]**

ii) Find the complete integral of $u_3 z(u_1 + u_2) + x + y = 0$ by Jacobi's Method. **[4]**

Total No. of Questions : 8]

SEAT No. :

PA-3118

[Total No. of Pages : 3

[5907]-31

M.A./M.Sc.

MATHEMATICS

MTUT - 131 : Functional Analysis

(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Symbols have their usual meanings.***

Q1) a) Let B and B' be two Banach spaces and T is a continious linear transformation of B onto B'. Prove that image of each open sphere centred at origine in B contains an open sphere centred at origine in B'. **[7]**

b) Let M be any closed subspace of normed linear space N. Define norm on N/M by $||x + M||$ $\inf\{||x + m|| : m \in M\}$ prove that N/M is a normed space and also show that if N is complete then N/M is a banach space.**[7]**

Q2) a) Let M and N be closed subspaces of Hilbert space H such that $M \perp N$. Prove that M + N is closed. **[5]**

b) Consider operator T on l_2 given by

$T(x_1, x_2,\text{-------}) = (0, x_1, x_2,\text{--------})$ show that T has no eigenvalues. **[4]**

c) Let T be an operator on a Hilbert H. Prove that following statement's are equivalent **[5]**

i) $T^* \circ T = I$

ii) $(T(x), T(y)) = (x, y)$ for all $x, y \in H$

iii) $||T(x)|| = ||x||$ for all $x \in H$

P.T.O.

Q3) a) Let x be a fixed vector in normed space N. Define Fx : $N^* \to F$ by $Fx(f) = f(x)$.

Show that Fx is linear and also show that $||F_x|| = ||x||$. **[5]**

b) Let M be a closed subspace of normed linear space N. Let $x_0 \notin M$. Prove that there exist's a functional f such that $f(M) = \{0\}$ and $f(x_0) \neq 0$. **[5]**

c) Prove that a non empty subset X of a normed linear space N is bounded if and only if $f(x)$ is bounded for each f in N*. **[4]**

Q4) a) Prove that if M is a closed subspace of Hilbert space H then $H = M \oplus M^{\perp}$. **[5]**

b) Let M be a proper closed subspace of Hilbert space H. Prove that there exist's a non zero vector z_0 in H such that $z_0 \perp H$. **[5]**

c) For any vector y in H define $f_y(x) = (x, y)$. Show that f_y is linear and also show that $||f_y|| \leq ||y||$. **[4]**

Q5) a) Prove that T is self adjoint aperator on Hilbert space H if and only if $(T(x), x)$ is real for all $x \in H$. **[4]**

b) If N_1 and N_2 are normal operator on Hilbert space H such that $N_1 \circ N_2^* = N_2^* \circ N_1$ and $N_2 \circ N_1^* = N_1^* \circ N_2$. Prove that N_1+N_2 is normal operator. **[5]**

c) State and prove the open mapping theorem. **[5]**

Q6) a) Let T be any operator on Hilbert space H such that $(T(x), x) = 0$ for all $x \in H$. Prove that T is identically zero function. **[5]**

b) Prove that a closed convex subset of Hilbert space H contains a unique vector of smallest norm. **[5]**

c) Prove that for any two vector's x and y in Hilbert space H, $|< x, y >| \leq ||x|| \cdot ||y||$. **[4]**

Q7) a) Let B = $\{e_i\}$ be an ordered basis for Hilbert space H and [T] denotes the matrix of operator T on H relative to basis B. Prove that mapping $T \to [T]$ is one - to - one homomorphism. **[6]**

b) Let T be any operator on Hilbert space H and N be a normal operator. Prove that if T commutes with N then T commutes with N^*. **[4]**

c) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be given by $T(x, y, z) = (x, y + z, y)$ find matrix of T with respect to basis B = {(1, 1, 1), (1, 1, 0), (1, 0, 0)}. **[4]**

Q8) a) Let M be a closed subspace of a Hilbert space H. Prove that M is invarient under T if and only if $M^{\perp}$ is invarient under T^*. **[5]**

b) Let T be a normal operator on a Hilbert space H. Let λ_i be eigenvalue of T and M_i be corresponding eigenspace for all i prove that M'S span H. **[5]**

c) Prove that λ is an eigenvalue of operator T on a Hilbert space H if and only if λ is a root of polynomial det $(T - \lambda I)$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3119

[Total No. of Pages : 2

[5907]-32
M.A./M.Sc.
MATHEMATICS
MTUT-132 : Field Theory
(2019 CBCS Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions of the following.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let F be a field and 'n' be a positive integer. Then show that there exist a primitive n^{th} root of unity in some extension E of F if and only if either $ch(F) = 0$ or $ch(F) \nmid n$. **[7]**

b) Show that the polynomial $x^5 - 9x + 3$ is not solvable by radicals. **[5]**

c) Express the polynomial $x_1^2 + x_2^2 + x_3^2$ as rational functions of the elementry symmetric function. **[2]**

Q2) a) Show that a regular n-gon is constructible if and only if $\phi(n)$ is a power of 2. **[7]**

b) Let E be the spliting field of a polynomial of degree n over a field F. Then show that $[E : F] \leq n!$. **[5]**

c) Define cyclic extension of a field. **[2]**

Q3) a) Show that the Galois group of $x^3 - 2 \in Q[x]$ is the group of symmetrices of the triangle. **[7]**

b) If E is a finite normal separable extension of a field F then prove that F is the fixed field of G(E/F). **[5]**

c) Define Galois extension of a field F. **[2]**

P.T.O.

Q4) a) Let F be a field of characteristic $\neq 2$ and $x^2 - a \in F[x]$ be an irreducible polynomial over F. Then show that Galois group of polynomial $x^2 - a$ is of order 2. **[5]**

b) Let $E = Q\left(\sqrt[3]{2}, W\right)$, Where $w^3 = 1$, $w \neq 1$ and G = {1,6}

Where $1:\begin{cases} \sqrt[3]{2} \to \sqrt[3]{2} \\ w \to w \end{cases}$ & $\sigma:\begin{cases} \sqrt[3]{2} \to \sqrt[3]{2} \\ w \to w^2 \end{cases}$ are automorphism on E, then find E_G.

[7]

c) Prove that $\sqrt{2}$ and $\sqrt{3}$ both are algebraic over Q. **[2]**

Q5) a) Give an example to show that an extension of a field F is algebraic but not finite. **[7]**

b) Let E be an extension of a field F and a $\in$ F has a minimal polynomial of odd degree over F, then prove that F(a) = F(a^2). **[5]**

c) Find the minimal polynomial of $3\sqrt{5}+5$. **[2]**

Q6) a) Let K be any field. Then show that K is algebraically closed if and only if every irreducible polynomial in K[x] is of degree 1. **[7]**

b) Show that the spliting field of $f(x) = x^4 - 2 \in Q[x]$ over Q is Q[$2^{1/4}$, i]. **[7]**

Q7) a) Let F be a finite field then prove that the characteristic of F is a prime and F contains a subfield isomorphic to Z_p. **[7]**

b) Show that every finite separable extension of a field F is simple. **[7]**

Q8) a) If $f(x) \in F[x]$ is irreducible polynomial over F then show that all roots of $f(x)$ have same multiplicity. **[7]**

b) If P(x) is an irreducible polynomial in F[x] then prove that there is an extension E of F in which P(x) has a root. **[5]**

c) If $Q\left(\sqrt{2}\right)$ a normal extension of Q? Justify. **[2]**

x x x

Total No. of Questions : 4]

SEAT No. :

PA-3120

[Total No. of Pages : 1

[5907]-33

M.A./M.Sc.-II

MATHEMATICS

MTUT 133 : Programming with Python

(CBCS 2019 Pattern) (Semester-III)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates:

1) ***Question 1 is compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Attempt any 2 questions from Q.2, Q.3 and Q.4.***

Q1) Attempt the following.

a) Write any 4 development environments which are available in Python. **[2]**

b) Write the names of 3 projects which are using python. **[2]**

c) What is the advantages of using python over other languages? **[3]**

Q2) Attempt the following.

a) i) Write a python program which accept positive integer and disply wheather it is multiple of 2 or not. **[5]**

ii) Explain the meaning of, " Tuples are immutable". **[2]**

b) Write a note on conditional statements in python with an example. **[7]**

Q3) Attempt the following.

