

Total No. of Questions : 8]

SEAT No. :

PE-5200

[Total No. of Pages : 3

[6508]-101

First Year M.A./M.Sc.

MATHEMATICS

MTUT-111 : Linear Algebra

(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Show that $\{a, b\}$ is a linearly dependent set if and only if $a = \lambda b$ or $b = \lambda' a$ for some λ or λ' in F . [7]

b) Let $S \in L(V, V)$ be given by [4]

$$S(u_1) = u_1 + u_2$$

$$S(u_2) = -u_1 - u_2$$

Where, $\{u_1, u_2\}$ is a basis for V .

i) Find the rank and nullity of S .

ii) Check S is invertible or not.

c) The set of all $f \in C(R)$ such that for some $\alpha, \beta, \gamma \in R$

$\alpha \frac{d^2 f}{dt^2} + \beta \frac{df}{dt} + \gamma f = 0$ is a subset of $C(R)$. If F subspace of $C(R)$? Justify your answer. [3]

Q2) a) Let $T \in L(V, V)$ for some finite dimensional vector space V over F . Then prove that the following statements are equivalents. [7]

i) T is invertible

ii) T is one to one

iii) T is onto

b) Let A, B, C be points in R_2 . Then prove that $\overline{AB} + \overline{BC} = \overline{AC}$. [4]

c) Test the linear transformation $T : R_3 \rightarrow R_2$ defined by the system of equation. [3]

$$y_1 = 3x_1 - x_2 + x_3$$

$$y_2 = -x_1 + 2x_3 \text{ is onto or not}$$

P.T.O.

- Q3)** a) Prove that, for all vectors $u, v, \in V$. $\|u + v\| \leq \|u\| + \|v\|$. [5]
- b) Prove that, every orthonormal set of vectors is linearly independent set. [6]
- c) For vectors $a = \langle \alpha_1, \alpha_2 \rangle, b = \langle \beta_1, \beta_2 \rangle$ define their inner product $(a \cdot b) = \alpha_1 \beta_1 + \alpha_2 \beta_2$. Show that, the inner product satisfies [3]
- i) $(\lambda a, b) = \lambda (a, b), \forall \lambda \in \mathbb{R}$
- ii) $(a, b + c) = (a, b) + (a, c)$
- Q4)** a) Let $T \in L(V, V)$ and let $f(x) \in F[x]$ then prove that, the set of all vectors $V \in V$ such that $f(T)(V) = 0$ is a T -invariant subspace. [5]
- b) Let S be a linear transformation on a vector space over the complex number such that $S(V_1) = V_1 + 2V_2$
 $S(V_2) = 4V_1 + 3V_2$
 Where, $\{V_1 = (1, 0), V_2 = (0, 1)\}$ is a basis for the vector space. Then find [7]
- i) Characteristic polynomial of S .
- ii) Minimal polynomial of S .
- iii) Eigenvalues of S .
- iv) Eigenvectors of S .
- c) Find the rational canonical form of a linear transformation on a vector space over the field of rational numbers $\mathbb{Q}$ whose elementary divisors are $x + 1, (x^2 + x + 1)^2$. [2]
- Q5)** a) Find an orthonormal basis for the subspace of $C(\mathbb{R})$ generated by the function $\{1, x, x^2\}$ with respect to the inner product [6]
- $$(f, g) = \int_0^1 f(t) \cdot g(t) dt$$
- b) Let T be a linear transformation on a finite dimensional vector space over F and let $\alpha \in F$. Then show that, α is a characteristic root of T if and only if the determinant $D(T - \alpha \cdot 1) = 0$, where 1 is the identity transformation on V . [6]
- c) Define quadratic form of matrix. [2]

- Q6)** a) Let V be a vector space over a field F and let Y be a subspace of V . Then prove that, the relation R on the set V defined by VRV' if $V - V' \in Y$ is an equivalence relation. [6]
- b) Let $T \in L(V, V)$ and $\{V_1, V_2, \dots, V_n\}$ be a basis of V and $\{f_1, f_2, \dots, f_n\}$ the dual basis of V^* . Let A be the matrix of T with respect to the basis $\{V_1, V_2, \dots, V_n\}$. Then prove that, the matrix of T^* with respect to the basis $\{f_1, f_2, \dots, f_n\}$ is the transpose matrix A^t . [6]
- c) Compute $A_1 \times B_1$, where $A_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. [2]
- Q7)** a) Let U and V be the finite dimensional vector space with bases $\{u_1, u_2, \dots, u_k\}, \{v_1, v_2, \dots, v_k\}$ respectively, then prove that $U_1 \{(u, 0) | u \in U\}$ and $V_1 \{(0, v) | v \in V\}$ are subspaces of $U + V$ and $U + V$ is the direct sum of $U_1 \oplus V_1$. [6]
- b) If V be a vector space over an algebraically closed field F , then prove that every irreducible invariant subspace W relative to $T \in L(V, V)$ has dimension 1. [6]
- c) Define symmetric tensor. [2]
- Q8)** a) Let T be an orthogonal transformation on a real vector space V with an inner product then prove that V is a direct sum of irreducible invariant subspaces $\{W_1, W_2, \dots, W_s\}$ for $s \geq 1$ such that vectors belonging to distinct subspaces W_i and W_j are orthogonal. [7]
- b) Let $\{E_1, E_2, \dots, E_s\}$ be a set of linear transformations of V such that $1 = \sum E_i$ and $E_i E_j = 0$ if $i \neq j$. Then prove that the $\{E_i\}$ are self-adjoint for $1 \leq i \leq s$ if and only if the subspaces $[E_i V]$ are mutually orthogonal. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5201

[Total No. of Pages : 3

[6508]-102

First Year M.A./M.Sc.

MATHEMATICS

MTUT-112 : Real Analysis
(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicates full marks.
- 3) Use of single memory, non-programmable calculator is allowed.

Q1) a) i) If there exists a set A in the collection $\mathcal{A}$ for which $m(A) < \infty$. Then show that $m(\phi) = 0$. [4]

ii) Prove that outer measure is translation invariant. [3]

b) If $\{E_k\}_{k=1}^{\infty}$ is a countable disjoint collection of measurable sets.

Prove that $\bigcup_{k=1}^{\infty} E_k$ is measurable and $m\left(\bigcup_{k=1}^{\infty} E_k\right) = \sum_{k=1}^{\infty} m(E_k)$ [7]

Q2) a) Let f be a simple function defined on E . Then prove that for each $\epsilon > 0$, there is a continuous function g on $\mathbb{R}$ and a closed set F contained in E for which $f = g$ on F and $m(E \setminus F) < \epsilon$. [7]

b) Let the function f have a measurable domain E , then prove that the following statements are equivalent. [7]

- i) for each real number C , the set $\{x \in E \mid f(x) > C\}$ is measurable.
- ii) for each real number C , the set $\{x \in E \mid f(x) \geq C\}$ is measurable.
- iii) for each real number C , the set $\{x \in E \mid f(x) < C\}$ is measurable.

P.T.O.

- Q3) a) i)** Let f be an increasing function on the closed bounded interval $[a, b]$. Prove that for $\alpha > 0$,

$$m^* \left\{ x \in (a, b) \mid \bar{D}f(x) \geq \alpha \right\} \leq \frac{1}{\alpha} [f(b) - f(a)]. \quad [4]$$

- ii) A function f defined on $\mathbb{R}$ as

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Find upper and lower derivatives of f at $x = 0$. [3]

- b) Let f be continuous on closed bounded interval $[a, b]$ and absolutely continuous on $[a, b]$. Prove that the family of divided difference function $\{\text{Diff}_h f\}_{0 < h \leq 1}$ is uniformly integrable over $[a, b]$. [7]

- Q4) a)** Let E be any subset of real numbers. Prove that the following statements are equivalent to the measurability of E . [7]

- i) For each $\varepsilon > 0$, there is an open set O containing E for which $m^*(O \setminus E) < \varepsilon$.
 ii) There is a G_δ set G containing E for which $m^*(G \setminus E) < 0$.

- b) Prove that Cantor set C is closed, uncountable set of measure zero. [7]

- Q5) a)** Let f be an extended real valued function on E . [7]

- i) If f is measurable on E and $f = g$ almost every where on E . Prove that g is measurable.
 ii) For a measurable subset D of E , prove that f is measurable on E if and only if the restrictions of f to D and $E \setminus D$ are measurable.

- b) State and prove Egoroff's theorem. [7]

- Q6) a)** Let function $f(x)$ defined on $[0, 1]$ by

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x^2}\right), & \text{for } 0 < x \leq 1 \\ 0, & \text{for } x = 0 \end{cases}$$

Show that f' is not integrable over $[a, b]$. [7]

- b) Let f be a integrable over the closed bounded interval $[a, b]$. Then prove that $f(x) = 0$ for almost all $x \in [a, b]$ if and only if $\int_{x_1}^{x_2} f = 0$ for all $(x_1, x_2) \subseteq [a, b]$. [7]

Q7) a) Let f be a absolutely continuous function on closed bounded interval $[a, b]$. Prove that f is differentiable almost everywhere on (a, b) and $\int_a^b f' = f(b) - f(a)$. [7]

b) Show that the function f defined by $f(x) = \frac{1}{x^3}, 0 < x \leq 1$ and $f(0) = 0$ is Lebesgue integrable over $[0, 1]$. [7]

Q8) a) i) Prove that countable set has measure zero. [4]

ii) Show that, translation of a F_σ set is also a F_σ set. [3]

b) Prove that every interval is measurable. [7]



SEAT No. :

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PE-5202

[Total No. of Pages : 2

[6508]-103

First Year M.A./M.Sc.

MATHEMATICS

MTUT-113 : Group Theory
(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of single memory, non-programmable calculator is allowed.*

- Q1)** a) In any group G , show that **[5]**
 i) There is only one identity element.
 ii) For any $a \in G$, there is a unique element b in G such that $ab = ba = e$.
 b) Prove that the center of a group G is a subgroup of G . **[5]**
 c) List all the subgroups of the group $\mathbb{Z}_6$. **[4]**

- Q2)**
- a) Let $\alpha = (1\ 2)(4\ 5)$ and $\beta = (1\ 6\ 5\ 3\ 2)$ be permutations in S_6 . Compute each of the following [5]
- i) α^{-1} i) $\beta\alpha$ i) $\alpha\beta$
- b) Let A_n denote the set of even permutations of S_n , then prove that A_n is subgroup of S_n of order $\frac{n!}{2}$. [5]
- c) Let $G = SL_2(\mathbb{R})$, the group of 2×2 matrices over $\mathbb{R}$ with determinant 1. If $\phi_m(A) = MAM^{-1}$, for A in G and for fix M in G . Then prove that ϕ_m is an isomorphism of $SL_2(\mathbb{R})$. [4]

- Q3)** a) State and prove Lagrange's theorem. **[5]**
 b) Let H and K be two subgroups of a group G . Define $HK = \{hk \mid h \in H, k \in K\}$. Then prove that $|HK| = \frac{|H||K|}{|H \cap K|}$. **[4]**
 c) Determine the number of elements of order 5 in $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$.

P.T.O.

- Q4)** a) Let G be a group and Let $Z(G)$ be the center of G . If $G/Z(G)$ is cyclic then prove that G is abelian. [5]
- b) Let H be a subgroup of G . $N(H)$ denote normalizer of G and $C(H)$ denote the centralizer of H . Then prove that $N(H)/C(H)$ is isomorphic to a subgroup of $\text{Aut}(H)$. [5]
- c) Show that A_4 has no subgroup of order 6. [4]
- Q5)** a) Show that $C_1(a) = \{a\}$ if and only if $a \in Z(G)$, where $Z(G)$ is center of G and $C_1(a)$ denote conjugacy class of a in G . [5]
- b) If G is a group of order pq , where p, q are primes with $p < q$ and p does not divide $q - 1$, then prove that G is cyclic and $G \cong \mathbb{Z}_{pq}$. [5]
- c) Let G be a group of order 99. Then prove that $G \cong \mathbb{Z}_{99}$ or $G \cong \mathbb{Z}_3 \times \mathbb{Z}_{33}$. [4]
- Q6)** a) Show that, every subgroup of cyclic group is cyclic. [5]
- b) Let S be a finite group and σ denotes a permutation of S . Prove that $\sigma = \alpha_1 \alpha_2 \dots \alpha_n$, where $\alpha_1, \alpha_2, \dots, \alpha_n$ are disjoint cycles. [5]
- c) Prove that every group of order p^2 is abelian. [4]
- Q7)** a) Let G and H be finite cyclic groups. Then $G \oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime. [5]
- b) Show that the group $SL(2, \mathbb{R})$ of 2×2 matrices with determinant 1 is a normal subgroup of the group $GL(2, \mathbb{R})$. [5]
- c) List all the elements of $\mathbb{Z}_8$ with their order. [4]
- Q8)** a) i) Prove that, for any a, b in G , [5]

$$(ab)^{-1} = b^{-1}a^{-1}$$
ii) Compute the order of the permutation α in S_6 , where

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix}$$
- b) Determine the number of cyclic subgroups of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$. [5]
- c) Determine the class equation of the quaternion group Q_8 . [4]



Total No. of Questions : 8]

SEAT No. :

PE-5203

[Total No. of Pages : 3

[6508]-104

First Year M.A./M.Sc.

