

Total No. of Questions : 4]

SEAT No. :

PE4398

[Total No. of Pages : 2

[6508]-1

First Year M.A./M.Sc.

MATHEMATICS

MTS-501 MJ : Linear Algebra

(2023 Credit Pattern) (Semester - I)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[6]

- a) Show that every finite dimensional inner product space has an orthonormal basis.
- b) Show that $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x, y) = (y, x)$ is a linear transformation.
- c) Prove that similar matrices have same characteristic polynomial.
- d) Prove that if ϕ is a bilinear form on V then $\phi(\alpha, 0) = 0 = \phi(0, \alpha)$ for all $\alpha \in V$.

Q2) Attempt any two of the following.

[10]

- a) Write all possible Jordan canonical forms if the characteristic polynomial is $(x + 3)^2 (x + 4)^2 (x + 5)$.
- b) If W_1 and W_2 are finite dimensional subspaces of a vector space V , then show that $W_1 + W_2$ is finite dimensional and
$$\dim W_1 + \dim W_2 = \dim (W_1 + W_2) + \dim (W_1 \cap W_2).$$
- c) Let V be a finite dimensional inner product space and f a linear functional on V , then prove that there exists a unique vector β in V such that $f(\alpha) = (\alpha|\beta)$ for all α in V .

PTO.

Q3) Attempt any two of the following.

[10]

- a) Let V and W be vector spaces over the field F and let T be a linear transformation from V into W . If T is invertible, then prove that the inverse function T^{-1} is a linear transformation from W onto V .
- b) Describe the bilinear forms on $\mathbb{R}^3$ which satisfy $f(\alpha, \beta) = -f(\beta, \alpha)$ for all α, β .
- c) Let $B = \{\alpha_1, \alpha_2, \alpha_3\}$ be the ordered basis for $\mathbb{R}^3$ consisting of $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$. Which are the coordinates of the vector (a, b, c) in the ordered basis B ?

Q4) a) Attempt any one of the following.

[4]

- i) Prove that a normal and nilpotent operator is zero operator.
- ii) If T is a linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (x, x + y)$ then find the matrix of T with respect to standard basis of $\mathbb{R}^2$.

b) Attempt any one of the following.

[5]

- i) Let F be a field and let B be an $n \times n$ matrix over F , then prove that, B is similar over the field F to one and only one matrix which is in rational form.
- ii) Let T be a linear operator on $\mathbb{R}^3$ which is represented in the standard

ordered basis by the matrix
$$\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}.$$

Prove that T is diagonalizable by exhibiting a basis for $\mathbb{R}^3$, each vector of which is a characteristic vector of T .



[6508]-2
First Year M.A./M.Sc.
MATHEMATICS
MTS-503 MJ : Group Theory
(NEP 2020) (2023 Credit Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicates full marks.*

Q1) Attempt any five of the following. **[10]**

- a) Write out the cycle decomposition of each element of order 3 in S_4 .
- b) Show that $GL_2(F_2)$ is non abelian.
- c) Prove that the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic.
- d) Let G and H be groups and let $\phi : G \rightarrow H$ be a homomorphism. Prove that $\ker \phi$ is a subgroup of G .
- e) Write a class equation of the group D_8 .
- f) What is order of $(2, 3)$ in $\mathbb{Z}_4 \times \mathbb{Z}_4$?
- g) Write the non isomorphic abelian groups of order 8.

Q2) Attempt any Two of the following. **[10]**

- a) Show that the order of S_n is $n!$.
- b) Let $\sigma = (1\ 3\ 5)(2\ 4)$ and $\tau = (1\ 5)(2\ 3)$. Find the orders and cycle decompositions of each of the permutations $\sigma^2, \sigma\tau, \tau\sigma, \tau^2\sigma$.
- c) Let G be a group, $x \in G$ and $a \in \mathbb{Z} - \{0\}$. If $|x| = n < \infty$ then prove that

$$|x^a| = \frac{n}{(n, a)}.$$

Q3) Attempt any two of the following. **[10]**

- a) Prove that if H and K are subgroups of G then so is their intersection $H \cap K$.
- b) Draw the lattice of subgroups of the group Q_8 .
- c) If a and b are commuting elements of G , prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$.

P.T.O.

Q4) Attempt any two of the following. **[10]**

- a) State and prove first isomorphism theorem.
- b) Let G be any group containing a subgroup H of index 2. Prove that $H \trianglelefteq G$.
- c) Let G be a finite group and let $g_1, g_2, \dots, g_r$ be representatives of the distinct conjugacy classes of G not contained in the centre $Z(G)$ of G .