a) i) Explain the difference between logical operators and Ternary operators in python with an example. **[5]**

ii) What is the output of test condition in python? **[2]**

b) Write a note on while statement in python with an example. **[7]**

Q4) Attempt the following.

a) i) Write a note on "File opening modes" in Python. **[5]**

ii) What is the meaning of "encoding a string in UTF Format"? **[2]**

b) Write a note "class in Python. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3121

[Total No. of Pages : 2

[5907]-34

M.A./M.Sc. - I

MATHEMATICS

MTUTO - 134 : Discrete Mathematics

(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that a graph G is Eulerian if and only if it has at most one nontrivial component and its vertices all have even degree. **[7]**

b) Show with generating functions that every positive integer can be written as a unique sum of distinct powers of 2. **[5]**

c) Find the number of integer solutions to the equation $x_1 + x_2 + x_3 = 11$, $x_1, x_2, x_3 \geq 0$. **[2]**

Q2) a) How many ways are there to pick two different cards from a standard 52-card deck such that **[7]**

i) the first card is an ace and the second card is not a Queen?

ii) the first card is a spade and the second card is not a Queen?

b) Prove that, an edge is a cut-edge if and only if it belongs to no cycle. **[7]**

Q3) a) How many arrangements of the letters in MATHEMATICS are there in which TH appear together but the TH is not immediately followed by an E (not THE)? **[7]**

b) Prove that the isomorphism relation defined on set of simple graphs is an equivalence relation. **[5]**

c) Find the coefficient of $\frac{x^r}{r!}$ in $\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+......\right)^n$. **[2]**

Q4) a) Let T be a tree with average vertex degree a. Determine n(T) in terms of a. **[5]**

b) If two vertices are nonadjacent in the Petersen graph, then prove that they have exactly one common neighbour. **[5]**

c) Find the connectivity and the edge connectivity of K_4. **[4]**

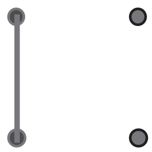
P.T.O.

Q5) a) State and prove Inclusion - Exclusion formula. **[7]**

b) Show by a combinatorial argument that **[7]**

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots\ldots+\binom{n}{n}=2^n$$

c) Find the complement of the following graph. **[2]**



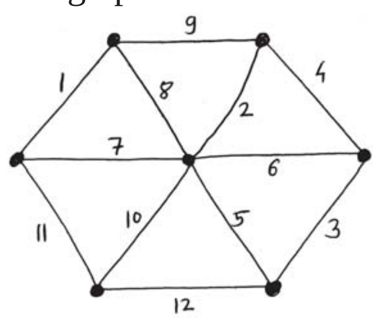
Q6) a) Prove that a graph is bipartite if and only if it has no odd cycle. **[7]**

b) Solve the following recurrence relation **[7]**

$a_n = 3\,a_{n-1} - 3a_{n-2} + a_{n-3},\ a_0 = a_1 = 1,\ a_2 = 2.$

Q7) a) Let $\tau(G)$ denote the number of spanning trees of a graph G. If $e \in E(G)$ is not a loop, then prove that $\tau(G) = \tau(G-e) + \tau(G\cdot e)$. **[6]**

b) Use Kruskal's Algorithm to find the minimum spanning tree for the following weighted graph **[6]**



c) Prove or disprove the following statement. 'If every vertex of a simple graph G has degree 2 then G is cycle.' **[2]**

Q8) a) What is the probability that an arrangement of a, b, c, d, e, f has *a* and *b* side by side? **[5]**

b) Use generating function to find the number of ways to select 10 balls from large pile of red, white and blue balls if the selection has at least two balls of each colour. **[5]**

c) Solve the following recurrence relation.

$a_n = 3a_{n-1} + 4a_{n-2},\ \ a_0 = 1,\ a_1 = 1.$ **[4]**

x x x

Total No. of Questions : 8]

SEAT No. :

PA-3122

[Total No. of Pages : 2

[5907]-35

M.A./M.Sc.-II
MATHEMATICS
MTUTO135 : Mechanics
(2019 Pattern) (Semester-III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the following terms. **[7]**

i) Linear Momentum.

ii) Generalised momentum.

iii) Angular momentum.

b) A particle of mass m moves in a plane under the action of a Conservative force F with components $F_x = -k^2(2x+y)$, $F_y = -k^2(x+2y)$, k is a constant Find total energy of the motion the Lagrangian and equations of motion of particle. **[7]**

Q2) a) Show that the total energy of particle moving in a conservative force field remains constant, if the potential energy is not an explicit function of time. **[5]**

b) Prove that if the lagrangian does not contain time t explicitly the total energy of the conservative system is conserved. **[5]**

c) Explain scleronomic constants with examples. **[4]**

Q3) a) A Particle is constrained to move on the surface of cylinder of fixed radius, obtain lagranges eq^n of motion. **[7]**

b) Show that the Euler-lagranges equation of the functional

$I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\,dx$ has first integral $f - y'\dfrac{\partial f}{\partial y'}$ =Constant if the integrand does not depend on x **[7]**

P.T.O.

Q4) a) Find Euler-lagranges equation satisfied by twice differentiable function $y(x)$ which extremizes the functional $I(y(x)) = \int_{x_1}^{x_2} f(x, y, y')\,dx$ where y is prescribed at the end points. **[7]**

b) Show that geodesic in a eudidian plane is a Straight line. **[7]**

Q5) a) Find the curve which generates a surface of revolution of minimum area when H is revolved about x–axis. **[5]**

b) Find the external of the functional $\int_0^1 (y'^2 - 12xy)\,dx$ $y(0)=1, y(1)=2$ **[5]**

c) Explain the basic lemma. **[4]**

Q6) a) Write a note on brachistochrone problem. **[7]**

b) Deduce newtons second law of motion from Hamilton's principle. **[7]**

Q7) a) Derive the Hamilton's canonical equations of motion from Hamiltonian function. **[7]**

b) Prove that central force motion is always motion in a plane. **[7]**

Q8) a) Prove that the Keplers first law of planetary motion. **[7]**

b) Find the central force under the action of which a particle will follow r = a (1+cos θ) orbit. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3123

[Total No. of Pages : 3

[5907]-36

M.A/ M.Sc

MATHEMATICS

MTUTO 136 : Advanced Complex Analysis

(2019 CBCS Pattern) (Semester - III)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Suppose f is holomorphic in an open set Ω If D is a disc centered at Z_0 and whose closure is contained in Ω, then prove that F has a power series expansion at Z_0, $f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n$ for all $Z \in$ D, and the coefficients are given by $a_n = \dfrac{f^{(n)}(z_0)}{n!}$ for all n$\geq$0 **[5]**

b) Prove that $\int_0^{\infty} \text{Sin}(x^2)\,dx = \dfrac{\sqrt{2\pi}}{4}$ **[5]**

c) If $\{f_n\}_{n=1}^{\infty}$ is a sequence of holomorphic functions that converges uniformly to a function f in every compact subset of Ω, then prove that f is holomorphic in Ω. **[4]**

Q2) a) Let F (z,s) be defined for $(z,s) \in \Omega \times [0,1]$ where Ω is an open set in $\mathbb{C}$. suppose F satisfies the following properties:

i) F(*z*,*s*) is holomorphic in z for each s.

ii) F is continuous on Ω × [0,1].