MATHEMATICS

MTUT-114 : Advance Calculus

(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Define directional derivative of a scalar field. By giving example show that the existence of all directional derivative need not imply continuity of a function. [6]

b) A scalar field f is defined on $\mathbb{R}^n$ by the equation $f(\bar{x}) = \bar{a} \cdot \bar{x}$ where $\bar{a}$ is a constant vector. Compute $f'(\bar{x}; \bar{y})$ for arbitrary $\bar{x}$ and $\bar{y}$. [4]

c) Let $\bar{f} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be linear transformation. Prove that $\bar{f}$ is continuous at each point $\bar{a}$ in $\mathbb{R}^n$. [4]

Q2) a) If a function f is continuous on a rectangle $Q = [a, b] \times [c, d]$ then prove that f is integrable on Q . Also show that, [6]

$$\iint_Q f = \int_c^d \left[\int_a^b f(x, y) dx \right] dy = \int_a^b \left[\int_c^d f(x, y) dy \right] dx.$$

b) Evaluate $\iiint_S dx dy dz$ where S is a solid sphere of radius a and center at origin by transforming to spherical co-ordinates. [4]

c) Use Green's theorem to compute the work done by the force field $\bar{f}(x, y) = (y + 3x)\bar{i} + (2y - x)\bar{j}$ in a moving a particle once around a ellipse $4x^2 + y^2 = 4$. [4]

P.T.O.

- Q3) a)** Prove that every function f which is bounded on a rectangle Q has a lower integral $\bar{I}(f)$ and upper integral $\bar{I}(f)$. When is f integrable? [8]
- b)** Explain the change of variables in double integral by polar co-ordinates. Evaluate the integral using polar co-ordinates $\iint_S \sqrt{a^2 - x^2 - y^2} \, dx dy$ where the region S is the first quadrant of the circular disk $x^2 + y^2 \leq a^2$. [6]
- Q4) a)** Let $f : S \rightarrow \mathbb{R}$ be a scalar field defined on a set S in $\mathbb{R}^n$. Let $\bar{a}$ be an interior point of S . Let f be differentiable at $\bar{a}$ with total derivative $T_{\bar{a}}$. Prove that the directional derivative $f'(\bar{a}, \bar{y})$ exists for every $\bar{y}$ in $\mathbb{R}^n$ and $T_{\bar{a}}(\bar{y}) = f'(\bar{a}, \bar{y})$. Also show that $f'(\bar{a}, \bar{y}) = \sum_{k=1}^n D_k f(\bar{a}) y_k$. [6]
- b)** If a scalar field $\bar{f}$ is differentiable at $\bar{a}$ then prove that $\bar{f}$ is continuous at $\bar{a}$. [4]
- c)** Find the gradient vector at each point at which it exists for the scalar $f(x, y) = x^2 + y^2 \sin(xy)$. [4]
- Q5) a)** Define the line integral and illustrate it by an example. State the basic properties of the line integral. [5]
- b)** A particle of mass m moves along a curve under the action of a force field $\bar{f}$. If the speed of the particle at time t is $v(t)$, its kinetic energy is defined to be $\frac{1}{2}mv^2(t)$. Prove that the change in kinetic energy in any time interval is equal to the work done by $\bar{f}$ during this time interval. [4]
- c)** Calculate the line integral of the vector field $\bar{f}(x, y, z) = x\bar{i} + y\bar{j} + (xz - y)\bar{k}$ from $(0, 0, 0)$ to $(1, 2, 4)$ along a line segment. [4]

- Q6)** a) State and prove the first fundamental theorem for line integral. [6]
- b) Let $\vec{f} = (f_1, f_2, \dots, f_n)$ be a continuously differentiable vector field on an open set S in $\mathbb{R}^n$. If $\vec{f}$ is a gradient on S then prove that the partial derivatives of the component of $\vec{f}$ are related by the equations $D_i f_j(\bar{x}) = D_j f_i(\bar{x})$ for $i, j = 1, \dots, n$ for every $\bar{x}$ in S . [4]
- c) Show that the line integral of a continuous gradient is zero around every piece wise smooth closed path in S . [4]
- Q7)** a) State the Stoke's theorem and show that the surface integrals which appears in Stoke's theorem in terms of the vector field. [8]
- b) Compute the area of hemisphere S of radius a and center at origin from the implicit representation $x^2 + y^2 + z^2 = a^2, z \geq 0$. [6]
- Q8)** a) State and prove Gauss divergence theorem. [8]
- b) Transform the surface integral $\iint_S (\text{curl} \vec{F}) \cdot \vec{n} \, ds$ to a line integral by the use of Stoke's theorem where $\vec{F}(x, y, z) = y^2 \vec{i} + xy \vec{j} + xz \vec{k}$, S is the hemisphere $x^2 + y^2 + z^2 = 1, z \geq 0$ and $\vec{n}$ is the unit normal with nonnegative z -component. [6]



Total No. of Questions : 8]

SEAT No. :

PE-5204

[Total No. of Pages : 2

[6508]-105

First Year M.A./M.Sc.

MATHEMATICS

MTUT-115 : Ordinary Differential Equations

(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Attempt the following:

- i) Show that the function $\phi(x) = e^{-\sin x}$ is the solution of the equation $y' + \cos x \cdot y = 0$. [2]
- ii) Solve the equation $3y' + y = 2e^{-x}$. [5]
- b) Explain the method of solving the equation $y' + ay = 0$, where a is a complex constant. [7]

Q2) Attempt the following:

- a) Solve the equation $Ly' + Ry = E$ where L , R and E are constants. Also show that every solution of it tends to E/R as $x \rightarrow \infty$. [7]
- b) Find the solution of the equation $y'' - 7y' + 6y = \sin x$. [7]

Q3) a) Attempt the following:

- i) Find all solution ϕ of the equation $y'' + y = 0$ satisfying $\phi(0) = 1$, $\phi(\pi/2) = 2$. [4]
- ii) Determine whether the differential equation $(3x^2 - 2xy) dx + (2y - x^2) dy = 0$ is exact or not. [3]
- b) Show that two solutions ϕ_1 and ϕ_2 of $y'' + a_1y' + a_2y = 0$ are linearly independent on an interval I if and only if $W(\phi_1, \phi_2) \neq 0, \forall x \in I$. [7]

P.T.O.

Q4) Attempt the following:

- a) Compute $W(\phi_1, \phi_2, \phi_3)(x)$ at a point $x = 0$ for the function $\phi_1 = e^x$, $\phi_2 = xe^x$ and $\phi_3 = x^2e^x$. [7]
- b) Verify that the function $\phi_1(x) = x^3$ satisfies the equation $x^2y'' - 7xy' + 15y = 0 (x > 0)$ and find the second independent solution. [7]

Q5) Attempt the following:

- a) Show that $\phi_1(x) = |x|^i$ and $\phi_2(x) = |x|^{-i}$ are linearly independent solutions of the equation $x^2y'' + xy' + y = 0$. [7]
- b) Explain the method of reduction of order for solving n^{th} order homogeneous equation with variable coefficients. [7]

Q6) Attempt the following:

- a) Determine that the equation $(\cos x \cos^2 y) dx - (\sin x \cdot \sin 2y) dy = 0$ is exact or not and solve. [7]
- b) i) Compute the indicial polynomial and its roots for the differential equation $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$ [4]
- ii) If Γ is gamma function show that $\overline{ZH} = Z\overline{Z}$. [3]

Q7) Attempt the following:

- a) Find all solutions of the equation $x^2y'' + xy' + 4y = 1$ for $|x| > 0$ [7]
- b) Let M, N be two real valued functions which have continuous first order partial derivatives on some rectangle $R : |x - x_0| \leq a, |y - y_0| \leq b$, then prove that the equation $M(x, y) dx + N(x, y) dy = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [7]

Q8) Attempt the following:

- a) Explain method of successive approximation. [7]
- b) Show that infinity is not a regular singular point for the equation $x^2y'' + xy' + (x^2 - \alpha^2)y = 0$. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5205

[Total No. of Pages : 4

[6508]-201

First Year M.A. / M.Sc.

MATHEMATICS

MTUT-121 : Complex Analysis
(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Prove that “Every non-constant polynomial in one variable with coefficients in $\mathbb{C}$ has atleast one root in $\mathbb{C}$. [5]

b) Let $P = \sum_{n \geq 0} a_n t^n$ be a power series over $\mathbb{C}$. [5]

Put $L = \limsup \sqrt[n]{|a_n|}$ and $R = \frac{1}{L}$ with the convention $\frac{1}{0} = \infty$ and $\frac{1}{\infty} = 0$.

Then prove that :

- i) For all $0 < r < R$ the series P is absolutely and uniformly convergent in $|z| \leq r$ and
- ii) For all $|z| > R$ the series is divergent.
- c) Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + \dots + a_0$, ($a_n \neq 0$) with $a_j \in \mathbb{R}$. Then show that $p(\bar{z}) = \overline{p(z)}$. Also prove that if z is a root of a polynomial P with real coefficients, then $\bar{z}$ is also a root of P . [4]

Q2) a) Let U be an open set in $\mathbb{R}^2$ and $f : U \rightarrow \mathbb{R}$ be a function having partial derivatives which are continuous at (x_0, y_0) . Then show that f is frechet differentiable at (x_0, y_0) . [5]

P.T.O.

b) Let $f(x, y) = \frac{x-y}{x+y}$ for $(x, y) \neq (0, 0)$. Show that two iterated limits exists

but are not equal. Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exists. [4]

c) Use Cauchy Riemann equations to determine the function $f(z) = \operatorname{Re}(z)$ is complex differentiable for $z \in \mathbb{C}$ or not. [3]

d) Compute the value of the integral $\int_{\omega} x dz$ where ω is the line segment from 0 to $1 + i$. [2]

Q3) a) Let Ω be a simply connected domain in $\mathbb{C}$ and f be a holomorphic function on it. Then show that for a simple closed contour γ in Ω ,

$$\int_{\gamma} f(z) dz = 0 \quad [5]$$

b) Discuss the continuity for the function [5]

$$f(x, y) = \frac{x^2 y}{x^2 + y^2}, \quad (x, y) \neq (0, 0)$$

$$= 0, \quad (x, y) = (0, 0)$$

at $(0, 0)$

c) Evaluate $\int_c \frac{\sin z}{z + 3i} dz$ where $c : |z - 2 + 3i| = 1$ traversed in counter clockwise direction. [4]

Q4) a) Let $f : \Omega \rightarrow \mathbb{C}$ be a non-constant complex differentiable function on a domain Ω . Then prove that there does not exists any point $\omega \in \Omega$ such that $|f(z)| \leq f(\omega) \forall z \in \Omega$. [5]

b) Given a complex differentiable function f such that $|f(z)| \leq 1$, find a variable upperbound for $|f^n(z)|$ in the disc $|z| < 1$ hence show that $|f^n(0)| \leq n!$ [4]

c) Let $f(z) = \frac{\cos z - 1}{z^2}$. Determine the nature of singularity at $z = 0$. If it is a removable singularity find the value of f at 0. If it is a pole, find the residue at 0. [3]

d) Determine the maximum modulus for the function [2]
 $f(z) = z^2 + z + 1$ for $z \in [-1, -1] \times [1, 1]$.

Q5) a) Let f be a holomorphic function in $B_r(a) \setminus \{a\}$ and let a be an essential singularity of f . Then prove that f takes values arbitrarily close to any arbitrary complex number inside arbitrary neighbourhood of a . [5]

b) Let f be a holomorphic function in a domain D . Suppose there is a point $a \in D$ such that $f^{(k)}(a) = 0$ for all $k \geq 0$. Then prove that $f \equiv 0$ on D . [5]

c) Compute the residues of all singular points of the function

$$f(z) = \frac{e^z}{(z + \pi i)^3} \quad [4]$$

Q6) a) Let a be a pole of order n of f and let $g(z) = (z - a)^n f(z)$ with g holomorphic and $g(a) \neq 0$. Then show that the residue of f at a is $R_a(f) = \frac{g^{(n-1)}(a)}{(n-1)!}$ [5]

b) Obtain the Laurent series expansion of the function $f(z) = \frac{1}{1+z^2}$ in the annulus $A = \{z/|z-1-i| < 1\}$. [5]

c) Use complex method to prove that $\int_0^{2\pi} \frac{d\theta}{5+4\cos\theta} = \frac{2\pi}{3}$. [4]

Q7) a) Prove that $\int_0^\pi e^{-R\sin\theta} d\theta < \frac{\pi}{R}, R > 0$. [5]

b) Find the value of the integral $\int_0^\infty \frac{\cos x}{x^2+a^2} dx$ $a > 0$ [7]

c) Find the Cauchy's principle value of $\int_{-\infty}^\infty \frac{1}{x-1} dx$ for $H =: |z| < |$. [2]

Q8) a) Let f and g be holomorphic function in an open set containing $\bar{D}$ closure of a disc D and satisfy the inequality $|f(z) - g(z)| < |g(z)| \forall z \in \partial D$. Then show that f and g have same number of zeros inside D . Hence determine the number of zeros of the polynomial $T^7 - 4T^3 + T - 1$ inside the unit circle. [7]

b) Let $F : A \rightarrow \mathbb{C}$ $Z_0 \in A$ $r > 0$ such that $B_r(z_0) \subset A$. Prove that f is complex differentiable at z_0 if and only if $\exists \alpha \in \mathbb{C}$ and a function. [5]

$Q : B_s(0) \setminus \{0\} \rightarrow \mathbb{C}$ $(0 < s \leq r)$ such that for all $h \in B_s(0) \setminus \{0\}$

$$f(z_0 + h) - f(z_0) = h\alpha + o(h); \lim_{h \rightarrow 0} o(h)/h = 0.$$

c) Find absolute value of $\frac{3}{3+4i}$. [2]



Total No. of Questions : 7]

SEAT No. :

PE-5206

[Total No. of Pages : 2

[6508]-202

First Year M.A./M.Sc.