Then prove that $|G| = |Z(G)| + \sum_{i=1}^r |G : C_G(g_i)|$.

Q5) Attempt any two of the following. **[10]**

- a) Let G be a group of order $p^\alpha m$, where P is a prime not dividing m . Prove that $\text{syl}_p(G) \neq \phi$.
- b) Let G be a group acting on the nonempty set A . Prove that the relation on A defined by $a \sim b$ if and only if $a = g \cdot b$ for some $g \in G$ is an equivalence relation.
- c) Prove that a subgroup N of the group G is normal if and only if it is the kernel of some homomorphism.

Q6) Attempt any two of the following. **[10]**

- a) Let G be a group, let $x, y \in G$ and let $H \leq G$. Then prove that $H \trianglelefteq G$ if and only if $[H : G] \leq H$.
- b) List the non isomorphic abelian group of order 32.
- c) Show that every subgroup of cyclic group is cyclic.

Q7) Attempt any two of the following. **[10]**

- a) Let $m, n \in \mathbb{Z}^+$. Prove that $\mathbb{Z}_m \times \mathbb{Z}_n \cong \mathbb{Z}_{mn}$ if and only if $(m, n) = 1$.
- b) Let G be a group, $x, y \in G$ prove that
 - i) $xy = yx [x, y]$
 - ii) $[y, x] = [x, y]^{-1}$
- c) If G is a group of prime order p , then prove that G is cyclic and $G \cong \mathbb{Z}_p$.



Total No. of Questions : 7]

SEAT No. :

PE4400

[Total No. of Pages : 3

[6508]-3

First Year M.A./M.Sc.

MATHEMATICS

MTS-504-MJ : Ordinary Differential Equations
(2023 Credit Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following.

[10]

- a) Verify that $\phi(x) = \sin x - 1$ is the solution of differential equation $y' + (\cos x)y = \sin x \cos x$.
- b) Solve the differential equation $y' + 5y = 2$.
- c) Determine whether the functions $\phi_1(x) = x$ and $\phi_2(x) = e^{rx}$, r is a complex constant, are linearly dependent or independent.
- d) Find solutions of the differential equation $y'' + y' - 6y = 0$.
- e) Find the singular points of Legendre's equation.
- f) Find the roots of the indicial polynomial associated with the differential equation $x^2 y'' + 2xy' - 6y = 0$.
- g) Determine whether the differential equation $x^2 y^3 dx - x^3 y^2 dy = 0$ is exact.

Q2) Attempt any two of the following.

[10]

- a) Explain the method of solving the differential equation $y' + a(x)y = b(x)$, where $a(x)$ and $b(x)$ are continuous functions on some interval I .
- b) Show that every solution of the equation $x^2 y' + 2xy = 1$ on $(0, \infty)$ tends to zero as $x \rightarrow \infty$.
- c) Let $\phi(x)$ be a function with continuous derivative on $0 \leq x \leq 1$ satisfying $\phi'(x) - 2\phi(x) \leq 1$ and $\phi(0) = 1$. Then show that $\phi(x) \leq \frac{3}{2}e^{2x} - \frac{1}{2}$.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Let ϕ_1 and ϕ_2 be the two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ where a_1 and a_2 are constants. Then prove that.

$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} \cdot W(\phi_1, \phi_2)(x_0)$, where $x_0 \in I$ (an interval where the solutions are defined).

- b) Find the solution of the initial value problem $y'' + (4i + 1)y + y = 0$; $y(0) = 0, y'(0) = 0$.
- c) Find the solution of $y''' - 4y' = 0$ and compute the Wronskian of three linearly independent solutions of this equation.

Q4) Attempt any two of the following.

[10]

- a) Explain the methods of variation of constants.
- b) Verify that the function $\phi_1(x) = x, x > 0$ satisfy the equation $x^2 y'' - xy' + y = 0$ and find the second independent solution.
- c) Find two linearly independent power series solutions of the equation $y'' - xy = 0$.

Q5) Attempt any two of the following.

[10]

- a) Show that if ϕ_1 is a solution of the equation $y'' + a_1(x)y' + a_2(x)y = 0$ on

I with $\phi_1(x) \neq 0$ on I then $\phi_2(x) = \phi_1(x) \int_{x_0}^x \frac{1}{[\phi_1(s)]^2} \exp \left[- \int_{x_0}^s a_1(t) dt \right] ds$ is

the second solution on I .