Then prove that the function f defined on Ω by $f(z)=\int_0^1 \mathrm{F}(z,s)\,ds$ is holomorphic **[5]**

P.T.O.

b) Suppose f is holomorphic in an open set Ω and $K \subset \Omega$ is compact. prove that there exists finitely many segments $\gamma_1, \gamma_2, \text{---}, \gamma_N$ in $\Omega - K$ such that, $f(z) = \sum_{n=1}^{N} \frac{1}{2\pi i} \int_{\gamma_n} \frac{f(\mathscr{E})}{(\mathscr{E} - z)} dz$ for all $z \in k$. **[5]**

c) State the following:

i) Symmetric principle.

ii) Schwarz reflection principle. **[4]**

Q3) a) Prove that the map $F : \mathbb{H} \to \mathbb{D}$ is a conformal map with inverse $G : \mathbb{D} \to \mathbb{H}$. **[5]**

b) Prove that $f(z) = -\frac{1}{2}\left(z + \frac{1}{z}\right)$ is a conformal map from the half-disc $\{z = x + iy \,\|\, z \,| < 1, y > 0\}$ to the upper half-plane. **[5]**

c) Let V and U be open sets in $\mathbb{C}$ and $F : V \to U$ a holomorphic, Function. If $u : U \to \mathbb{C}$ is a harmonic function, then prove that $u \circ F$ is harmonic on V. **[4]**

Q4) a) If f is an automorphism of the disc, then prove that there exist $\theta \in \mathbb{R}$ and $\alpha \in \mathbb{D}$ such that $f(z) = e^{i\theta} \frac{\alpha - z}{1 - \bar{\alpha} z}$ **[5]**

b) Show that if $f : D(O,R) \to \mathbb{C}$ is holomorphic, with $|f(z)| \le M$ for some $M > O$, then $\left| \frac{f(z) - f(o)}{M^2 - \overline{f(o)} f(z)} \right| \le \frac{|z|}{MR}$ **[5]**

c) if $f : \mathbb{D} \to \mathbb{D}$ is analytic and has two distinct fixed points, then f is the identity function. **[4]**

Q5) a) If Ω is a connected open subset of $\mathbb{C}$ and $\{f_n\}$ a sequence of injective holomorphic functions on Ω that converges uniformly on every compact subset of Ω to a holomorphic function f, then prove that f is either injective or constant. **[5]**

b) Prove that the complex plane slit along the union of the rays $\bigcup_{K=1}^{n} \{A_k + iy \mid y \le 0\}$ is simply connected. **[5]**

c) i) Define uniformly bounded family $\mathcal{F}$ on compact subsets of Ω. **[2]**

ii) Define equicontinuous family $\mathcal{F}$ on a compact set K. **[2]**

Q6) a) Suppose $S(z)=\int_0^z \frac{d\mathscr{E}}{(\mathscr{E}-A_1)^{\beta_1}....(\mathscr{E}-A_n)^{\beta_n}}$

where $A_1 < A_2 < ..... < A_n$ are n distinct points on the real axis arranged in increasing order and the exponents β_k will be assumed to satisfy the conditions $\beta_k<1$ for each K and $1<\sum_{K=1}^{n}\beta_K$. If $\sum_{K=1}^{n}\beta_K=2$ and P denotes the polygon whose vertices are given by a_1,----, a_n, then prove that S maps the real axis onto $P-\{a_\infty\}$. Also prove that the point a_∞ lies on the segment $[a_n,a_1]$ and is the image of the point at infinity and the angle at the vertex q_k is $\alpha_K\pi$ where $\alpha_k=1-\beta_k$. **[7]**

b) Prove that the conformal map F extends to a continuous function from the closure of the disc to the closure of the polygon. **[5]**

c) Define the elliptic integrals. **[2]**

Q7) a) For each 0 < r < ½, let C_r denote the circle centered at Z_0 of radius r. suppose that for all sufficiently small r we are given two points z_r and z'_r in the unit disc that also lie on C_r. If we let ϱ (r) = | f (z_r) – f (z'_r) |, then prove that there exists a sequence $\{r_n\}$ of radii that tends to zero, and such that $\lim_{n\to\infty}\varrho(r_n)=0$ **[7]**

b) If F is a conformal map from the upper half-plane to the polygonal region p and maps the points A_1,---, A_{n-1}, ∞ to the vertices of P then prove that there exist constant C_1 and C_2 such that

$$F(z)=C_1\int_0^z \frac{d\mathscr{E}}{(\mathscr{E}-A_1)^{B_1}---(\mathscr{E}-A_{n-1})^{B_{n-1}}}+C_2$$ **[7]**

Q8) a) Prove that the weierstrass p function is an elliptic function that has periods 1 and T, and double Poles at the lattice points. **[7]**

b) Prove that every even elliptic function F with periods 1 and T is a rational function of weierstrass P function. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3124

[Total No. of Pages : 3

[5907]-37

M.A./M.Sc. - II

MATHEMATICS

MTUTO - 137 : Integral Equations

(2019 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Derive an equivalent volterra integral equation to following initial value problem. **[5]**

$$y'' - \sin x\, y' + e^x y = x, \;\; y(0) = 1, \;\; y'(0) = -1$$

b) Verify $u(x) = \sin x$ is a solution of the integral equation **[5]**

$$u(x) = x - \int_0^x (x-t)u(t)\, dt.$$

c) Convert the volterra integral equation $u(x) = 1 + x + \int_0^x (x-t)^2 u(t)\, dt$ to an equivalent initial value problem. **[4]**

Q2) a) Solve the following Fredholm integral equation by using the Adomian decomposition method $u(x) = x + \cos x - 2x\int_0^{\frac{\pi}{6}} u(t)\, dt$. **[4]**

b) Solve the following Fredholm integral equation by using the direct computation method $u(x) = xe^x - x + \int_0^1 x\; u(t)\, dt$. **[5]**

c) Find the non trival solution for following Homogeneous Fredholm integral equation by using the eigen value and eigen function **[5]**

$$u(x) = \lambda \int_0^{\frac{\pi}{2}} \cos x \cdot \sin t \cdot u(t)\, dt \cdot$$

P.T.O.

Q3) a) Solve the volterra integral equation $u(x)=1+2\sin x-\int_0^x u(t)\,dt$ by using series solution method. **[5]**

b) Solve the following volterra integral equation by converting to equivalent initial value problem $u(x)=1-3\int_0^x u(t)\,dt$. **[4]**

c) Solve the volterra equation $u(x)=1-\int_0^x 2t\,u(t)\,dt$ by the successive approximation method. **[5]**

Q4) a) Solve the volterra integral equation of the first kind $xe^{-x}=\int_0^x e^{t-x}\,u(t)\,dt$ by using modified decomposition method. **[5]**

b) Solve the Fredholm integro-differential equation $u'(x)=\frac{x}{21}-\int_0^1 xt\,u(t)\,dt$, $u(0)=\frac{1}{6}$ by using the direct computation method. **[4]**

c) Solve the Fredholm integro-differential equation **[5]**

$u'(x)=1-\frac{x}{3}+\int_0^1 xt\,u(t)\,dt,\ u(0)=0$ by using the Adomian decomposition method.

Q5) a) Solve the volterra integro-differential equation **[4]**

$u'(x)=2\cos x-\frac{x^2}{2}+\int_0^x u(t)\,dt,\ u(0)=0$ by using the variational iteration method.

b) Solve the following volterra-differential equation by converting the problem to volterra integral equation **[5]**

$$u''(x)=1+\int_0^x (x-t)\,u(t)\,dt,\ u(0)=1,\ u'(0)=0$$

c) Solve the volterra integro differential equation **[5]**

$u'(x)=e^x-\int_0^x u(t)\,dt,\ u(0)=1$ by converting the problem to an initial value problem.

Q6) a) Solve the following Abel's integral equation $\pi(x+1)=\int_0^x \frac{1}{\sqrt{x-t}}\, u(t)\ dt$. **[7]**

b) Solve the following weakly-singular volterra integral equation of the second kind $u(x)=\sqrt{x}-\pi x+2\int_0^x \frac{1}{\sqrt{x-t}}\, u(t)\ dt$. **[7]**

Q7) a) Verify $u(x)=x+\frac{1}{24}$ is a solution of $u(x)=x+\int_0^{\frac{1}{4}} u(t)\ dt$. **[4]**

b) Derive the equivalent Fredholm integral equation of the boundary value problem. **[5]**

$y''+4y=\sin x,\ 0<x<1,\ y(0)=y(1)=0$

c) Solve the following volterra integral equation $u(x)=1-\int_0^x u(t)\ dt$. **[5]**

Q8) a) Use the variation iteration method to solve the Fredholm integro-differential equation. **[7]**

$u'(x)=2-\sin x+\int_0^{\pi} t\ u(t)\ dt,\ u(0)=1$

b) Solve the following volterra integral equation by the successive substitution method $u(x)=x+\int_0^x u(t)\ dt$. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3125

[Total No. of Pages : 3

[5907]-38

M.A./M.Sc. (Mathematics)