MATHEMATICS

MTUT-122 : General Topology
(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicates full marks.

Q1) Solve any five of the following :

- a) Let A be a set. Prove that there is no injective map $f: P(A) \rightarrow A$ and there is no surjective map $g: A \rightarrow P(A)$. [5]
- b) If C is an infinite subset of $\mathbb{Z}_+$, then C is countably infinite. [5]
- c) Show that, the set $\mathbb{Q}_+$ of positive rational numbers is countably infinite. [4]

Q2) a) Let X be a topological space. Then the following condition holds: [6]

- i) ϕ and X are closed
- ii) Arbitrary intersection of closed sets are closed
- iii) Finite union of closed sets are closed
- b) Define closed set. Also show that the subset $[1, 3]$ of $\mathbb{R}$ is closed. [4]
- c) Write any four topologies on $X = \{a, b, c\}$. [4]

Q3) a) Prove that, every finite point set in a Hausdorff space X is closed. [6]

- b) Let X be a set. Let τ_f be the collection of all subsets U of X such that $X - U$ either is finite or is all of X . Then show that, τ_f is topology on X . [6]
- c) State pasting lemma. [2]

P.T.O.

- Q4)** a) Let A be connected subspace of X . If $A \subset B \subset \bar{A}$, then show that B is also connected. [5]
- b) Show that every path connected space is connected. [5]
- c) Prove that the union of a collection of connected subspaces of X that have a point in common is connected. [4]
- Q5)** a) Prove that every compact subspace of a Hausdorff space is closed. [6]
- b) Show that the interval $(0,1]$ is not compact in $\mathbb{R}$. [4]
- c) State [4]
- i) Tube lemma
- ii) Finite intersection property
- Q6)** a) Show that a subspace of Hausdorff space is Hausdorff. [5]
- b) Prove that every regular space with a countable basis is normal. [7]
- c) State Urysohn lemma. [2]
- Q7)** a) Prove that a subspace of regular space is regular. [6]
- b) Prove that, every metrizable space is normal. [6]
- c) Define Lindelöf space. [2]
- Q8)** a) State and prove pasting lemma. [5]
- b) Prove that, the image of connected space under a continuous map is connected. [5]
- c) Define: [4]
- i) Regular space
- ii) Normal space



Total No. of Questions : 8]

SEAT No. :

PE-5207

[Total No. of Pages : 2

[6508]-203

First Year M.A./M.Sc.

MATHEMATICS

MTUT-123 : Ring Theory

(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right side indicate full marks.

Q1) a) If R is commutative with 1, then prove that $A \in \text{Mn}(R)$ is a unit if and only if determinant, $\det(A)$ is a unit in R . [7]

b) Let R be an integral domain with 1. Prove that a polynomial

$P(X) = a_0 + a_1x + a_2x^2 + \cdots + a_rx^r$ is a unit in $R[X]$ if and only if a_0 is unit in R and $r = 0$. [7]

Q2) a) Let R be a commutative ring with 1, and I be an ideal in R , then prove that R/I is an integral domain if and only if I is prime ideal in R . [5]

b) Prove that the set of all nilpotent elements in a commutative ring R with 1 is the intersection of all prime ideals. [7]

c) Define simple ring with suitable example. [2]

Q3) a) For $2 \leq n$, $n \in \mathbb{N}$, the ring $\mathbb{Z}/n\mathbb{Z}$ is a field then prove that $\mathbb{Z}/n\mathbb{Z}$ is an integral domain if and only if n is a prime number. [7]

b) If R is a local ring with M_R its ideal of non-units, then prove that the quotient R/M_R is a division ring. [7]

Q4) a) Suppose I and J are 2-sided ideals in a ring R then prove that the natural map $\int IJ : R/I \rightarrow R/J$ defined by $a + I \rightarrow a + J$ is well defined if and only if $I \subseteq J$ moreover, ι_{IJ} is an epimorphism whose Kernel is J/I . Then prove that $(R/I) / (J/I)$ is isomorphic to R/J . [7]

P.T.O.

- b) For any commutative integral domain R with 1 , $Q(R)$ is the smallest field containing R as a subring, having the same identity as R , smallest in the sense that if K is a field containing R as a subring, then prove that $K \geq Q(R)$ as a subfield. This $Q(R)$ is unique upto isomorphism called the field of fraction of R . [7]
- Q5)** a) Let ' a ' be a non-zero non-unit in a commutative integral domain R then prove that the element ' a ' is irreducible in R if and only if the ideal (a) is maximal among all principal ideals other than R . [7]
- b) Prove that the ring $\mathbb{Z}[i]$ of Gaussian integers is Euclidean. [7]
- Q6)** a) Prove that every PID is a factorisation domain. [7]
- b) Prove that domain R is a UFD if and only if R is an factorisation domain in which every irreducible element is a prime. [7]
- Q7)** a) State and prove Eisenstein's Criterion. [7]
- b) Show that $1 + i\sqrt{3}$ is irreducible in $\mathbb{Z}[i\sqrt{3}]$. Is it a prime? [5]
- c) Show that the polynomial $2 + 2x + 3x^3 \in \mathbb{Q}[x]$ is irreducible. [2]
- Q8)** a) Prove that an R -module M is simple if and only if $M \cong R/I$ for some maximal left ideal in R . [7]
- b) If P, N are submodules of module M . Then prove that there exist natural isomorphism $\frac{P+N}{P} \cong \frac{N}{N \cap P}$. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5208

[Total No. of Pages : 4

[6508]-204

First Year M.A./M.Sc.

MATHEMATICS

MTUT-124 : Advanced Numerical Analysis

(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Determine the corresponding rate of convergence for the function

$$f(x) = \frac{2^n + 3}{2^n + 7}. \quad [4]$$

- b) Prove that if $g : [a, b] \rightarrow [a, b]$ is continuous function on $[a, b]$, differentiable on (a, b) and there exists a constant $k < 1$ such that, $|g'(x)| \leq k < 1, \forall x \in (a, b)$ then,

Prove that [5]

- i) The sequence $\{p_n\}$ generated by $p_n = g(p_{n-1})$ converges to the fixed point p for any $p_0 \in [a, b]$.

ii) $|p_n - p| \leq \frac{k^n}{1-k} |p_1 - p_0|$

- c) Use the secant method to determine P_5 , the fifth approximation to root of $f(x) = x^7 - 3$ in $(1, 2)$ with $P_0 = 1$ and $P_1 = 2$. [5]

- Q2) a)** The sequence listed below was obtained from fixed point iteration applied to the function $g(x) = e^{-x}$, which has a unique fixed point. Applying Aitken Δ^2 - method to the given sequence. Find $\hat{p}_3, \hat{p}_4$ and $\hat{p}_5$. [6]

1	1.0000000
2	0.3678794412
3	0.6922006276
4	0.5004735006
5	0.6062435351

- b) Show that when Newton's method is applied to the equation $\frac{1}{x} - a = 0$, the resulting iteration function is $g(x) = x(2 - ax)$. [4]
- c) Define: [4]
- Order of convergence
 - Round off error

- Q3) a)** Solve the following system of equations by using Gaussian Elimination with scaled partial pivoting. [5]

$$3x_1 + x_2 + 4x_3 - x_4 = 7$$

$$2x_1 - 2x_2 - x_3 + 2x_4 = 1$$

$$5x_1 + 7x_2 + 14x_3 - 8x_4 = 20$$

$$x_1 + 3x_2 + 2x_3 + 4x_4 = -4$$

- b) Show that the matrix [4]

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$$

has no LU Decomposition.

- c) Solve the following system by Jacobi method starting with vector $x_0^{(0)} = [0 \ 0 \ 0]^T$ perform two iteration. [5]

$$3x_1 + x_2 = -1$$

$$-x_1 + 2x_2 + x_3 = 3$$

$$x_2 + 3x_3 = 4$$

- Q4) a)** Solve the following system of linear equations by SOR method, start with $x^{(0)} = [0 \ 0 \ 0]^T$ and $w = 0.9$ (perform 3 iteration). [5]

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

- b)** Solve the following system of non-linear algebraic equations by using Broyden's method start with $x^{(0)} = [1 \ 1 \ 1]^T$ (perform 3 iteration) [5]

$$x_1^3 - 2x_2 - 2 = 0$$

$$x_1^3 - 5x_3^2 + 7 = 0$$

$$x_2 x_3^2 - 1 = 0$$

- c)** Let, A be an $n \times n$ matrix with eigen-values $\lambda_1, \lambda_2, \dots, \lambda_n$ and associated eigen vectors $v_1, v_2, \dots, v_n$ then prove that, If $\beta = a_0 I + a_1 A + a_2 A^2 + \dots + a_m A^m = P(A)$ where, P is the polynomial $p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_m x^m$, then the eigenvalue of B are $p(\lambda_1), p(\lambda_2), \dots, p(\lambda_n)$ with associated eigen vector $v_1, v_2, \dots, v_n$. [4]

- Q5) a)** Use Euler's method to solve Initial value problem [7]

$$\frac{dx}{dt} = t^2 + x, 0 \leq t \leq 0.03, x(0) = 1.$$

- b) i)** Solve the initial value problem [7]

$$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \leq t \leq 1.5, x(1) = 1, h = 0.25$$

By using Taylor method of order $N = 2$

- ii)** Solve the initial value problem

$$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \leq t \leq 2, x(1) = 2, h = 0.5$$

By using Taylor method of order $N = 4$.

- Q6) a)** Explain the QR algorithm for finding the eigen value of symmetric tridiagonal matrix. [7]

- b)** Construct the House Holder Matrix H for $W = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix}^T$ [4]

- c)** If $f(x) = \ln x$, find $f'(z)$ for $h = 0.1, 0.01, 0.001$. [3]

Q7) a) Evaluate $\int_{-2}^2 \frac{x}{5+2x} dx$ by using Trapezoidal rule by dividing the interval $[-2, 2]$ into five equal subintervals. [3]

b) Define: [4]

i) Relative error

ii) Triangular matrix

c) Derive the difference equation for the four-step Adams-Bash fourth method: [7]

$$\frac{w_{i+1} - w_i}{h} = \frac{55}{24} f(t_i, w_i) - \frac{59}{24} f(t_{i-1}, w_{i-1}) + \frac{37}{24}$$

$$f(t_{i-2}, w_{i-2}) - \frac{9}{24} f(t_{i-3}, w_{i-3})$$

Also, derive the associated truncation error:

$$\tau_i = \frac{251h^4}{720} y^{(5)}(\xi)$$

Q8) a) Use the Runge Kutta method of order $N = 4$ to solve initial value problem

$$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \leq t \leq 6, x(1) = 1 \quad [7]$$

b) Derive the following difference approximation for the first derivative

$$f'(x_0) \approx \frac{f(x_0 + 2h) - f(x_0 - h)}{3h} \quad [4]$$

c) Show that [3]

If f is continuous on $[a, b]$, g is integrable on $[a, b]$ and $g(x)$ does not change sign on $[a, b]$, then there exist a number $\xi \in [a, b]$ such that,

$$\int_a^b f(x)g(x)dx = f(\xi) \int_a^b g(x)dx.$$



[6508]-205

First Year M.A./M.Sc.