- b) Find two solutions of Bessel's equation $x^2 y'' + xy' + x^2 y = 0$ of order zero.
- c) Compute indicial polynomial and its roots for the differential equation $x^2 y'' + (x + x^2)y' - y = 0$.

Q6) Attempt any two of the following.

[10]

- a) Show that a function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ if and only if it is a solution of the integral equation

$$y - y_0 + \int_{x_0}^x f(t, y) dt \text{ on } I.$$

- b) Determine whether the equation $(\cos x \cdot \cos^2 y)dx - (\sin x \cdot \sin 2y)dy = 0$ is exact and find its solution.
- c) Compute the first four successive approximations ϕ_0, ϕ_1, ϕ_2 and ϕ_3 of the solution of the initial value problem $y' = 1 + xy, y(0) = 1$.

Q7) Attempt any two of the following.

[10]

- a) Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \leq a, |y - y_0| \leq b$ then the equation $M(x, y) + N(x, y)y' = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R .
- b) Prove that the Wronskian of two solutions of the homogeneous system of differential equation $\bar{x}' = A\bar{x}$, where A is a 2×2 matrix, is either always zero or never zero.

- c) Find the general solution of $\bar{x}' = A\bar{x}$ where $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ -2 & 0 & 2 \end{bmatrix}$.



[6508]-4

First year M.A/M.Sc.

MATHEMATICS

MTS - 506(A) - MJ : Advanced Numerical Analysis
(2023 Credits Pattern) (Semester - I) (NEP 2020)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Use of single, non-programmable scientific calculator is allowed.
- 3) Figures to the right indicate full marks.

Q1) Attempt any THREE of the following. [6]

- a) Show the order of convergence of Newton's method is two.
- b) Verify that $x = \sqrt{a}$ is a fixed point of the function $g(x) = \frac{x^3 + 3xa}{3x^2 + a}$.
- c) Find the vector valued function F associated with the following system and compute the Jacobian of F

$$x^3 - 20x - y + 15 = 0, x + xy^2 - 30y + 1 = 0$$
- d) If $f(x) = 1 + x + x^3$. Find $f'(2)$ for $h = 0.01, 0.001$.

Q2) Attempt any TWO of the following. [10]

- a) For the matrix, $A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$ with initial vector $x^{(0)} = [1 \ 0 \ 1]^T$

Perform 3 iterations of the power method to find the dominant eigen value and corresponding eigen vector.

- b) Solve the following system of the linear equations by Guass - Seidal method, start with $x^{(0)} = [0 \ 0 \ 0]^T$ Perform 3 iterations.

$$4x_1 - x_2 = 2,$$

$$-x_1 + 4x_2 - x_3 = 1,$$

$$-x_2 + 4x_3 = 10.$$

P.T.O.

- c) Let g is a continuous function on the interval $[a, b]$ with $g: [a, b] \rightarrow [a, b]$ and suppose that g' is continuous on the interval (a, b) with $|g'(x)| \leq k < 1$ for all $x \in (a, b)$ if $g'(p) \neq 0$ then for any $p_0 \in [a, b]$ the sequence $p_n = g(p_{n-1})$ converges only linearly to the fixed point p .

Q3) Answer any TWO of the following. [10]

- a) Solve the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \leq t \leq 2, x(1) = 1, h = 0.5$ using Taylor Series method of order $N=2$.
- b) Derive the closed Newton cotes formula with $n = 2$
- $$\int_a^b f(x)dx = \left(\frac{b-a}{6}\right) \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$
- c) Determine the Newton form of the interpolating polynomial for the following data set then use this polynomial to estimate the value of y when $x = 1.5$

x	-1	0	1	2
y	5	1	1	11

Q4) A) Attempt any ONE of the following: [4]

- i) Construct the Householder matrix H for $w = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 3 & 3 \end{bmatrix}^T$
- ii) Solve the following system of equations using Gaussian elimination with scaled partial pivoting
- $$3x_1 + x_2 + 4x_3 - x_4 = 7, 2x_1 - 2x_2 - x_3 + 2x_4 = 1,$$
- $$5x_1 + 7x_2 + 14x_3 - 8x_4 = 20, x_1 + 3x_2 + 2x_3 + 4x_4 = -4$$

B) Attempt any ONE of the following. [5]

- i) Use Euler's method to solve initial value problem

$$\frac{dx}{dt} = tx^3 - x, 0 \leq t \leq 1, x(0) = 1 \text{ perform 4 iterations.}$$

- ii) Find the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 6 & 4 \\ -1 & -2 & 3 \end{bmatrix}$ by

using crout decomposition method.