MTUTO - 138 : DIFFERENTIAL MANIFOLDS

(2019 CBCS Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $g : A \to B$ be a diffeomorphism of open sets in $\mathbb{R}^k$. Let $\beta : B \to \mathbb{R}^n$ be a map of class C^r, Let $Y = \beta(B)$. Let $\alpha = \beta \circ g$; then $\alpha : A \to \mathbb{R}^n$ and $Y = \alpha(A)$. If $f : Y \to \mathbb{R}$ is a continuous function, then prove that f is integrable over Y_β if and only if it is integrable over Y_α; in this case

$$\int_{Y_\alpha} f\, dv = \int_{Y_\beta} f\, dv$$ **[5]**

b) Let W be a linear subspace of $\mathbb{R}^n$ of dimension k. Then prove that there is an orthogonal transformation $h : \mathbb{R}^n \to \mathbb{R}^n$ that carries W onto the subspace $\mathbb{R}^k \times O$ of $\mathbb{R}^n$. **[5]**

c) Let $x_1, x_2, \ldots\ldots, x_k$ be vectors in $\mathbb{R}^n$. Show that $V(x_1, x_2, \ldots\ldots, \lambda xi, \ldots. x_k) = |\lambda| V(x_1, x_2, \ldots\ldots, x_k)$. **[4]**

Q2) a) Let M be a k-manifold in $\mathbb{R}^n$, of class C^r. If ∂M is non-empty, then prove that ∂M is a k-1 manifold without boundary in $\mathbb{R}^n$ of class C^r. **[5]**

b) Let A be open in $\mathbb{R}^k$; let $\alpha : A \to \mathbb{R}^n$ be of class C^r; let $Y = \alpha(A)$. Suppose $h : \mathbb{R}^n \to \mathbb{R}^n$ is an isometry ; Let $z = h(Y)$ and Let $\beta = h \circ \alpha$. Show that Y_α and Z_β have the same volume. **[5]**

c) Show that if $T : V \to W$ is a linear transformation and if $f \in A^k(W)$, then

$T^*f \in A^k(V)$. **[4]**

Q3) a) Let M be a manifold in $\mathbb{R}^n$, and let $\alpha : U \to V$ be a coordinate patch on M. If U_0 is a subset of U that is open in U, then prove that the restriction of α to U_0 is also a co-ordinate patch on M. **[5]**

b) Prove that if the support of f can be covered by a single co-ordinate patch, the integral $\int_M f\ dv$ is well-defined, independent of the choice of co-ordinate patch. **[5]**

c) Let M be a compact k-manifold in $\mathbb{R}^n$. Let $h : \mathbb{R}^n \to \mathbb{R}^n$ be an isometry; Let N=h(M). Let $f : N \to \mathbb{R}$ be a continuous function. Show that N is a k-Manifold in $\mathbb{R}^n$, and $\int_N f\ dv = \int_M (foh)\, dv$. **[4]**

Q4) a) Determine which of the following are tensors on $\mathbb{R}^4$, and express those that are in terms of the elementary tensors on $\mathbb{R}^4$. **[7]**

$$f(x, y) = 3x_1y_2 + 5x_2x_3$$
$$g(x, y) = x_1y_2 + x_2y_4 + 1$$
$$h(x, y) = x_1y_1 - 7x_2y_3$$

b) Let k > 1, If M is an orientable k - manifold with non-empty boundary, then prove that ∂M is orientable. **[7]**

Q5) a) Let f be a k-tensor on V; Let $\sigma, \tau \in S_k$. Prove that the tensor f is alternating if and only if $f^\sigma = (\text{sgn}\sigma) f$ for all σ. If f is alternating and if $V_p = V_q$ with $p \neq q$, then prove that $f(V_1, V_2, \ldots, V_k) = 0$. **[5]**

b) Let W be a k-form on the open set A of $\mathbb{R}^n$. Then prove that W is of class C^r if and only if its component functions b, are of class C^r on A. **[5]**

c) Is $f(x,y) = x_1y_2 - x_2y_1 + x_1y_1$ alternating tensors in $\mathbb{R}^4$? Why? **[4]**

Q6) a) Let f be a k-tensor on Vi let $\sigma, \tau \in S_k$ prove that the transformation $f \mapsto f^\sigma$ is a linear transformation of $L^k(V)$ to $L^k(V)$. Also show that it has the property that for all σ, τ, $(f^\sigma)^\tau = f^{\tau o \sigma}$. **[5]**

b) If $\sigma \in S_k$, then prove that σ equals a composite of elementary permutation. **[5]**

c) Let $\alpha : \mathbb{R} \to \mathbb{R}^2$ be the map $\alpha(x) = (x, x^2)$; let M be the image set of α. Show that M is a 1 - manifold in $\mathbb{R}^2$ covered by the single coordinate patch α. **[4]**

Q7) a) Consider the forms **[5]**

$w = xydx + zdy - yzdz,$

$n = xdx - yz^2dy + 2xdz,$

in $\mathbb{R}^3$, verify by direct computation that $d(dw) = 0$ and

$d(w \wedge n) = (dw) \wedge n - w \wedge dn.$

b) Let $A = \mathbb{R}^2 - 0$ consider the 1-form in A defined the equation **[5]**

$w = (xdx + ydy) / (x^2+y^2)$

Show that w is closed, also show that w is exact on A.

c) Let $r : \mathbb{R} \to \mathbb{R}^n$ be of class C^r. Show that the velocity of r corresponding to the parameter value t is the vector $r_*(t\ ;\ e_1)$. **[4]**

Q8) a) Let A be open in $\mathbb{R}^k$, Let $\alpha : A \to \mathbb{R}^n$ be of class C^∞. If w is an l - form defined in an open set of $\mathbb{R}^n$ containing $\alpha(A)$, then prove that $\alpha^*(dw) = d\ (\alpha^* w)$. **[7]**

b) State and prove stoke's theorem. **[7]**

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Total No. of Questions : 8]

SEAT No. :

PA-3126

[Total No. of Pages : 4

[5907]-41

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MTUT141 : Fourier Series and Boundary Value Problems

(2019 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Suppose that a function g(u) is piecewise continuous on the interval $0 < u < \pi$ and that the right - hand derivative $g_R'(0)$ exists. Then prove that

$$\lim_{N\to\infty}\int_0^{\pi} g(u)D_N(u)du = \frac{\pi}{2}g(0,+)$$

Where $D_N(u)$ is Dirichlet Kernel **[6]**

b) Find the Fourier series for function **[5]**

$$f(x)=\begin{cases} \dfrac{-\pi}{2} & \text{when } -\pi<x<0 \\ \dfrac{\pi}{2} & \text{when } 0<x<\pi \end{cases}$$

c) Find the Fourier sine series for the function $f(x) = x$ $(0 < x < \pi)$. **[3]**

Q2) a) If $f \in C_P(0, \pi)$, then prove that the fourier sine series coefficient b_n tends to zero as n tends to infinity. **[7]**

b) Find the Fourier cosine series for the function $f(x) = \pi - x$ $(0 < x < \pi)$ **[5]**

P.T.O.

c) Show that the function defined by the equations. **[2]**

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & \text{when } x\neq 0 \\ 0 & \text{when } x=0 \end{cases}$$

is continuous at x = 0 but that neither $f_R'(0)$ nor $f_L'(0)$ exists.

Q3) a) Let f denote a function such that **[6]**

i) f is continuous on the interval $-\pi \leq x \leq \pi$

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f′ is piecewise continuous on the interval $-\pi < x < \pi$.