MATHEMATICS

MTUT-125 : Partial Differential Equations
(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Explain Charpit's method for $f(z, p, q) = 0$ that means given partial differential equation does not involves x and y explicitly. [4]

b) Attempt the following:

- i) Find a complete integral of the equation $zpq - p - q = 0$ by Charpits method. [5]
- ii) Solve the equation $xu_x + yu_y = u^2_z$ by Jacobi's method. [5]

Q2) a) Prove that the necessary and sufficient condition for the integrability of equation $dz = \phi(x, y, z) dx + \psi(x, y, z)dy$ is

$$[f, g] = \frac{\partial(f, g)}{\partial(x, p)} + p \frac{\partial(f, g)}{\partial(z, p)} + \frac{\partial(f, g)}{\partial(y, q)} + q \frac{\partial(f, g)}{\partial(z, q)} = 0 \quad [6]$$

b) Attempt the following:

- i) Show that the equations [5]
 $f(x, y, z, p, q) = xp - yq - x = 0$ and
 $g(x, y, z, p, q) = x^2p + q - xz = 0$ are compatible.
- ii) Verify that the equation $z = x + ax^2y^2 + b$ is a solution of the partial differential equation $xp - yq - x = 0$ [3]

Q3) a) Explain the method of second order partials differential equation $R_r + S_s + T_t + f(x, y, z, p, q) = 0$ to a canonical form if $S^2 - 4RT < 0$. [5]

b) Attempt the following:

- i) Reduce the equation $u_{xx} + x^2u_{yy} = 0$ to a canonical form. [5]
- ii) Solve by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ [4]

P.T.O.

- Q4) a)** If f and g be arbitrary functions of their respective arguments, show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \text{ provided } \alpha^2 = 1 - \frac{v^2}{c^2}. \quad [5]$$

- b)** Attempt the following: [7]

- i) Verify that the equations

$$1) \quad z = \sqrt{2x + a} + \sqrt{2y + b}$$

$$2) \quad z^2 - \mu = 2(1 + \lambda^{-1})(x + \lambda y)$$

are both complete integral of the partial differential equation

$z = \frac{1}{p} + \frac{1}{q}$ and show further, that the complete integral. (b) is the envelope of the one parameter subsystem obtained by taking

$$b = \frac{-a}{\lambda} - \frac{\mu}{1 + \lambda} \text{ is the solution.}$$

- ii) Define the terms: [2]

- 1) Complete integral of first order p.d.e.
- 2) Singular solution of first order p.d.e.

- Q5) a)** Explain the method of complementary function of $F(D, D') Y = f(x, y)$, if it's auxiliary equation has distinct and real roots. [6]

- b)** Attempt the following:

- i) Find the complementary functions of the partial differential equation

$$1) \quad (D^3 - 4D^2D' + 4DD'^2) Z = 4 \sin(2x + y) \quad [3]$$

$$2) \quad (D^2 - 6DD' + 9D'^2) Z = 6x + 2y \quad [3]$$

- ii) Classify the following equations [2]

$$1) \quad u_{xx} + 2u_{xy} + u_{yy} = 0$$

$$2) \quad 3u_{xx} + 10u_{xy} + 3u_{yy} = 0$$

- Q6) a)** Find the solution of two dimensional Laplace equation $\Delta^2 u = 0$ by using separation of variables method. [7]

- b)** Attempt the following: [7]

- i) Show that $T(x, t) = \frac{1}{\sqrt{4\pi\alpha t}} \exp\left[-\frac{(x - \xi)^2}{4\alpha t}\right]$ is a solution of

$$\text{differential equation } \frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, -\infty < x < \infty, t > 0.$$

Q7) a) Prove that the solution $u(x, t)$ of the differential equation

$$\frac{\partial u}{\partial t} - K \frac{\partial^2 u}{\partial x^2} = F(x, t), 0 < x < l, t > 0. \text{ Satisfying the initial condition}$$

$u(x, 0) = f(x), 0 < x < l$ and boundary conditions $u(0, t) = u(l, t) = 0, t > 0$ is unique. [5]

b) Attempt the following: [6]

1) Solve $K \left(\frac{\partial^2 u}{\partial x^2} \right) = \frac{\partial u}{\partial t}$

for $0 < x < \pi, t > 0$ if $u_x(0, t) = u_x(\pi, t) = 0$ and $u(x, 0) = \sin x$.

2) If $u = f(x + iy) + g(x - iy)$ where f and g are arbitrary functions, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad [3]$$

Q8) a) Reduce one dimensional wave equation $u_{tt} = c^2 u_{xx}$ to canonical form. [4]

b) Attempt the following:

i) Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq L, t > 0$

subject to conditions

$$u(0, t) = 0, t > 0 \quad u(L, t) = 0, t > 0. \quad [8]$$

ii) Find solution of $(D^2 - D'^2 + D - D')Z = 0$ [2]



Total No. of Questions : 8]

SEAT No. :

PE-5210

[Total No. of Pages : 2

[6508]-301

S.Y. M.A./M.Sc.

MATHEMATICS

MTUT-131 : Functional Analysis
(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.
- 3) Symbols have their usual meanings.

Q1) a) i) Define Hilbert space. [2]

ii) The space l_2 with the inner product of the vectors $x = \{x_1, x_2, \dots, x_n, \dots\}$

and $y = \{y_1, y_2, \dots, y_n, \dots\}$ defined by $(x, y) = \sum_{n=1}^{\infty} x_n \bar{y}_n$

Show that l_2 is Hilbert space. [5]

b) State and prove Schwarz inequality. [5]

c) If H is a Hilbert space and $x, y \in H$ then prove that [2]

$$\|x + y\|^2 + \|x - y\|^2 = 2\|x\|^2 + 2\|y\|^2$$

Q2) a) Let S be the set of all normal operator on a Hilbert space H . Then prove that S is closed subset of $B(H)$ which contains the set of all self adjoint operators and is closed under scalar multiplication. [7]

b) If T is an operator on H , then prove that the following conditions are all equivalent to one another [7]

i) $T^*T = T$

ii) $\langle Tx, Ty \rangle = \langle x, y \rangle$ for all x and y .

iii) $\|Tx\| = \|x\|$, for all x .

P.T.O.

- Q3)** a) If P is a projection on Hilbert space H with range M and null space N then prove that $M \perp N$ if and only if P is self adjoint and $N = M^\perp$. [7]
- b) State and prove Bessel's inequality. [7]
- Q4)** a) Let M be a closed linear subspace of a normed linear space N . If the norm of a coset $x + M$ in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| / m \in M\}$ then prove that N/M is a normed linear space. [7]
- b) Prove that T is a self-adjoint operator on H if and only if $\langle T(x), x \rangle \in \mathbb{R}$ for all $x \in H$. [4]
- c) Prove that for any two vectors x and y in a Hilbert space H , $|\langle x, y \rangle| \leq \|x\| \|y\|$. [3]
- Q5)** a) State and prove Open Mapping theorem. [7]
- b) Define Banach space. [2]
- c) State and prove Holder's Inequality. [5]
- Q6)** a) State and prove closed graph theory. [7]
- b) If N is a normed linear space and x_0 is a non-zero vector in N , then prove that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$. [7]
- Q7)** a) An operator T on Hilbert space H is singular if and only if there exist a non-zero vector $x \in H$ such that $Tx = 0$. [7]
- b) If $T^k = 0$ for some k_1 then prove that $\sigma(T) = \{0\}$, if $T^k = 0$ then T is nilpotent operator with index k . [4]
- c) Let H be two dimensional Hilbert space with $B = \{e_1, e_2\}$ as a basis, then find the spectrum of T on H which is defined by $T(e_1) = e_2$ and $T(e_2) = -e_2$. [3]
- Q8)** a) State and prove spectrum theorem. [7]
- b) If T is a normal operator on H then prove that eigen values spaces M_i 's of T are pairwise orthogonal. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5211

[Total No. of Pages : 2

[6508]-302

S.Y. M.A./M.Sc.

MATHEMATICS

MTUT-132 : Field Theory

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions of the following.
- 2) Figures to the right indicate full marks.

Q1) a) Prove that a regular n -gon is constructible if and only if $\phi(n)$ is power of 2. [7]

b) Construct a field with g elements. [5]

c) Define cyclic extension. [2]

Q2) a) State and prove Eisenstein criterion. [7]

b) Prove that $\mathbb{Q}(\sqrt{7}) - \mathbb{Q}$ is an algebraic extension. [5]

c) Define simple extension. [2]

Q3) a) If E is finite extension of field F then prove that $|G(E \setminus F)| \leq [E : F]$. Verify it on $G(\mathbb{Q}\sqrt{2}) \setminus \mathbb{Q}$. [7]

b) State fundamental theorem of Galois theory. [5]

c) Find minimal polynomial of $\alpha = \sqrt{2} + 5$ over $\mathbb{R}$. [2]

Q4) a) Prove that group of automorphisms of a field F with p^n elements is cyclic of order n and generated by ϕ where $\phi(x) = x^p, x \in F$. [7]

b) State any four ancient geometric problem. [5]

c) Define Galois extension of a field F . [2]

P.T.O.

- Q5)** a) Find splitting field of $x^4 - 2 \in \mathbb{Q}[x]$. [7]
 b) Is $\mathbb{Q} \setminus \mathbb{R}$ normal? Justify. [5]
 c) Explain the terms algebraically closed field. [2]
- Q6)** a) Give an example to show that an extension of a field F is algebraic but not finite. [7]
 b) Find all subfields of field F with cardinality 7^{16} . [5]
 c) Is $\mathbb{Z}_7[x]$ field? Justify. [2]
- Q7)** a) Prove that each field has an algebraic closure. [7]
 b) Prove that splitting field of $x^3 + x^2 + 1$ over $\mathbb{Z}_2$ is field with 8 elements. [7]
- Q8)** a) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals. [7]
 b) Define cyclotomic polynomial $\phi_n(x)$ and find it for $n = 1, 2, 3, 4, 5, 6$. [7]



Total No. of Questions : 4]

SEAT No. :

PE-5212

[Total No. of Pages : 2

[6508]-303

S.Y. M.A./M.Sc.

MATHEMATICS

MTUT-133 : Programming with Python
(2019 Pattern) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *Question 1 is compulsory.*
- 2) *Attempt any two questions from Q 2,3 and 4.*
- 3) *Figures to the right indicate full marks.*

Q1) Attempt the following:

- a) Explain the difference between logical and relational operators in Python with an example. **[4]**
- b) Explain any three features of Python. **[3]**

Q2) Attempt the following:

- a) Write a program to generate the following pattern in Python. **[4]**
*
* *
* * *
* * * *
- b) Write a Python program which accept a positive integer 'n' and display its factorial value. **[4]**
- c) Write a note on operator overloading in Python. **[6]**

Q3) Attempt the following:

- a) Write a python program which accept positive integer and check whether it is prime or not. **[5]**
- b) Write a note on file handling in Python. **[6]**
- c) Explain how an object is created in Python? **[3]**

P.T.O.

Q4) Attempt the following:

- a) What is a class? What are the essential components of a class? Define attributes and functions of a class. [7]
- b) What is a constructor? State and explain the different types of constructors in Python. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5213

[Total No. of Pages : 3

[6508]-304

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO134 : Discrete Mathematics
(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) Prove that every even graph decomposes into cycles. [5]

b) Prove that k -regular graph with n vertices has $\frac{nk}{2}$ edges. [5]

c) Show by a combinatorial argument that $\binom{2n}{2} = 2\binom{n}{2} + n^2$. [4]

Q2) a) How many arrangements of the letters in MATHEMATICS are there in which TH appear together but TH is not immediately followed by an E (not THE)? [7]

b) Prove that an edge is a cut-edge if and only if it belongs to no cycle. [7]

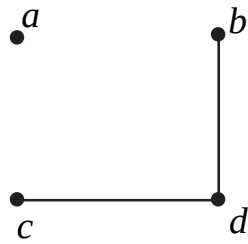
Q3) a) Use generating functions to show that [7]

$$\sum_{k=0}^r \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}$$

b) Find the number of ways to place 25 people into three rooms with atleast one person in each room. [5]

P.T.O.

- c) Find the complement of the following graph. [2]



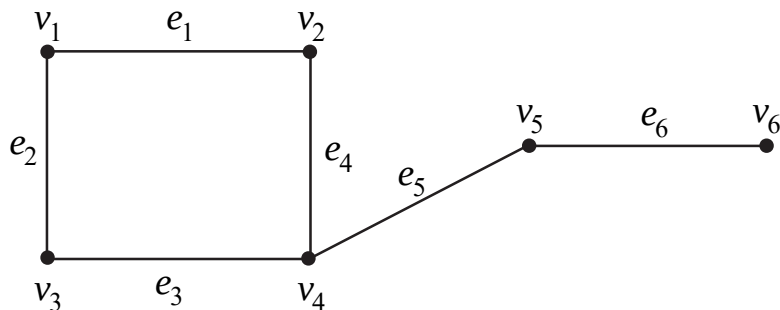
- Q4)** a) Let G be an n -vertex graph with $n \geq 1$. Prove that following statements are equivalent. [7]

- i) G is connected and has no cycles.
 - ii) G has no loops and for each $u, v \in V(G)$, there exists exactly one u, v - path.
- b) Show with the generating functions that every positive integer can be written as unique sum of distinct powers of 2. [5]
- c) Find the number of integer solutions to the equation, $x_1 + x_2 + x_3 = 15, x_1, x_2, x_3 \geq 0$. [2]

- Q5)** a) Show that $\sum_{k=0}^n k(k-1) \binom{n}{k} = n(n-1)2^{n-2}$ [5]

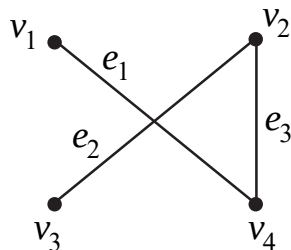
where n is a non-negative integer.