Prove that the Fourier series

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$ for F, with coefficients

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

converges absolutely & uniformly to F(x) on the interval $-\pi \leq x \leq \pi$.

b) Find the Fourier series for the function **[5]**

$$F(x)=\begin{cases} 0 & \text{when } -\pi < x \leq 0 \\ x & \text{when } 0 < x < \pi \end{cases}$$

c) If $f(x)=\sqrt[3]{x}$ $(-\pi < x < \pi)$, then show that $f(x)$ is piecewise continuous on the interval $-\pi < x < \pi$ but $f'(0+)$ and $f'(0-)$ does not exists. **[3]**

Q4) a) Solve the following linear boundary value problem. **[7]**

$u_t(x, t) = \mathrm{K}u_{xx}(x, t) \quad (0 < x < \mathrm{C}, t > 0)$

$u_x(0, t) = 0, \quad u_x(\mathrm{C}, t) = 0 \ (t > 0)$

$u(x, 0) = f(x)$

b) Solve the following boundary value problem. **[7]**

$y_{tt}(x, t) = a^2 y_{xx}(x, t) \quad (0 < x < \mathrm{C}, t > 0)$

$y(0, t) = 0, \quad y(\mathrm{C}, t) = 0, y_t(x, 0) = 0,$

$y(x, 0) = f(x)$

Q5) a) Solve the following boundary value problem. **[7]**

$\rho^2 \mathrm{u}_{\rho\rho}(\rho, \phi) + \rho u_\rho(\rho, \phi) + u_{\phi\phi}(\rho, \phi) = 0 \ (0 < \rho < a) \ (0 < \phi < \pi)$

satisfying homogeneous conditions.

$\mathrm{u}_\phi(\rho, 0) = 0, \quad \mathrm{u}_\phi(\rho, \pi) = 0 \ (0 < \rho < a)$

non homogeneous condition

$u(a, \phi) = f(\phi) \ (0 < \phi < \pi)$

b) Solve the following boundary value problem. **[7]**

$u_t(x, t) = \mathrm{K}u_{xx}(x, t) + \mathrm{g(t)} \quad (0 < x < \pi, t > 0)$

$u(0, t) = 0, \quad u(\pi, t) = 0$

and $u(x, 0) = f(x)$

Q6) a) The boundary value problem **[6]**

$y_{tt}(x, t) = y_{xx}(x, t) + \mathrm{A}\sin wt \quad (0 < x < 1, t > 0)$

when A is constant

$y(0, t) = 0, y(1, t) = 0, y(x, 0) = 0 \ y_t(x, 0) = 0$

describes transeverse displacement in a stretched string. Show that resonance occurs when w has one of the values. $w_n = (2n - 1)\pi$ (n = 1, 2, ...)

b) Let C_n ($n = 1, 2, 3, ...$) be the Fourier constants for a function f in $C_p(a, b)$ with respect to an orthogonal set $\{\phi_n(x)\}$ ($n = 1, 2, 3, ...$) in that space. Then prove that all possible linear combination of the function $\phi_1(x)$, $\phi_2(x)$, ... $\phi_N(x)$ the combination $C_1\phi_1(x) + C_2\phi_2(x) + ... + C_N\phi_N(x)$ is the best approximation in the mean to $f(x)$ on the fundamental interval $a < x < b$. **[5]**

c) Find the eigen values and normalized eigen function of sturm - liouville problem. **[3]**

$X''(x) + \lambda X(x) = 0,\ X(0) = 0,\ X(1) - X'(1) = 0$

Q7) a) If C_n ($n = 1, 2, ...$) are the fourier constants for a function f in $C_p(a, b)$ with respect to an orthonormal set in that space, then prove that $\lim\limits_{n\to\infty} C_n = 0$. **[7]**

b) Show that the function $\psi_1(x) = 1$ and $\psi_2(x) = x$ are orthogonal on the interval $-1 < x < 1$ and determine constants A and B such that the function $\psi_3(x) = 1 + Ax + Bx^2$ is orthogonal to both $\psi_1(x)$ and $\psi_2(x)$ on the interval. **[5]**

c) If $L = x$ and $M = \dfrac{\partial}{\partial x}$ are linear operators on $C_p(a, b)$ then show that the product LM and ML are not always same. **[2]**

Q8) a) Let λ be an eigen value of the regular Sturm - Liouville problem. **[7]**

$(rX')' + (g + \lambda P)X = 0 \quad (a < x < b)$

$a_1X(a) + a_2\, X'(a) = 0$

$b_1X(b) + b_2\, X'(b) = 0$

If the condition's $g(x) \le 0$ ($a \le x \le b$) and $a_1a_2 \le 0$, $b_1b_2 \ge 0$ are satisfied, then prove that $\lambda \ge 0$.

b) Find eigen values and normalized eigenfunction of sturm - Liouville problem. **[5]**

$X''(x) + \lambda X(x) = 0,\ X(0) = 0,\ X'(1) = 0$

c) If m and n are positive integers, then show that **[2]**

$$\int_0^{\pi} \cos(mx)\cos(nx)\,dx = \begin{cases} 0 & \text{when } m \ne n \\ \dfrac{\pi}{2} & \text{when } m = n \end{cases}$$

Total No. of Questions : 8]

SEAT No. :

PA-3127

[Total No. of Pages : 4

[5907] - 42
M.A./M.Sc.
MATHEMATICS
MTUT - 142 : Differential Geometry
(2019 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and Prove Lagranges Multiplier Theorem. **[7]**

b) Let S be the hyperboloid $-x_1^2+x_2^2+x_3^2=1$ oriented by the unit normal vector field $\vec{N}(p)=\left(p,\frac{-x_1}{\|p\|},\frac{x_2}{\|p\|},\frac{x_3}{\|p\|}\right)$, $p=(x_1,x_2,x_3)\in S$ then find normal curvature of S at P = (0, 0, 1). **[5]**

c) Define terms : **[2]**

i) Level Curve.

ii) Graph of function.

Q2) a) Let S be an n-surface in R^{n+1}, let $p,q,\in S$, and let α be a piecewise smooth parametrized curve from p to q. Prove that parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product. **[7]**

b) Define complete vector field. Determine whether vector field $\vec{X}(x_1,x_2)=(x_1,x_2,1,0)$, on $U=R^2-\{(0,0)\}$ is complete? **[5]**

P.T.O.

c) Let $\vec{X}$ and $\vec{Y}$ be smooth vector fields. Prove that $\left(\vec{X} \dot{+} \vec{Y}\right) = \dot{\vec{X}} + \dot{\vec{Y}}$. **[2]**

Q3) a) Let $S \subset R^{n+1}$ be a connected n-surface in R^{n+1}. Prove that on S there exist exactly two smooth unit normal vector fields $\vec{N}_1$ and $\vec{N}_2$ with $\vec{N}_2(p) = -\vec{N}_1(p)$ for all $p \in S$. **[7]**

b) Find the integral curve through $p = (1, 0)$ of the vector field $\vec{X}(p) = \left(p, X(p)\right)$ where $X(x_1, x_2) = (-x_2, x_1)$. **[5]**

c) Sketch Vector Field on R^2: **[2]**

$\vec{X}(p) = (p, X(p))$ where $X(p) = (0,1)$.

Q4) a) Let C be a connected oriented plane curve and $\beta : I \longrightarrow C$ be unit speed parametrization of C. Prove that β is either one to one or periodic. Also prove that β is periodic if and only if C is compact. **[7]**

b) Define normal curvature of surface. Let S be sphere $x_1^2 + --- + x_{n+1}^2 = r^2$ of radius r oriented by inward normal $\vec{N}(p) = \left(p, \dfrac{-p}{\|p\|}\right)$. Then find normal curvature of S at $p \in S$ in direction $\vec{v} \in Sp$. **[5]**

c) Show that geodesic have constant speed. **[2]**

Q5) a) Show that gradient of f at $p \in {}^{f^{-1}}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p. **[5]**

b) Let V be a finite dimensional vector space with dot product and let $L : V \to V$ be a self-adjoint linear transformation on V. Let $S = \{v \in V : v.v = 1\}$ and define $f : S \to R$ by $f(v) = L(v).v$. Suppose f is stationary at $v_0 \in S$. Then show that $L(v_0) = f(v_0)v_0$. **[5]**

c) Let $\alpha(t)=\left(x(t), y(t)\right)(t \in \mathrm{I})$ be a local parametrization of oriented plane curve C. Show that

$$k o \alpha=\frac{\left(x' y''-y' x''\right)}{\left(x'^{2}+y'^{2}\right)^{3/2}} .$$ **[4]**

Q6) a) Let S be an n-surface in R^{n+1}, oriented by unit normal vector field $\vec{\mathrm{N}}$. Let $p \in \mathrm{S}$ and $\vec{v} \in \mathrm{S}_p$. Then show that for every parametrized curve $\alpha: \mathrm{I} \rightarrow \mathrm{S}$ with $\alpha:(t_0)=\vec{v}$ for some $t_0 \in \mathrm{I}$, $\ddot{\alpha}(t_0) . \vec{\mathrm{N}}(p)=\mathrm{L} p(\vec{v}) . \vec{v}$ · **[5]**

b) Under what condition n-plane. $a_1 x_1+---+a_{n+1} x_{n+1}=b$ is surface. **[5]**

c) Show that for each a, b, c, d $\in$ R the parametrized curve

$\alpha(t)=(\cos(at+b), \sin(at+b),(t+d)$ is geodesic in the cylinder $x_1^2+x_2^2=1$ in R^3. **[4]**

Q7) a) Let $\vec{\mathrm{X}}$ be a smooth vector field on an open set $\mathrm{U} \subset \mathrm{R}^{n+1}$ and let $p \in \mathrm{U}$. Prove that there exists an open interval I containing 0 and an integral curve $\alpha: \mathrm{I} \rightarrow \mathrm{U}$ of $\vec{\mathrm{X}}$ such that **[7]**

i) $\alpha(0)=p$.

ii) If $\beta: \tilde{\mathrm{I}} \rightarrow \mathrm{U}$ is any other integral curve of $\vec{\mathrm{X}}$ with $\beta(0)=p$ then show that $\tilde{\mathrm{I}} \subset \mathrm{I}$ and $\beta(t)=\alpha(t)$ for all $t \in \tilde{\mathrm{I}}$.

b) Find global parametrization of circle C $\left(x_1-a\right)^2+\left(x_2-b\right)^2=r^2$. Also find curvature k for C oriented by outward normal. **[7]**

Q8) a) Let S be an oriented n-surface in R^{n+1} and let $\vec{v}$ be unit vector in Sp, $p \in \mathrm{S}$. Show that there exists an open set $\mathrm{V} \subset \mathrm{R}^{n+1}$ containing p such that $\mathrm{S} \cap \mathrm{N}(\vec{v}) \cap \mathrm{V}$ is plane curve. Also show that curvature at p of this curve is equal to normal curvature $k(\vec{v})$. **[7]**

b) Describe the spherical image of given 1-surface oriented by $\dfrac{\nabla f}{\|\nabla f\|}$ where f is function associated to given surface-cone $-x_1^2 + x_2^2 = 0 \;\; x_1 > 0$. **[7]**

Total No. of Questions : 4]

SEAT No. :

PA-3128

[Total No. of Pages : 1

[5907]-43
M.A./M.Sc.
MATHEMATICS
MTUT - 143 : Introduction to Data Science
(2019 Pattern) (Semester - IV)

Time : 2 Hours] *[Max. Marks : 35*

Instructions to the candidates :

1) ***Question 1 is compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Attempt any two questions from Q.2, 3 and 4.***

Q1) What is Data Science? Describe all steps involved in Data Science. **[7]**

Q2) a) Give any five facets of data. **[5]**

b) Explain in detail process of data cleansing. **[5]**

c) State steps involved in data exploration. **[4]**

Q3) a) What is machine learning? Describe types of machine learning in detail. **[5]**

b) What is Spark? Give it's four components. **[5]**

c) Write a short note on confusion matrix. **[4]**

Q4) a) Describe Hadoop and it's components. **[5]**

b) What is text mining? State techniques to handle text mining. **[5]**

c) State four Python packages that are used for text mining. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3129

[Total No. of Pages : 3

[5907]-44

M.A. / M.Sc.

MATHEMATICS

MTUTO-144 : Number Theory

(2019 Pattern) (Semester - IV)

Time : 3 Hours] ***[Max. Marks : 70***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicates full marks.***

Q1) a) If an irreducible polynomial $p(x)$ divides a product $f(x)\, g(x)$ then prove that $p(x)$ divides at least one of the polynomials $f(x)$ and $g(x)$. **[5]**

b) Let a, b, and $m > 0$ be given integers, and $d = (a, m)$. Then prove that the congruence $ax \equiv b$ (mod m) has a solution if and only if d/b. If x_0 is a solution then prove that the other solutions are given by

$$x_0 + \frac{m}{d},\ x_0 + \frac{2m}{d},\ ,............,\ x_0 + (d-1)\frac{m}{d}.$$ **[7]**

c) Find the remainder obtained by dividing 1! + 2! + 3! + ...+ 100! by 12. **[2]**

Q2) a) Prove that the product of two primitive polynomials is primitive. **[8]**

b) Find all integers that satisfy simultaneously : **[6]**

$x \equiv 3$ (mod 4)

$x \equiv 4$ (mod 5)

$x \equiv 6$ (mod 7)

P.T.O.

Q3) a) Find units in $\mathbb{Z}[i]$, the ring of Gaussian integers. **[5]**

b) Let p be a prime. Then for any nonzero integers a, b prove that $\text{Ord}_p\, ab = \text{Ord}_p\, a + \text{Ord}_p\, b$. **[5]**

c) For all odd integers n, show that $8|n^2 - 1$ and also show that if $3|n$ then $6|n^2-1$. **[4]**

Q4) a) State and prove fermats little theorem. **[5]**

b) If p is an odd prime then prove the following : **[5]**

i) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right)=\left(\frac{ab}{p}\right)$

ii) $\left(\frac{a^2}{p}\right)=1$ provided $(a, p) = 1$.

c) State wilson's theorem and using it show that $p|((p-2)! -1)$ where p is a prime. **[4]**

Q5) a) Let $a, b, m \in \mathbb{Z}$ and $m \neq 0$. Then prove the following : **[4]**

i) If $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$ then $ab \equiv bd \pmod{m}$.

ii) if $a \equiv c \pmod{m}$ and $d|m$, $d > 0$ then $a \equiv c \pmod{d}$

b) Let $\mu(n)$ be the Möbius mu function then prove that $\mu(n)$ is multiplicative and $\sum_{d/n}\mu(d)=\begin{cases}1 & if\ n=1\\ 0 & if\ n>1\end{cases}$. **[6]**

c) Find $\left(\frac{-42}{61}\right)$. **[4]**

Q6) a) If x any y are real numbers then prove that : **[6]**

i) $[x + m] = [x] + m$, if m is an integer.

ii) $\left[\frac{[x]}{m}\right]=\left[\frac{x}{m}\right]$, if m is a positive integer.

b) Check following congruences are solvable or not, justify your answer. **[4]**

i) $x^2 \equiv 5 \pmod{227}$

ii) $x^2 \equiv 5 \pmod{229}$

c) For any positive integer n, prove that $\underset{d/n}{\pi} d = n^{\frac{d(n)}{2}}$. **[4]**

Q7) a) Let $f(n)$ be a multiplicative function and $f(n) = \sum_{d/n} f(d)$. Then prove that $f(n)$ is multiplicative. **[5]**

b) If a, b and c are integers such that $a|bc$ and a, b both are relatively prime then prove that $a|c$. **[4]**

c) Prove that if p is an odd prime then $x^2 \equiv 2 \pmod{p}$ has solutions if and only if $p \equiv 1$ or $7 \pmod{8}$. **[5]**

Q8) a) If α and β are algebraic numbers then prove that $\alpha + \beta$ and $\alpha\beta$ are also algebraic numbers. **[7]**

b) Find highest power of 7 that divides 1000!. **[4]**

c) Find $d(12)$, $\sigma(12)$ and $\Omega(12)$. **[3]**

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Total No. of Questions : 8]

SEAT No. :

PA-3130

[Total No. of Pages : 2

[5907]-45
S.Y. M.A./M.Sc.
MATHEMATICS
MTUTO 145 : Algebraic Topology
(2019 Pattern) (Semester - IV) (CBCS)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define homotopy relation between two continuous function. Prove that this relation is an equivalence relation. **[5]**

b) Show that if y is contractable, then for any x, the set $[x, y]$ has single element. **[5]**

c) Find the star convex set that is not convex. **[4]**

Q2) a) Prove that in a simply connected space x any two paths having the same initial and final points are path homotopic. **[5]**

b) Define a covering map. Show that a covering map is a Local homeomorphism. **[5]**

c) Give an example of non identity covering map from S' on to S'. **[4]**

Q3) a) If $P : E \to B$ and $P' : E' \to B'$ are covering map then

$P \times P'$; $E \times E' \to B \times B'$ is covering map. **[5]**

b) Let $q : x \to y$ and $r : y \to z$ be covering maps, Let $p = \mathrm{ro}q$, show that $r^{-1}(z)$ is finite for each $z \in Z$ then p is covering map. **[5]**

c) Compute the fundamental group of the "solid torus" $S^1 \times B^2$ and the product space $S^1 \times S^2$. **[4]**

P.T.O.