- b) Define the term ‘perfect matching’. Find the perfect matching for the following graph. [5]



- c) Find the connectivity and the edge connectivity of K_4 . [4]

- Q6)** a) Let $\tau(G)$ denote the number of spanning trees of a graph G . If $e \in E(G)$ is not a loop then prove that $\tau(G) = \tau(G - e) + \tau(G \cdot e)$. [6]
- b) Use generating function to find the number of ways to select 10 balls from large pile of red, white and blue balls if the selection has atleast two balls of each colour. [5]
- c) How many ways are there to pick a man and a woman who are not husband and wife from a group of n married couples? [3]
- Q7)** a) Solve the following recurrence relation.
 $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}, a_0 = a_1 = 1, a_2 = 2$. [7]
- b) Prove that a graph is bipartite if and only if it has no odd cycle. [7]
- Q8)** a) State Inclusion-Exclusion formula. Use it to find the number of ways to select 6-card hand from a regular 52-card deck such that the hand contains atleast one card in each suit. [7]
- b) Prove that every graph with n vertices and k edges has atleast $n - k$ components. [5]
- c) Find the incidence matrix for the following graph. [2]



Total No. of Questions : 8]

SEAT No. :

PE-5214

[Total No. of Pages : 2

[6508]-305

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-135 : Mechanics

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

- Q1)** a) Explain scleronomic constraints with examples. [5]
b) The length of simple pendulum changes with time such that $l = a + bt$, where a and b are constants. Find the Lagrangian equation of motion. [5]
c) Explain the principle of virtual work. [4]

- Q2)** a) Obtain Lagrange's equation of motion from D'Alembert's principle. [7]
b) Prove that if the Lagrangian does not contain time t explicitly the total energy of the conservative system is conserved. [7]

- Q3)** a) Find Lagrangian and equation of motion for a particle of mass m constrained to move on the surface of cone $y^2 + z^2 = x$. [7]
b) Show that the geodesic $\phi = \phi(\theta)$ on the surface of sphere of radius r is an arc of great circle. [7]

- Q4)** a) Show that the geodesic on right circular cylinder is helix. [5]
b) Find the curve for which functional

$$\pm(y(x)) = \int_0^{\pi/4} (y^2 - y'^2) dx$$

can have extrema, given that $y(0) = 0$, while right end point can vary along line $x = \pi/4$. [5]

- c) Explain Basic Lemma. [4]

P.T.O.

Q5) a) Find the solid of revolution with given surface area and maximum volume. [7]

b) Show that the Hamilton's principle $\delta \int_{t_0}^{t_1} L dt = 0$ also holds for non conservative system. [7]

Q6) a) State Hamilton's principle for non conservative system and hence derive from it the Lagrange's equations of motion for non conservative holonomic system. [7]

b) Prove the Kepler's third law of planetary motion. [7]

Q7) a) Prove that central force motion is always motion in a plane. [5]

b) Prove Kepler's second law of planetary motion. [5]

c) If the particles attract each other according to inverse square law of force prove that $\frac{d}{dt}(2t + v) = 0$ where t is total kinetic energy of the particles and V is the potential energy. [4]

Q8) a) A Lagrangian for a particle of unit mass is given by

$$L = \frac{1}{2} \dot{x}^2 - \frac{1}{2} \omega^2 x^2 - \alpha x^3 + \beta x \dot{x}^2.$$

Find Hamiltonian and hence show that H represents the constant of motion and also it represent total energy. [7]

b) Prove that the extremal of the isoperimetric problem $t = \int_1^4 y'^2 dx$ subject to the condition $\int_1^4 y dx = 36$ and $y(1) = 3, y(4) = 24$ is a parabola. [7]



Total No. of Questions : 8]

SEAT No. :

PE-5215

[Total No. of Pages : 3

[6508]-306

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-136 : Advanced Complex Analysis

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

Q1) a) If $\{f_n\}_{n=1}^{\infty}$ is a sequence of holomorphic functions that converges uniformly to a function f on every compact subset of Ω , then prove that f is holomorphic in Ω . [5]

b) Suppose f is holomorphic in an open set Ω and $K \subset \Omega$ is compact. Prove that there exist finitely many segments $\gamma_1, \dots, \gamma_N$ in $\Omega - K$ such that

$$f(z) = \sum_{n=1}^N \frac{1}{2\pi i} \int_{\gamma_n} \frac{f(\zeta)}{\zeta - z} d\zeta, \text{ for all } z \in K. \quad [5]$$

c) Prove that $\int_0^{\infty} \sin(x^2) dx = \int_0^{\infty} \cos(x^2) dx = \frac{\sqrt{2\pi}}{4}$. [4]

Q2) a) Prove that every non-constant polynomial $P(z) = a_n z^n + \dots + a_0$ with complex coefficients has a root in $\mathbb{C}$. [5]

b) Suppose f and g are holomorphic in a region Ω and $f(z) = g(z)$ for all z in some sequence of distinct points with a limit point in Ω . Then prove that $f(z) = g(z)$ throughout Ω . [5]

c) State the following: [4]

- i) Symmetry principle
- ii) Schwarz reflection principle

P.T.O.

- Q3)** a) Let V and U be open sets in $\mathbb{C}$ and $F : V \rightarrow U$ be a holomorphic function. If $u : U \rightarrow \mathbb{C}$ is a harmonic function, then prove that $u \circ F$ is harmonic on V . [5]
- b) Show that the map $f(z) = e^{iz}$ takes the half-strip $\{z = x + iy : -\pi/2 < x < \pi/2, y > 0\}$ conformally to the half-disc $\{w = u + iv : |w| < 1, u > 0\}$. [5]
- c) If $f : U \rightarrow V$ is holomorphic and injective, then prove that $f'(z) \neq 0$ for all $z \in U$. [4]
- Q4)** a) If f is an automorphism of the disc, then prove that there exist $\theta \in \mathbb{R}$ and $\alpha \in \mathbb{D}$ such that $f(z) = e^{i\theta} \frac{\alpha - z}{1 - \bar{\alpha}z}$. [5]
- b) Prove that the only automorphisms of the unit disc that fix the origin are the rotations. [5]
- c) Let Ω be an open subset of $\mathbb{C}$
- i) Define a uniformly bounded family $\mathcal{F}$ on compact subsets of Ω . [2]
- ii) Define an equicontinuous family $\mathcal{F}$ on a compact set $K \subset \Omega$. [2]
- Q5)** a) State and prove Montel's theorem. [7]
- b) Prove that any two proper simply connected open subsets in $\mathbb{C}$ are conformally equivalent. [5]
- c) State the Riemann mapping theorem. [2]
- Q6)** a) Suppose $S(z) = \int_0^z \frac{d\zeta}{(\zeta - A_1)^{\beta_1} \dots (\zeta - A_n)^{\beta_n}}$, where $A_1 < A_2 < \dots < A_n$ are n distinct points on the real axis arranged in increasing order, and the exponents β_k satisfy the conditions $\beta_k < 1$ for each k and $1 < \sum_{k=1}^n \beta_k$. If $\sum_{k=1}^n \beta_k = 2$ and P denotes the polygon whose vertices are given in order by $a_1, \dots, a_n$, then prove that S maps the real axis onto $P - \{a_\infty\}$. Also, prove that the point a_∞ lies on the segment $[a_n, a_1]$ and is the image of the point at infinity, and the interior angle at the vertex a_k is $\alpha_k \pi$, where $\alpha_k = 1 - \beta_k$. [7]
- b) Prove that the total number of poles of an elliptic function in P_0 is always ≥ 2 . [7]

- Q7) a)** If F is a conformal map from the upper half-plane to the polygonal region P and maps the points $A_1, \dots, A_{n-1}, \infty$ to the vertices of boundary polygon p , then prove that there exist constants C_1 and C_2 such that

$$F(z) = C_1 \int_0^z \frac{d\zeta}{(\zeta - A_1)^{B_1} \dots (\zeta - A_{n-1})^{B_{n-1}}} + C_2 \quad [7]$$

- b) If $F(z) = \int_1^z \frac{d\zeta}{(1 - \zeta^n)^{2/n}}$, then show that F maps the unit disc conformally

onto the interior of a regular polygon with n sides and perimeter

$$2^{\frac{n-2}{n}} \int_0^\pi (\sin \theta)^{-2/n} d\theta. \quad [5]$$

- c) Does there exist a holomorphic surjection from the unit disc to $\mathbb{C}$? [2]

- Q8) a)** Prove that every even elliptic function F with periods 1 and τ is a rational function of the Weierstrass $\wp$ function. [7]

- b) Prove that the conformal map F extends to a continuous function from the closure of the disc to the closure of the polygon. [5]

- c) Define elliptic integrals. [2]



Total No. of Questions : 8]

SEAT No. :

PE-5216

[Total No. of Pages : 3

[6508]-307

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-137 : Integral Equations

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions from the following.
- 2) Figures to the right side indicate full marks.

Q1) a) Explain the method of successive substitutions to find the solution of Fredholm integral equation. [5]

b) Convert the following initial value problem to an equivalent Volterra integral equation [5]

$$y^{(iv)} - y = 0; y(0) = 1, y'(0) = 0, y''(0) = -1, y'''(0) = 0$$

c) Check whether $u(x) = x + e^x$ is a solution of following Fredholm integro-differential equation or not. [4]

$$u''(x) = e^x - \frac{4}{3}x + \int_0^1 xtu(t)dt; u(0) = 1, u'(0) = 2$$

Q2) a) Find the solution $u(x)$ of the following Fredholm integral equation using Adomian Decomposition method. [5]

$$u(x) = \cos x + 2x + \int_0^\pi xtu(t)dt$$

b) Solve the following Volterra integral equation by using noise term phenomenon. [5]

$$u(x) = \sinh x + x \sinh x - x^2 \cosh x + \int_0^x xtu(t)dt$$

c) Find the initial value problem equivalent to the following Volterra integral equation

$$u(x) = x^3 + \int_0^x (x-t)^2 u(t)dt \quad [4]$$

P.T.O.

- Q3) a)** Solve the following Volterra integral equation by using the method of successive approximations [5]

$$u(x) = 1 - 9 \int_0^x (x-t)u(t)dt$$

- b) Find the solution of following homogeneous Fredholm integral equation. [5]

$$u(x) = \lambda \int_0^1 (6x-2t)u(t)dt$$

- c) Solve the following Fredholm integral equation by using direct computation method. [4]

$$u(x) = \sec^2 x - 1 + \int_0^{\pi/4} u(t)dt$$

- Q4) a)** Explain the method of Adomian decomposition to solve Volterra integral equation. [5]

- b) Solve the following Fredholm integro-differential equation by converting it to a standard Fredholm integral equation. [5]

$$u'(x) = 1 - \frac{1}{3}x + x \int_0^1 tu(t)dt, u(0) = 0$$

- c) If $u(x) = e^{-x^2}$ is a solution of the Volterra integral equation

$$u(x) = 1 - \alpha \int_0^x tu(t)dt \text{ then find the value of } \alpha. [4]$$

- Q5) a)** Solve the following Volterra integro-differential equation by converting it to a standard Volterra integral equation. [5]

$$u''(x) = -1 + \int_0^x (x-t)u(t)dt, u(0) = 1, u'(0) = 0$$

- b) Solve the following Fredholm integro-differential equation by using direct computation method. [5]

$$u'''(x) = \sin x - x - \int_0^{\pi/2} xtu'(t)dt; u(0) = 1, u'(0) = 0, u''(0) = -1$$

- c) Explain the noise term phenomenon for Fredholm integral equation. [4]

- Q6) a)** Solve the following Volterra integral equation by method of successive substitutions. [5]

$$u(x) = \frac{1}{3!}x^3 - \int_0^x (x-t)u(t)dt$$

- b) Derive an equivalent Fredholm integral equation from the following boundary value problem. [5]

$$y'' + y = x, 0 < x < \pi, y(0) = 1, y(\pi) = \pi - 1$$

- c) Find the unknown function $f(x)$ of the following Volterra integral equation

$$\text{if } u(x) = 2\cos x - 1 \text{ is a solution of } u(x) = 1 - \frac{1}{2}f(x) - \int_0^1 (x-t)u(t)dt.$$

[4]

- Q7) a)** Explain direct computation method for solving Fredholm integral equation. Hence find the solution of following Fredholm integral equation. [7]

$$u(x) = \frac{5}{6}x + \frac{1}{2}\int_0^1 xtu(t)dt$$

- b) Solve the following Volterra integral equation by converting it to an equivalent initial value problem [7]

$$u(x) = x^2 + \frac{1}{12}x^4 + \int_0^x (t-x)u(t)dt$$

- Q8) a)** Explain the series solution method for Volterra integro-differential equation. Hence find the solution of following equation. [7]

$$u''(x) = x \cosh x - \int_0^x tu(t)dt, u(0) = 0, u'(0) = 1$$

- b) Explain Leibnitz rule and hence convert the following Volterra integral equation to an equivalent initial value problem. [7]

$$u(x) = x + \int_0^x (t-x)u(t)dt$$



[6508]-308

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-138 : Differential Manifolds

(2019 Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicates full marks.

Q1) a) Let W be a k -dimensional linear subspace of $\mathbb{R}^n$. Then show that there is an orthogonal transformation $h: \mathbb{R}^n \rightarrow \mathbb{R}^n$ that carries W onto the subspace $\mathbb{R}^k \times 0$ of $\mathbb{R}^n$. [5]

b) Let $X = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{vmatrix}$ [4]

i) Find $X^{tr} \cdot X$

ii) Find $V(x)$

c) Let S be a subset of $\mathbb{R}^k$; Let $f: S \rightarrow \mathbb{R}^n$. Prove that if for each $x \in S$, there is a neighbourhood U_x of x and a function $g_x: U_x \rightarrow \mathbb{R}^n$ of class C^r that agrees with f on $U_x \cap S$, then f is of class C^r on S . [5]

Q2) a) Let $g: A \rightarrow B$ be a diffeomorphism of open sets in $\mathbb{R}^k$. Let $\beta: B \rightarrow \mathbb{R}^n$ be a map of class C^r ; Let $\alpha = \beta \circ g$; then $\alpha: A \rightarrow \mathbb{R}^n$ and $\gamma = \alpha(A)$. If $f: \gamma \rightarrow \mathbb{R}$ is a continuous function, then prove that f is integrable over γ_β if and only if it is integrable over γ_α ; in this case $\int_{\gamma_\alpha} f dv = \int_{\gamma_\beta} f dv$. [7]

P.T.O.

- b) Let X be an $n \times k$ matrix with $k \leq n$. Then prove that

$$V(X) = \left[\sum_{[I]} \det^2 X_I \right]^{1/2}, \text{ where the symbol } [I] \text{ indicates that the summation}$$

extend over all ascending k -tuples from the set $\{1, 2, \dots, n\}$. [7]

- Q3)** a) Let $a_1, a_2, \dots, a_n$ be a basis for V . If $f, g : V^k \rightarrow \mathbb{R}$ are k -tensor on V , and if $f(a_{i_1}, a_{i_2}, \dots, a_{i_k}) = g(a_{i_1}, a_{i_2}, \dots, a_{i_k})$ for every k -tuple $I = (i_1, i_2, \dots, i_k)$ of integers from the set $\{1, 2, \dots, n\}$ then show that $f = g$. [5]

- b) Determine which of the following tensor on $\mathbb{R}^4$ [4]

i) $f(x, y) = 3x_1y_2 + 5x_2x_3$

ii) $h(x, y) = x_1y_1 - 7x_2y_3$

- c) Let Y_I be an elementary alternating tensor on $\mathbb{R}^n$ corresponding to the usual basis for $\mathbb{R}^n$, where $I = (i_1, i_2, \dots, i_k)$. Given vectors $x_1, x_2, \dots, x_k$ of $\mathbb{R}^n$, Let $X = [x_1, x_2, \dots, x_k]$. Then prove that $Y_I(x_1, x_2, \dots, x_k) = \det X_I$ where X_I denotes the matrix whose successive rows are rows $i_1, i_2, \dots, i_k$ of X . [5]

- Q4)** a) Let A be open in $\mathbb{R}^k$ or H^k ; let $\alpha \rightarrow \mathbb{R}^m$ be of class C^r . Let B be an open set of $\mathbb{R}^m$ or H^m containing $\alpha(A)$; Let $B : B \rightarrow \mathbb{R}^n$ be of class C^r . Then prove that $(\beta \circ \alpha)_* = \beta_* \circ \alpha_*$ [5]

- b) Let A be an open set in $\mathbb{R}^n$. There exists a unique linear transformation $d : \Omega^k(A) \rightarrow \Omega^{k+1}(A)$ defined for $k \geq 0$. Show that for every form w , $d(dw) = 0$. [5]

- c) If $w = x^2yz dx + e^x yz dy + ze^x dz$ and $\eta = y \sin x dx + xy dy + 2yz dz$, then find $(w \wedge \eta)$. [4]

Q5) a) Let M be a K -manifold in $\mathbb{R}^n$, of class C^r . Let $\alpha_0 : U_0 \rightarrow V_0$ and $\alpha_1 : U_1 \rightarrow V_1$ be co-ordinate patches on M , with $W = V_0 \cap V_1$, non empty. Let $W_i = \alpha_i^{-1}(W)$. Then show that the map $\alpha_1^{-1} \circ \alpha_0 : W_0 \rightarrow W_1$ is of class C^r , and its derivative is non-singular. [7]

b) Show that the unit circle S^1 is a 1-manifold in $\mathbb{R}^2$. [3]

c) Let M be a compact K -manifold in $\mathbb{R}^n$, of class C^r . Let $f, g : M \rightarrow \mathbb{R}$ be continuous. Then prove that $\int_M (af + bg) dV = a \int_M f dV + b \int_M g dV$. [4]

Q6) a) In $\mathbb{R}^3$, Let $W = xy dx + 2z dy - y dz$. Let $\alpha : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be given by equation $\alpha(u, v) = (uv, u^2, 3u + v)$. [7]

i) Find dw

ii) Find α^*w

iii) Find $\alpha^*(dw)$

b) Let $T : V \rightarrow W$ be linear transformation and if f and g are alternating tensor on W , then prove that $T^*(f \wedge g) = T^*f \wedge T^*g$. [5]

c) Give an example of closed form. [2]

Q7) a) Let M be a 1-manifold in $\mathbb{R}^n$; Let f be a 0-form defined in an open set about M . If M is an arc with initial point P and final point q . Then prove that [5]

$$\int_M df = f(q) - f(p)$$

b) Let M be a compact oriented 1-manifold in $\mathbb{R}^n$; Let T be unit tangent vector to M corresponding to the orientation. Let $F(x) = (x; f(x)) = (x; \sum f_i(x)e_i)$ be a vector field defined in an open set of $\mathbb{R}^n$ containing M ; it corresponds to 1-form $W = \sum f_i dx_i$. Then prove that [5]

$$\int_M w = \int_M \langle F, T \rangle ds.$$

c) Show that Klein bottle is non-orientable 2-manifold. [4]

Q8) a) Let A be open in $\mathbb{R}^k$; let $\alpha : A \rightarrow \mathbb{R}^n$ be of class C^∞ ; Let $Y = \alpha(A)$. Let x denote the general point of A ; and Let z denote the general point of $\mathbb{R}^n$.

If $w = f dz_I$ is k -form defined in an open set of $\mathbb{R}^n$ containing Y , then show that $\int_{Y_\alpha} w = \int_A (f \circ \alpha) \det(\partial \alpha_I / \partial x) dx$. [5]

b) Let $\alpha : (0,1)^2 \rightarrow \mathbb{R}^3$ be given by $\alpha(u, v) = (u + v, uv, u^2 - v^2 + 1)$. Let Y be the image set of α . [5]

Evaluate $\int_Y x_2 dx_1 \wedge dx_3 + x_2 x_3 dx_1 \wedge dx_3$

c) Let M be a compact 1-manifold in $\mathbb{R}^n$; let T be unit tangent vector field to M . Let f be a C^∞ function defined in an open set about M . If ∂M is empty, then prove that $\int_M \langle \text{grad } f, T \rangle ds = 0$. [4]



Total No. of Questions : 8]

SEAT No. :

PE-5218

[Total No. of Pages : 4

[6508]-401

S.Y. M.A. / M.Sc.

MATHEMATICS

**MTUT-141 : Fourier Series and Boundry Value Problems
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicate full marks.*

- Q1)** a) Let f denote a function that is piecewise continuous on the interval $-\pi < x < \pi$ and periodic with period 2π on entire x axis. Then prove that Fourier series converges to the mean value $\frac{f(x^+) + f(x^-)}{2}$ of the one sided derivatives $f'_R(0)$ and $f'_L(0)$ exists. [7]
- b) Find the Fourier cosine series for the function $f(x) = x^2$, $0 < x < \pi$. [5]
- c) Find the Fourier sine series for the function $f(x) = 1$, $0 < x < \pi$. [2]

- Q2)** a) If the function $G(u)$ is piecewise continuous on the interval $0 < u < p$ then prove that [7]

$$\lim_{N \rightarrow \infty} \int_0^{\pi} G(u) \sin \frac{u}{2} + Nu \, du = 0$$

Where N denotes positive integers.

- b) Find the Fourier series for the function $f(x) = -x$, $-\pi < x < \pi$. [5]
- c) Let $f(x) = \begin{cases} 0, & \text{when } x < 0 \\ 1, & \text{when } x \geq 0 \end{cases}$ [2]

Show that the given function is not differentiable at point zero.

P.T.O.

Q3) a) Let f denote a function such that

i) f is continuous on the interval $-\pi \leq x \leq \pi$.

ii) $f(-\pi) = f(\pi)$

iii) It's derivative f' is piecewise continuous on the interval $-\pi < x < \pi$.
If a_n and b_n are the Fourier coefficients

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

for, f , then prove that the series $\sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2}$ converges. [7]

b) Find the Fourier sine series for the function [5]

$$f(x) = x(\pi^2 - x^2) \quad (0 < x < \pi)$$

c) $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ find $f'_R(0)$ and $f'_L(0)$ [2]

Q4) a) Solve the following boundry value problem. [7]

$$u_t(x, t) = k U_{xx}(x, t), \quad (0 < x < c, t > 0)$$

$$u_x(0, t) = 0, U_x(c, t) = 0, \quad (t > 0)$$

$$u(x, 0) = f(x)$$

b) Solve the following boundry value problem - [7]

$$y_{tt}(x, t) = a^2 y_{xx}(x, t), \quad (0 < x < c, t > 0)$$

$$y(0, t) = 0, y(c, t) = 0, \quad y_t(x, 0) = 0$$

$$y(x, 0) = f(x)$$

- Q5) a)** Solve the following boundary value problem. [7]
- $$u_{xx}(x, y) + u_{yy}(x, y) = 0, (0 < x < a, 0 < y < b)$$
- $$u(0, y) = 0, u(a, y) = 0, (0 < y < b)$$
- $$u(x, 0) = f(x), u(x, b) = 0, (0 < x < a)$$
- b)** Solve the following boundary value problem. [7]
- $$\rho^2 u_{\rho\rho}(s, \phi) + \rho u_{\rho}(s \cdot \phi) + u_{\phi\phi}(s, \phi) = 0, (1 < \rho < b, 0 < \phi < \pi)$$
- $$u(\rho, 0) = 0, u(\rho, \pi) = 0, (1 < \rho < b)$$
- $$u(1, \phi) = 0, u(b, \phi) = u_0, (0 < \phi < \pi)$$
- Q6) a)** If C_n ($n = 1, 2, 3, \dots$) be the Fourier constants for a function f in $C_p(a, b)$ with respect to an orthogonal set $\{\phi_n(x)\}$ ($n = 1, 2, 3, \dots$) in that space. Then prove that all possible linear combinations of the function $\phi_1(x), \phi_2(x), \dots, \phi_n(x)$ the combinations $C_1\phi_1(x) + \dots + C_n\phi_n(x)$ is the best approximation in the mean to $f(x)$ on the fundamental interval $a < x < b$. [7]
- b)** Solve the following boundary value problem. [7]
- $$u_t(x, t) = u_{xx}(x, t) + x_p(t) (0 < x < 1, t > 0)$$
- $$u(0, t) = 0, u(1, t) = 0, u(x, 0) = 0$$
- Q7) a)** If C_n ($n = 1, 2, \dots$) are the Fourier constants for a function f in $C_p(a, b)$ with the respect to an orthonormal set in that space, then prove that
- $$\lim_{n \rightarrow \infty} C_n = 0. \quad [7]$$
- b)** If $\phi_0(x) = \frac{1}{\sqrt{2\pi}}, \phi_{2n-1}(x) = \frac{1}{\sqrt{\pi}} \cos nx, \phi_{2n}(x) = \frac{1}{\sqrt{\pi}} \sin nx$ ($n = 1, 2, \dots$) then show that the set $\{\phi_n(x)\}$ ($n = 0, 1, 2, \dots$) is orthonormal on the interval $-\pi < x < \pi$. [5]
- c)** If $L = x$ and $M = \frac{\partial}{\partial x}$ are linear operators on $C_p(a, b)$ then show that the product LM and ML are not always same. [2]

Q8) a) Let λ be an eigen value of the regular Sturm-Liouville problem.

$$(r X')' + (q + \lambda p)X = 0 \quad (a < x < b)$$

$$a_1 X(a) + a_2 X'(a) = 0$$

$$b_1 X(b) + b_2 X'(b) = 0$$

If the condition's $q(x) \leq 0$ ($a \leq x \leq b$) and $a_1 a_2 \leq 0$, $b_1, b_2 \geq 0$ are satisfied then prove that $\lambda \geq 0$. **[7]**

b) If λ_m and λ_n are distinct eigen values of the Sturm-Liouville problem.

$$[r(x) X'(x)]' + [q(x) + \lambda p(x)]X(x) = 0$$

($a < x < b$) under condition

$$a_1 X(a) + a_2 X'(a) = 0,$$

$b_1 X(b) + b_2 X'(b) = 0$, then prove that corresponding eigen function's $X_m(x)$ and $X_n(x)$ are orthogonal with respect to weight function $p(x)$ on the interval $a < x < b$. **[7]**



Total No. of Questions : 8]

SEAT No. :

PE-5219

[Total No. of Pages : 2

[6508]-402

S.Y. M.A./M.Sc.