Q4) a) Show that $\mathbb{R}$ and $\mathbb{R}^n$ are not homeomorphic if $n > 1$. **[5]**

b) Let x be the union of two copies of S^2 having a single point is common, what is fundamental group of x. **[5]**

c) State seifert-van kampen Theorem. **[4]**

Q5) a) State and prove fundamental theorem of algebra. **[10]**

b) State and prove Borsuk-ulam theorem for S^2. **[4]**

Q6) a) Let $f : x \rightarrow y$ be a continous, let $f(x_0) = y_0$. If f is a homotopy equivalence then $f_* : \pi_1(x, x_0) \rightarrow \pi_1(y, y_0)$ is an isomorphism. **[5]**

b) Show that if $G = G_1 \oplus G_2$, where G_1 and G_2 are cyclic of order m and n resp. then m and n are not uniquely determined by G in general. **[5]**

c) Let x be the wedge of circle s_α for $\alpha \in J$, then x is normal. **[4]**

Q7) a) Let $\pi : E \rightarrow X$ be a closed quotient map. If E is normal then so X. **[6]**

b) Show that if x is an infinite wedge of circles, then x does not satify the first countability axiom. **[4]**

c) Prove that fundamental group of the torus has a presentation consisting of two generators α, β and a single relation $\alpha\beta\alpha^{-1}\beta^{-1}$. **[4]**

Q8) a) Show that if $n > 1$, every continuous map $f : S^n \rightarrow S'$ is nullhomotopic. **[5]**

b) Find a continuous map of the torus into S' that is not nullhomotopic. **[4]**

c) Let $P : E \rightarrow B$ be a covering map, with E simply connected show that Given any covering map $r : Y \rightarrow B$, there is covering map $q : E \rightarrow Y$ such that roq $= p$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

PA-3131

[Total No. of Pages : 3

[5907]-46

M.A. / M.Sc.

MATHEMATICS

MTUTO-146 : Representation Theory of Finite Groups

(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***

2) ***Figures to the right indicate full marks.***

Q1) a) Define following terms : **[4]**

i) Co ordinate vector

ii) Unitary operator

b) State and prove Cauchy - Schwartz inequality. **[6]**

c) Define $\phi : \frac{Z}{nz} \to GL_2(c)$ by $\phi(m) = \begin{bmatrix} \cos\left(\frac{2\pi m}{n}\right) & -\sin\left(\frac{2\pi m}{n}\right) \\ \sin\left(\frac{2\pi m}{n}\right) & \cos\left(\frac{2\pi m}{n}\right) \end{bmatrix}$ Which is matrix for rotation by $\frac{2\pi m}{n}$ and $\psi : Z/nz \to GL_2(c)$ by $\psi(m) = \begin{bmatrix} e^{\frac{2\pi mi}{n}} & 0 \\ 0 & e^{\frac{-2\pi mi}{n}} \end{bmatrix}$. Then show that $\phi \sim \psi$. **[4]**

Q2) a) Prove that every representation of a finite group G is equivalent to a unitary representation. **[6]**

b) Define $\phi : Z \to GL_2(c)$ by $\phi(n) = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ show that this representation is in decomposable. **[4]**

c) State and prove schur orthogonality relations. **[4]**

Q3) a) Let $T : V \to W$ be in Hom $G(\phi, e)$, then prove that ker T is a G variant sub space of V and T(V) = Im T is a G - invariant subspace of W. **[5]**

b) Let G be the abelian group. Then prove that any irreducible representation of G has degree one. **[5]**

c) The set $B = \{\delta \mid c \in Cl(G)\}$ is a basis for $Z\{L(G)\}$. Prove that dim $Z\{L(G)\} = |Cl(G)|$. **[4]**

Q4) a) Prove that there are atmost $|Cl(G)|$ equivalence classes of irreducible representation of G. **[5]**

b) Let $\phi = \rho \oplus \psi$. Then prove that $\chi_\phi = \chi_\rho + \chi_\psi$. **[5]**

c) Define following terms : **[4]**

i) Regular representation

ii) Character table

Q5) a) For $g, h \in G$, prove that $\delta_g * \delta_h = \delta_{gh}$. **[4]**

b) Prove that $f : G \to C$ is a class function if and only if $a * f = f * a$ for all $a \in L(G)$. **[5]**

c) Prove that the Fourier transform satisfies $\widehat{a * b} = \hat{a} \cdot \hat{b}$. **[5]**

Q6) a) Write standard basic for C^n. **[2]**

b) Define an inner product. **[2]**

c) State the properties of determinant. **[2]**

d) Let $\phi : G \to G(L(V))$ be equivalent to a decomposable representation. Then prove that ϕ is decomposable. **[8]**

Q7) a) Write an example of an irreducible representation. **[5]**

b) Show that Z(L(G)) is subspace of L(G). **[5]**

c) If ϕ and ρ are equivalent representations then prove that $\chi_\phi = \chi_\rho$. **[4]**

Q8) a) Define dual group $\hat{G}$ and product on $\hat{G}$ via point wise multiplication. **[4]**

b) Let G be a finite abelian group. Define $\eta = G \rightarrow \hat{\hat{G}}$ by $\eta(g(\chi) = \chi(g)$. prove that η is an isomorphism. **[5]**

c) Verify that L(G) satisfies distribution laws. **[5]**

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Total No. of Questions : 8]

SEAT No. :

PA-3132

[Total No. of Pages : 3

[5907]-47
M.A./M.Sc
MATHEMATICS
MTUTO 147 : Coding Theory
(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicates full marks.*

Q1) a) Let C be an [n,k,d] - linear code over finite field F_q then prove that **[5]**

i) For all $u \in F_q^n$, $|C + u| = |C| = q^k$

ii) There are q^{n-k} different cosets of C.

b) Let $x^6-1 = (1 + x)^2 (1 + x + x^2)^2 \in F_2[x]$. Then find number of cyclic codes in F_2^6. Also find cyclic code generated by $(1 + x + x^2)^2$. **[5]**

c) If C = {000, 111, 011, 100} then find extended code $\overline{C}$ and its distance $d(\overline{C})$. Where $C \subseteq F_2^3$. **[4]**

Q2) a) If code $C = \langle S \rangle$ with S = {12101, 20110, 01122, 11010} $\subseteq F_3^5$ then find basis of C. **[5]**

b) For an integer $q > 1$ and integers n, d such that $1 \le d \le n$. Prove that

$$Aq(n,d) \le \frac{q^n}{\sum_{i=0}^{\left[\frac{d-1}{2}\right]} \binom{n}{i} (q-1)^i}$$

where $[x]$ denotes greatest integer less than or equal to x. **[5]**

c) Show that distance of binary Hamming codes is 3. **[4]**

P.T.O.

Q3) a) Let C = {0000, 1011, 0101, 1110} $\subseteq F_2^4$ be a linear code with parity check matrix $H = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$ and coset leaders {0000, 0001, 0010, 1000}. Decode

i) w = 1101

ii) w = 1111

by using syndrome decoding. **[7]**

b) Prove that each monic divisor of $x^n - 1$ is the generator polynomial of some cyclic code in F_q^n. **[5]**

c) List elements of sphere S_A (1100, 3) in F_2^4. **[2]**

Q4) a) Prove that

i) The dimension of a q-ary BCH code of length $q^m - 1$ generated by $g(x) = l\text{cm}\{M^{(a)}(x), M^{(a+1)}(x), ..., M^{(a+\delta-2)}(x)\}$

is independent of the choice of the primitive element α.

ii) A q-ary BCH code of length $q^m - 1$ with designed distance δ has dimension at least $q^m - 1 - m(\delta - 1)$.