MATHEMATICS

**MTUT-142 : Differential Geometry
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicates full marks.*

- Q1)** a) Show that the velocity vector field along a parametrized curve $\alpha(t)$ in an n -surface S is parallel if and only if $\alpha(t)$ is geodesic. [5]
- b) Show that the graph of any function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a level set for same function $g : \mathbb{R}^{n+1} \rightarrow \mathbb{R}$. [5]
- c) Define level sets and explain how it is used in geography. [4]
- Q2)** a) Explain why an integral curve cannot cross itself as does the parametrized curve. [5]
- b) Define complete vector field and check wheather the vector field $X(x_1, x_2) = (x_1, x_2, 1 + x_1^2, 0)$ is complete or not. [4]
- c) Sketch the level sets for the function $f(x_1, x_2) = x_1^2 + x_2^2$ with $c = 0, 1, 2, 3, 4$. [4]
- Q3)** a) Define gradient of function. Hence, show that the gradient of f at a point $P \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at P . [6]
- b) Show that a parametrized curve $\alpha : I \rightarrow S$ is geodesic in S if and only if it's covariant acceleration $\alpha'(t)$ is zero along $\alpha(t)$. [6]
- c) Define terms: [4]
- i) Gauss Map
 - ii) Weingarten Map

P.T.O.

- Q4)** a) Show that Möbius band is an unorientable 2-surface. [5]
 b) State and prove any two properties of Levi-Civita parallel. [5]
 c) Show by an example that, the set of vectors tangent at point 'P' of level sets need not in general be a vector subspace of $\mathbb{R}_p^{n+1}$. [4]
- Q5)** a) Find the Global parametrization of the curve [5]
 $(x_1 - a)^2 + (x_2 - b)^2 = r^2$
 b) If $\alpha(t) = (x(t), y(t)) (t \in I)$ is a local parametrization of the oriented plane curve C. Then show that $k \circ \alpha = (x'y'' - y'x'') / (x'^2 + y'^2)^{3/2}$. [5]
 c) Show that the cylinder $x_1^2 + x_2^2 = 1$ in $\mathbb{R}^3$ can be represented as a level set of the function, $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ by $f(x_1, x_2, x_3) = 2x_1^2 + 2x_2^2 + \sin(x_1^2 + x_2^2)$. [4]
- Q6)** a) State and prove Lagrange's multiplier theorem. [7]
 b) If S is compact connected oriented n-surface in $\mathbb{R}^{n+1}$ exhibited as a level set $f^{-1}(c)$ of a smooth function $f: \mathbb{R}^{n+1} \rightarrow \mathbb{R}$ with $\nabla f|_p \neq 0, \forall p \in S$. Then show that the Gauss map maps S onto the unit sphere S^n . [7]
- Q7)** a) Show that a parametrized curve $\alpha(t)$ in the unit sphere $x_1^2 + x_2^2 + \dots + x_{n+1}^2 = 1$ is a geodesic if and only if it is of the form $\alpha(t) = (\cos at)e_1 + (\sin at)e_2$. [5]
 b) Show that if $\alpha: I \rightarrow \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then $\dot{\alpha}(t) \perp \ddot{\alpha}(t)$ for all $t \in I$. [5]
 c) Find the velocity, acceleration and the speed of the parametrized curve $\alpha(t) = (t, t^2)$ [4]
- Q8)** a) If C is a connected oriented plane curve and $\beta: I \rightarrow C$ be unit speed global parametrization of C. Then show that β is either one to one or periodic. Moreover, β is periodic if and only if C is compact. [7]
 b) Let S be an n-surface in $\mathbb{R}^{n+1}$ and $\alpha: I \rightarrow S$ be a parametrized curve in S. If $t_0 \in I$ and $\bar{V} \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field $\bar{V}$ tangent to S along $\alpha(t)$ which is parallel and has $\bar{V}(t_0) = \bar{V}$. [7]



Total No. of Questions : 4]

SEAT No. :

[Total No. of Pages : 1

PE-5220

[6508]-403

S.Y. M.A./M.Sc.

MATHEMATICS

MTUT-143 : Introduction to Data Science

(2019 Pattern) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *Question No.1 is compulsory.*
- 2) *Figures to the right side indicate full marks.*
- 3) *Attempt any two questions from Q.No. 2, 3 and 4.*

Q1) What is machine learning? Describe types of machine learning in detail. [7]

Q2) a) Describe all steps involved in Data Science Process. [5]

b) Explain any five facets of data. [5]

c) State details to be covered in a project charter. [4]

Q3) a) State any five techniques used to handle the missing data. [5]

b) Write a short note on confusion matrix. [5]

c) State the characteristics of big data. [4]

Q4) a) Explain in detail concept of Hadoop and its components. [5]

b) What is text mining? Give packages in python which are used for text mining. [5]

c) What are the applications of machine learning? [4]



Total No. of Questions : 8]

SEAT No. :

PE-5221

[Total No. of Pages : 3

[6508]-404

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-144 : Number Theory
(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to right indicates full marks.

- Q1)** a) If $a, b \in \mathbb{Z}$ and $b > 0$ then show that there exist $q, r \in \mathbb{Z}$ such that $a = qb + r$ with $0 \leq r < b$. [5]
- b) If R is a Euclidean Domain and $I \subseteq R$ is an ideal, then show that there is an element $a \in R$ such that $I = Ra = \{ra / r \in R\}$. [5]
- c) Let $a, b, c \in \mathbb{Z}$. Show that the equation $ax + by = c$ has solutions in integers if and only if $(a, b) \mid c$. [4]
- Q2)** a) Show that $61! + 1 \equiv 63! + 1 \equiv 0 \pmod{71}$ [5]
- b) Find the last digit in the ordinary decimal representation of 2^{400} . [5]
- c) Prove that $a^{\phi(m)} \equiv 1 \pmod{m}$ where $(a, m) = 1$. [4]
- Q3)** a) Find all integers that satisfies simultaneously. [7]
 $x \equiv 5 \pmod{7}$
 $x \equiv 7 \pmod{11}$
 $x \equiv 3 \pmod{13}$
- b) Find the number of positive integers less than or equal to 3600 that have a factor greater than 1 in common with 3600. [4]
- c) Evaluate the Legendre Symbol: $\left(\frac{-42}{61}\right)$. [3]

P.T.O.

Q4) a) If p and q are distinct primes then prove that [7]

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left(\frac{p-1}{2}\right)\left(\frac{q-1}{2}\right)}$$

b) If p is an odd prime and n denote the least positive quadratic nonresidue modulo p then show that $n < 1 + \sqrt{p}$. [4]

c) Determine whether $x^2 \equiv 10 \pmod{89}$ is solvable or not. [3]

Q5) a) If x and y are real numbers then prove that: [6]

i) $[x] + [-x] = \begin{cases} 0, & \text{if } x \text{ is an integer} \\ -1, & \text{otherwise} \end{cases}$

ii) $-[-x]$ is the least integer $\geq x$

b) Find the highest power of 6 dividing $100!$. [4]

c) If n is a positive integer, then prove that $d(n) = \prod_{p^\alpha \parallel n} (\alpha + 1)$. [4]

Q6) a) Prove that Mobius mu function $\mu(n)$ is multiplicative and [6]

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & \text{if } n = 1 \\ 0, & \text{if } n > 1 \end{cases}$$

b) Find $d(200)$, $\sigma(200)$ and $\Omega(200)$. [6]

c) Evaluate $\sum_{j=1}^{\infty} \mu(j!)$ [2]

Q7) a) If $f(n) = \sum_{d|n} \mu(d)F(n/d)$ for every positive integer n , then prove that

$$F(n) = \sum_{d|n} f(d). \quad [7]$$

b) Prove that the integers of any algebraic number field form a ring. [5]

c) Prove that if p is a prime and $a^2 \equiv b^2 \pmod{p}$ then $p|(a+b)$ or $p|(a-b)$. [2]

- Q8)** a) If ξ is algebraic number of degree n then prove that every number $Q(\xi)$ can be written uniquely in the form $a_0 + a_1\xi + \cdots + a_{n-1}\xi^{n-1}$ where a_i are rational numbers. [7]
- b) Show that the minimal equation of an algebraic integer is monic with integral coefficients. [4]
- c) Find the remainder obtained by dividing $1! + 2! + 3! + \dots + 100!$ by 12. [3]



Total No. of Questions : 8]

SEAT No. :

PE-5222

[Total No. of Pages : 2

[6508]-405

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-145 : Algebraic Topology
(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicate full marks.

- Q1)** a) Given spaces X and Y , let $[X, Y]$ denote the set of homotopy classes of maps of X into Y . Let $I = [0, 1]$. Show that for any X , the set $[X, I]$ has a single element. [5]
- b) Prove that path homotopy relation is an equivalence relation. [5]
- c) A space X is said to be contractible if the identity map $i_X : X \rightarrow X$ is null homotopic. Show that if Y is contractible, then for any X , the set $[X, Y]$ has a single element. [4]
- Q2)** a) Show that if A is star convex, then A is simply connected. [5]
- b) Let α be a path in X from x_0 to x_1 and β be a path in X from x_1 to x_2 . Show that if $\gamma = \alpha * \beta$, then $\hat{\gamma} = \hat{\beta} \circ \hat{\alpha}$. [5]
- c) Let $A \subset X$ and suppose $r : X \rightarrow A$ is a retraction, i.e. a continuous map such that $r(a) = a$ for each $a \in A$. If $a_0 \in A$, show that $r_* : \pi_1(X, a_0) \rightarrow \pi_1(A, a_0)$ is surjective. [4]
- Q3)** a) Let Y have the discrete topology. Show that if $p : X \times Y \rightarrow X$ is projection on the first coordinate, then p is a covering map. [5]
- b) Let $p : E \rightarrow B$ be a covering map. If $p(e_0) = b_0$, then prove that the homomorphism $p_* : \pi_1(E, e_0) \rightarrow \pi_1(B, b_0)$ is a monomorphism. [5]
- c) Let A be a 3 by 3 matrix of positive real numbers. Then prove that A has a positive real eigenvalue. [4]

P.T.O.

- Q4)** a) Show that $\mathbb{R}^1$ and $\mathbb{R}^n$ are not homeomorphic if $n > 1$. [5]
 b) Let $f : X \rightarrow Y$ be continuous and let $f(x_0) = y_0$. If f is a homotopy equivalence, prove that $f_* : \pi_1(X, x_0) \rightarrow \pi_1(Y, y_0)$ is an isomorphism. [5]
 c) Show that if A is a retract of B^2 , then every continuous map $f : A \rightarrow A$ has a fixed point. [4]

- Q5)** a) Define a covering map. Show that a covering map is a local homeomorphism. Is the converse true? Justify. [7]
 b) State and prove the Borsuk-Ulam Theorem for S^2 . [4]
 c) State the general lifting lemma. [3]

- Q6)** a) Let p, q and r be continuous maps with $p = r \circ q$ as shown in the following diagram.

$$\begin{array}{ccc} X & \xrightarrow{q} & Y \\ p \downarrow & & \swarrow r \\ Z & & \end{array}$$

- Prove that, if p and r are covering maps, so is q . [7]
 b) Let $\pi : E \rightarrow X$ be a closed quotient map. Prove that if E is normal, then so is X . [7]
- Q7)** a) Define the following terms: [6]
 i) Free group
 ii) Wedge of the circles
 iii) Commutator subgroup
 b) Let X be the wedge of circles S_α for $\alpha \in J$. Prove that X is normal. [4]
 c) State the Seifert-van Kampen theorem. [4]

- Q8)** a) Let $p : E \rightarrow B$ and $p' : E' \rightarrow B$ be covering maps. Let $p(e_0) = p'(e'_0) = b_0$. Prove that, there is an equivalence $h : E \rightarrow E'$ such that $h(e_0) = e'_0$ if and only if the groups $H_0 = p_*(\pi_1(E, e_0))$ and $H'_0 = p'_*(\pi_1(E', e'_0))$ are equal. [5]
 b) Let $p : E \rightarrow B$ be a covering map, with E simply connected. Show that, given any covering map $r : Y \rightarrow B$, there is a covering map $q : E \rightarrow Y$ such that $r \circ q = p$. [5]
 c) Find a continuous map of the torus into S^1 that is not null homotopic. [4]



Total No. of Questions : 8]

SEAT No. :

PE-5223

[Total No. of Pages : 3

[6508]-406

S.Y. M.A./M.Sc.