[8]

b) Let C and D be two linear codes over F_q of same length. Is CUD a linear code over F_q? Justify. **[3]**

c) Is C = {000, 100, 010, 001} $\subseteq F_2^3$ Cyclic code? Justify. **[3]**

Q5) a) For any $q > 1$, any positive integers n and any integer d such that $1 \leq d \leq n$ prove that $A_q(n, d) \leq q^{n-d+1}$. **[5]**

b) Suppose that codewords from the binary code {000, 100, 111} are being sent over Binary symmetric channel with crossover probability P = 0.03. Use the maximum likelihood decoding rule to decode word w = 010. **[5]**

c) Find information rate and relative minimum distance of repetition code

C = $\{\lambda\ (1,1,......., 1) \mid \lambda \in F_q\}$ of length n. **[4]**

Q6) a) Let I be a non-zero ideal in $F_q[x] \mid (x^n-1)$ and $g(x)$ be a non-zero monic polynomial of the least degree in I then prove that $g(x)$ is a generator of I and it divides x^n-1. **[6]**

b) Find number of distinct bases for vector space $V = \langle S \rangle$ over F_2 where $S = \{0001, 0010, 0100\}$ **[5]**

c) Let C be an $[n,k,d]$ – linear code and H be a parity check matrix for C. For $u,v \in F_q^n$, Prove that $s(u) = s(v)$ if and only if u and v are in same coset of C. **[3]**

Q7) a) Prove that for all $r \geq 0$, a sphere of radius r in A^n contains exactly $V_q^n(r)$ vectors. Where A is an alphabet of size $q > 1$. **[6]**

b) Let C be a binary linear code with parity check matrix **[4]**

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Find $d(c)$

c) Construct the incomplete maximum likelihood decoding table for binary code $C = \{101, 011, 111\}$ **[4]**

Q8) a) Prove that a code C is u-error-detecting if and only if $d(C) \geq u + 1$. **[5]**

b) For $S = \{1020, 0201\} \subseteq F_3^4$, find F_3 – linear span $\langle S \rangle$ and it's compliment $S^{\perp}$. **[5]**

c) For ternary code C = {00122, 12201, 20110, 22000}. Decode

i) 01122

ii) 22022

by using nearest neighbour decoding rule. **[4]**

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Total No. of Questions : 8]

SEAT No. :

PA-3133

[Total No. of Pages : 4

[5907]-48
M.A./M.Sc.
MATHEMATICS
MTUTO-148 : Probability and Statistics
(2019 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 70*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***Use of scientific calculator is allowed.***

Q1) a) Define following terms : **[4]**

i) Probability of an event

ii) Conditional probability of event B given A.

b) An experiment consists of tossing a die and then flipping a coin once if the number on the die is even. If the number on the die is odd, the coin is flipped twice. Using the notation 4H, for example to denote the out come that the die comes up 4 and then the coin comes up Heads and 3HT to denote the outcome that the die comes up 3 followed by head and then tail on the coin. Construct tree diagram to show the 18 elements of the samples. **[5]**

c) Suppose that the error in the reaction temperature in ºC for a controlled laboratory experiment is a continuous random variable X having the probability density function $f(x) = \begin{cases} \dfrac{x^2}{3} & -1 < x < 2 \\ 0 & elsewhere \end{cases}$ **[5]**

i) Verify that $f(x)$ is density function.

ii) Find $p(0 < X \leq 1)$

P.T.O.

Q2) a) Prove that the variance of a random variable X is $\sigma^2 = E(X^2) - \mu^2$. **[4]**

b) Suppose that the number of cars X that pass trough a car wash between 4 : 00 p.m. to 5:00 p.m. on any sunny friday has the following probability distribution.

x	4	5	6	7	8	9
p(X = x)	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

Let g(x) = 2x – 1 represent the amount of money paid to the attendant by the manager. Find the attendant's expected earning for this particular time period. **[5]**

c) A random variable X has mean $\mu = 8$, a variance $\sigma^2 = 9$ and an unknown probability distribution find : **[5]**

i) $P(-4 < x < 20)$

ii) $P|x - 8| \geq 6$.

Q3) a) Prove that the mean and variance of the binomial distribution $b(x; n, p)$ are $\mu = np$ and $r^2 = npq$. **[4]**

b) A home owner plants 6 bulbs selected at random from a box containing 5 tulip bulbs and 4 daffodil bulbs. What is the probability that he planted two daffodil bulbs and 4 tulip bulbs. **[5]**

c) Find the probability that a person flipping a coin gets : **[5]**

i) The third head of seventh flip.

ii) The first head on the fourth flip.

Q4) a) Prove that the mean and variance of $n(x; \mu, \sigma)$ are μ and σ^2 respectively. Further show that the standard deviation is σ. **[4]**

b) Given that X has a normal distribution with $\mu = 300$ and $\sigma = 50$. Find the probability that X assumes the value greater than 362. **[5]**

c) It is known, from previous data, that the length of time in months between customer complaints about a certain product is a gamma distribution, with $\alpha = 2$ and $\beta = 4$ changes were made to tighten quality control requirements. Following these changes, 20 months passed before the first complaint. Does it appear as if the quality control tightening was effective? **[5]**

Q5) a) Prove that an unbiased estimate of σ^2 is $S^2 = \dfrac{S_{yy} - b_1 S_{xy}}{n-2}$. **[4]**

b) The following data represent the mathematical grades for random sample of 12 freshmen at a certain college along with their scores on an intelligence test administered while they were still seniors in highschool **[5]**

Student	Test score x	Mathematical grades y
1	65	85
2	50	74
3	55	76
4	65	90
5	55	85
6	70	87
7	65	94
8	70	98
9	55	81
10	70	91
11	50	76
12	55	74

Compute and interprete sample correlation coefficient.

c) Compute and interprete the correlation coefficient for following data : **[5]**

Mathematics grade	70	92	80	74	65	83
English grade	74	84	63	87	78	90

Q6) a) From past experience, a stock broker believes that under present economic conditions a customer will invest in taxfree bonds with a probability of 0.6, will invest in mutual funds with a probability of 0.3 and will invest in both tax free bonds and mutual funds with a probability of 0.15. At this time, find the probability that the customer will invest. **[4]**

i) In either tax free bonds or mutual fund .

ii) In neither taxfree bonds nor mutual fund.

b) If $x_1, x_2, x_3 \dots\dots\dots x_n$ are mutually independent random variables that have respectively, chi-squared distributions with $v_1, v_2, v_3 \dots\dots\dots v_n$ degrees of freedom then prove that the random variable $Y = x_1 + x_2 + \dots\dots\dots + x_n$ has a chi-squared distribution with $V = v_1 + v_2 + v_3 + \dots\dots\dots + v_n$ degrees of freedom. **[5]**

c) An electrical firm manufactures a 100 watt light bulb, which according to specification written on the package, has a mean life of 900 hours with a standard deviation of 50 hours. At most, what percentage of bulb fail to last even 700 hours? Assume that the distribution is symmetric about the mean. **[5]**

Q7) a) Let X any Y be two random variables with moment generating function $M_X(t)$ and $M_Y(t)$ respectively. If $M_X(t) = M_Y(t)$ for all values of t then prove that X and Y have the same probability distribution. **[4]**

b) Find the moment generating function of the binomial random variable X and verify that $\mu = np$, $\sigma^2 = npq$. **[5]**

c) Hospital administrators in large cities anguish about traffic in emergency rooms. At a particular hospital in a large city, the staff on hand cannot accommodate the patient traffic. If there are more than 10 emergency cases in a given hour. It is assumed that patient arrival follows Poisson process and historical data suggest that, on the average 5 emergencies arrive per hour : **[5]**

i) What is the probability that in a given hour the staff cannot accommodate the patient traffic.

ii) What is the probability that more than 20 emergencies arrive a 3-hour's shift?

Q8) a) Prove that mean and variance of gamma distribution are $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$. **[4]**

b) Show in case of least squares fit to the simple linear regression model

$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, 2, \ldots\ldots n$ that $\sum_{i=1}^{n}(y_i - \hat{y}_i) = \sum_{i=1}^{n} e_i = 0$.

[5]

c) The pressure p of a gas corresponding to various volumes v is recorded and data are given as follows :

$V(cm^3)$	50	60	70	90	100
$p(kg/cm^2)$	64.7	51.3	40.5	25.9	7.8

The ideal gas law is given by the functional form $pv^\gamma = c$, where γ & c are constants. Estimate the constants c and γ. **[5]**

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