MATHEMATICS

**MTUTO-146 : Representation Theory of Finite Groups
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicate full marks.*

- Q1)** a) What are the possible dimensions of irreducible representations of the Quaternions group Q_4 ? [5]
- b) Show that eigenvalues of unitary matrix lies on unit circle in $\mathbb{C}$. Also set of $n \times n$ unitary matrices forms a subgroup of $GL_n(\mathbb{C})$. [5]
- c) Show that minimal polynomial of a matrix divides its characteristic polynomial. [4]
- Q2)** a) Give standard representations of S_n . Find all S_n - invariant subspaces of $GL_n(\mathbb{C})$. [7]
- b) If $\phi : G \rightarrow GL(V)$ is a representation of degree 2 then show that is irreducible if and only if there is no common eigenvector v to all ϕ_g with $g \in G$. [7]
- Q3)** a) Let $\phi : G \rightarrow GL(V)$ be equivalent to a decomposable representation. Then show that it is decomposable. [5]
- b) Let $\phi : G \rightarrow GL(V)$ be a unitary representation of a group. Then show that it is either irreducible or decomposable. [5]
- c) Give an indecomposable representation of $\mathbb{Z}$, which is not irreducible. [4]

P.T.O.

- Q4)** a) Let $\chi: G \rightarrow \mathbb{C}^*$ be a degree 1 representation of G . Define a map $\phi^\chi: G \rightarrow GL_n(\mathbb{C})$ by $\phi_g^\chi = \chi(g)\phi_g$. Show that ϕ^χ is a representation. Give an example showing that ϕ and ϕ^χ do not have to be equivalent. [5]
- b) Let $\phi, \psi: G \rightarrow \mathbb{C}^*$ be one-dimensional representations. Show that is equivalent to ψ if and only if $\phi = \psi$. [5]
- c) Let G be an abelian group. Then show that any irreducible representation of G has degree one. [4]
- Q5)** a) Let $Z(L(G))$ be a space of class functions of a group G . Then show that set of all irreducible characters is an orthonormal basis for $Z(L(G))$. [5]
- b) Let G be a group. Show that the regular representation is a unitary representation of G . [5]
- c) Compute the character table of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$. [4]
- Q6)** a) Let χ be a non-trivial irreducible character of a finite group G . Show that $\sum_{g \in G} \chi(g) = 0$. [5]
- b) Define character χ_ϕ of a representation ϕ of a group G . Show that every character is constant on conjugacy classes of a group G . [5]
- c) Let G be an abelian group. Then show that any irreducible representation of G has degree one. [4]
- Q7)** a) Let G_1 and G_2 be abelian groups. Let $\chi_1, \dots, \chi_m$ and $\phi_1, \dots, \phi_n$ be the irreducible representations of G_1, G_2 , respectively. Then show that the functions $\alpha_{ij}: G_1 \times G_2 \rightarrow \mathbb{C}^*$ with $1 \leq i \leq m$ and $1 \leq j \leq n$ given by $\alpha_{ij}(g_1, g_2) = \chi_i(g_1)\phi_j(g_2)$ form a complete set of irreducible representations of $G_1 \times G_2$. [5]
- b) Show that if G is a finite group and g is a non-trivial element of G , then there is an irreducible representation ϕ with $\phi(g) \neq I$. [5]
- c) Let $A \in GL_m(\mathbb{C})$ be a matrix of finite order. Then show that A is diagonalizable. [4]

- Q8)** a) Let G' be commutator subgroup of a group G and $\phi : G \rightarrow G/G'$ be the canonical homomorphism given by $\phi(g) = gG'$. Prove that every degree one representation $\rho : G \rightarrow \mathbb{C}^*$ is of the form $\psi \circ \phi$, where $\psi : G/G' \rightarrow \mathbb{C}^*$ is a degree one representation of the abelian group G/G' . [5]
- b) Compute Character table of D_4 . [5]
- c) Compute Character table of A_4 [4]



Total No. of Questions : 8]

SEAT No. :

PE-5524

[Total No. of Pages : 3

[6508]-407

S.Y. M.A./M.Sc.

MATHEMATICS

MTUTO-147 : Coding Theory
(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) Attempt any five questions.
- 2) Figures to the right indicates full marks.

- Q1)** a) Prove that a code C is u -error-detecting if and only if $d(c) \geq u + 1$ [5]
b) For the ternary code $C = \{00122, 12201, 20110, 22000\}$, use the nearest neighbour decoding rule to decode the following received words: [6]
i) 01122
ii) 10021
iii) 20120
c) Let x, y, z be words of length n over an alphabet A . Then show that $d(x, z) \leq d(x, y) + d(y, z)$. [3]
- Q2)** a) Let C be a linear code and let H be a parity-check matrix for C . Then prove that C has distance $\geq d$ if and only if any $d - 1$ columns of H are linearly independent. [5]
b) Find a generator matrix and parity-check matrix for the binary linear code $C = \langle S \rangle$, Where $S = \{11101, 10110, 01011, 11010\}$. [7]
c) Find $\dim(C)$ of a binary linear code $C = \{000, 001, 100, 101\}$. [2]
- Q3)** a) Let $g(x)$ be the generator polynomial of q -ary $[n, k]$ - cyclic code C . Put $h(x) = \frac{x^n - 1}{g(x)}$. Then show that $h_0^{-1}h_R(x)$ is the generator polynomial of $C^\perp$, where h_0 is the constant term of $h(x)$. [7]

P.T.O.

- b) Let $g(x) = 1 + x^4 + x^6 + x^7 + x^8 \in F_2[x]$ be the generator polynomial of a binary code $[15, 7]$ cyclic code C . Write down a generator matrix and parity-check matrix for C . Construct a generator matrix of the form (I_7/A) . [7]

- Q4)** a) Show that narrow-sense q -ary BCH code of length $q^m - 1$ with designed distance δ has dimension exactly $q^m - 1 - m(\delta - 1)$ if $q \neq 2$ and $\gcd(q^m - 1, e) = 1$ for all $1 \leq e \leq \delta - 1$. [7]
 b) Determine the generator polynomial and dimension of the smallest cyclic code containing $1000111 \in F_2^7$. [7]

- Q5)** a) Let C be a linear code of length n over F_q . Then show that [7]

- i) $|C| = q^{\dim(C)}$
 ii) $C^\perp$ is a linear code
 iii) $(C^\perp)^\perp = C$

- b) Assign message to the words in F_2^3 as follows: [7]

000	100	010	001	110	101	011	111
A	C	D	E	G	I	N	O

Let C be the binary code with generator matrix $G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$

use G to encode the message ENCODING.

- Q6)** a) Suppose d is odd. Show that binary (n, M, d) - code exists if and only if a binary $(n + 1, M, d + 1)$ code exists. [7]
 b) Find optimal binary code with $n = 3$ and $d = 2$. [5]
 c) Find size and distance of the binary code of all the words w of length 3. [2]

- Q7)** a) Prove that for an integer $q > 1$ and integers n, d such that $1 \leq d \leq n$, we

$$\text{have } A_q(n, d) \leq \frac{q^n}{\sum_{i=0}^{\lceil (d-1)/2 \rceil} \binom{n}{i} (q-1)^i} \quad [6]$$

- b) Prove that $A_q(n, d) \leq q A_q(n-1, d)$. [4]
 c) Determine whether the binary code with parameters $(8, 29, 3)$ exists. [4]

- Q8)** a) Show that for any prime p and integer $n \geq 1$, there exists a unique finite field of p^n elements. [5]
- b) Find all cyclotomic cosets of 2 mod 15. [5]
- c) Find all irreducible quadratic and cubic polynomials over F_2 . [4]



Total No. of Questions : 8]

SEAT No. :

PE-5225

[Total No. of Pages : 4

[6508]-408

S.Y. M.A./M.Sc.

MATHEMATICS

**MTUTO-148 : Probability and Statistics
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *Attempt any five questions.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of Scientific calculator is allowed.*

- Q1) a)** The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, then what is the probability that **[5]**
- i) Atleast 10 survive
 - ii) From 3 to 8 survive
- b)** One bag contains 4 white balls and 3 black balls, second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from second bag is black? **[5]**
- c)** If $X_1, X_2, \dots, X_n$ are mutually independent random variables that have respectively chi squared distributions with $V_1, V_2, \dots, V_n$ degrees of freedom, then prove that the random variable $Y = X_1 + X_2 + \dots + X_n$ has a chi squared distribution with $V = V_1 + V_1 + \dots + V_n$ degrees of freedom. **[4]**
- Q2) a)** In an NBA championship series, the team that wins four games out of seven is the winner. Suppose that the teams A and B face each other in the championship games and team A has probability 0.55 of winning a game over team B. **[5]**
- i) What is the probability that the team A will win the series in 6 games?
 - ii) What is the probability that the team A will win the series?

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- b) Show that the mean and variance of the binomial distribution $b(x; n, p)$ are $\mu = np$ and $\sigma^2 = npq$ respectively. [6]
- c) In how many ways can 8 people be seated at a round table if two particular person must not sit next to each other? [3]

- Q3)** a) i) Given a standard normal distribution, find the value of K such that $P(Z > K) = 0.3015$.
- ii) Given a random variable X having a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that X assumes a value between 45 and 62. [5]
- b) Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, then find the joint probability function $f(x, y)$. [5]
- c) Suppose that the number of cars X that pass through a car wash between 4:00 pm and 5:00pm on any sunny Friday has the following probability distribution: [4]

X	4	5	6	7	8	9
$P(X = x)$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{6}$

Let $g(x) = 2x - 1$ represent the amount of money in dollars, paid to the attendant, find the attendant's expected earnings for this particular time period.

- Q4)** a) Show that the covariance of two random variables X and Y with mean μ_x and μ_y respectively is [5]
 $\sigma_{xy} = E(XY) - (\mu_x)(\mu_y)$
- b) A random variable X has mean $\mu = 8$ and variance $\sigma^2 = 9$ and an unknown probability distribution, find [5]
- i) $P(-4 < X < 20)$
- ii) $P(1X - 8 \geq 6)$
- c) Attempt following: [4]
- i) Define probability density function
- ii) State Baye's theorem

- Q5) a)** Compute and interpret the correlation coefficient for the following data. [5]

Mathematics grade	English grade
70	74
92	84
80	63
74	87
65	78
83	90

- b) Determine the value of C so that the following functions represent probability distribution of the discrete random variable X. [5]

i) $f(x) = c(x^2 + 4)$ for $x = 0, 1, 2$

ii) $f(x) = c \binom{2}{x} \binom{3}{3-x}$ for $x = 0, 1, 2$

- c) i) A coin is tossed twice. What is the probability that at least 1 head occurs?
 ii) Define the moment generating function of the random variable X. [4]

- Q6) a)** A manufacturing firm employees three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times in fact plant 1, 2 and 3 are used for 30%, 20% and 50% of the products respectively. The defect rate is different for 3 procedures as follows

$$P(D/P_1) = 0.01, P(D/P_2) = 0.03, P(D/P_3) = 0.02$$

where $P(D/P_i)$ is the probability of the defective product given plan i . If a random product was observed and found to be defective, which plan was observed most likely used and thus responsible. [6]

- b) A committee of K people is to be chosen from a set of seven women and four men. How many ways are there to form the committee if [5]
 i) The committee consists of three women and men
 ii) The committee can be any positive size but must have equal numbers of women and men?
- c) List the elements of each of the following sample spaces: [3]
 i) The set of integers between 1 and 50 divisible by 8
 ii) The set $S = \{x \mid x^2 + 4x - 5 = 0\}$
 iii) The set $S = \{x \mid 2x - 4 \geq 0 \text{ and } x < 1\}$

- Q7) a)** Suppose that a large conference room at a certain company can be reserved for no more than 4 hours. Both long and short conferences occur quite often. In fact, it can be assumed that the length X of a conference has a uniform distribution on the interval $[0, 4]$. [5]
- What is the probability that any given conference lasts atleast 3 hours?
 - What is the probability that any given conference lasts atmost 3 hours?
- b)** Let X and Y be two random variables with moment generating functions $M_x(t)$ and $M_y(t)$ respectively. If $M_x(t) = M_y(t)$ for all values of t , then prove that X and Y have the same probability distribution. [5]
- c)**
- Define the conditional probability of B given A .
 - If A and A' are complementary events, then show that $P(A) + P(A') = 1$. [4]

- Q8) a)** Given the joint density function [5]

$$f(x, y) = \begin{cases} \frac{x(1+3y^2)}{4}, & 0 < x < 2, 0 < y < 1 \\ 0 & , \text{elsewhere} \end{cases}$$

Find $f(x/y)$ and $P\left(\frac{1}{4} < X < \frac{1}{2} \mid Y = \frac{1}{4}\right)$.

- b)** Let the random variable X represents the number of automobiles that are used for official business purposes on any give workday. The probability distribution for company A is

x	1	2	3
$f(x)$	0.3	0.4	0.3

and for company B is

x	0	1	2	3	4
$f(x)$	0.2	0.1	0.3	0.3	0.1

Show that the variance of the probability distribution for company B is greater than that of company A. [6]

- c)** If a and b are constants, then show that $E(aX + b) = aE(X) + b$ where X is random variable. [3]