[6508]-5
F.Y.M.A./M.Sc.
MATHEMATICS
MTS-506(B)-MJ : Number Theory
(2023 Credit Pattern) (Semester - I) (NEP - 2020)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following. **[6]**

- a) Prove that $(a, a + 2) = 1$ or 2 for every integer a .
- b) For $n = 3600$, Evaluate $d(n)$ and $\sigma(n)$.
- c) Find the minimal polynomial of the algebraic number $\frac{1 + \sqrt[3]{7}}{2}$.
- d) Find all primitive pythagorean triples for which $0 < z < 30$.

Q2) Attempt any two of the following. **[10]**

- a) Solve the system of congruences $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, $x \equiv 2 \pmod{7}$
- b) Let a, b be integers, $a > 0$. Show that there exist unique integers q and r such that $b = aq + r$, $0 \leq r < a$.
- c) If p and q are distinct odd primes then show that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2} \frac{q-1}{2}}$.

Q3) Attempt any two of the following. **[10]**

- a) For what real number x is true that
 - i) $[x] + [x] = [2x]$?
 - ii) $[x + 3] = 3 + [x]$?
 - iii) $[9x] = 9$?
- b) Find all solutions of $10x - 7y = 17$.
- c) Prove that product of two primitive polynomial is again primitive.

P.T.O.

Q4) A) Attempt any one of the following.

[4]

a) Let p be an odd prime. Then prove that

$$\text{i) } \left(\frac{a}{p} \right) = a^{\frac{p-1}{2}} \pmod{p}$$

$$\text{ii) } \left(\frac{a}{p} \right) \left(\frac{b}{p} \right) = \left(\frac{ab}{p} \right)$$

b) Prove that for any positive integer n ,

$$\sigma(n) = \prod_{p^{\alpha} \parallel n} \left(\frac{p^{\alpha+1} - 1}{p - 1} \right)$$

B) Attempt any one of the following.

[5]

a) Prove that if ξ is an algebraic number of degree n , then number in $\mathbb{Q}(\xi)$ can be written uniquely in form $a_0 + a_1\xi + \dots + a_{n-1}\xi^{n-1}$ where a_i are rational numbers.

b) Prove that set of all algebraic numbers forms a field and set of all algebraic integer forms a ring.



Total No. of Questions : 4]

SEAT No. :

PE4403

[Total No. of Pages : 2

[6508]-6

First Year M.A./M.Sc.

MATHEMATICS

MTS - 506(C) - MJ : Combinatorics

(2023 Credit Pattern) (Semester - I)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[6]

- a) How many nonempty collections of letters can be formed from three A'S and five B'S?
- b) How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 12$, with $x_i \geq 0$?
- c) Find the coefficient of x^{18} in $(x^2+x^3+x^4+x^5+x^6+x^7)^4$.
- d) Show that $\sum_{k=0}^n 2^k \binom{n}{k} = 3^n$.

Q2) Attempt any two of the following.

[10]

- a) By a combinatorial argument show that

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2 = \binom{2n}{n}$$

- b) How many arrangements of "MISSISSIPPI" are there with no pair of consecutive S's?
- c) Solve the recurrence relation $a_n = -na_{n-1} + n!$ when $a_0=1$.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Show that the number of ways to select ' r ' objects from ' n ' types of objects is $\binom{r+n-1}{r}$.
- b) Find the coefficient of x^{25} in $(1 + x^3 + x^8)^{10}$
- c) An elf has a staircase of ' n ' stairs to climb. Each step it takes can cover either one stair or two stairs. Find and solve the recurrence relation for a_n , the number of different ways for the elf to ascend the n -staircase.

Q4) A) Attempt any one of the following.

[5]

- a) How many ways are there to assign 6 cars, denoted $c_1, c_2, c_3, c_4, c_5, c_6$ to six men $m_1, m_2, m_3, m_4, m_5, m_6$, if man m_1 will not drive cars c_2 and c_4 ; man m_2 will not drive cars c_1 or c_5 ; if man m_3 drives all cars; man m_4 will not drive c_2 or c_5 ; man m_5 will not drive c_4 and man m_6 will not drive c_6 ?
- b) How many ways are there for a child to take 10 pieces of Candy with four types of candy if the child does not take exactly two pieces of any type of candy?

B) Attempt any one of the following.

[4]

- a) State and prove Inclusion - Exclusion formula.
- b) How many ways are there to distribute 25 identical balls into six distinct boxes with at most 6 balls in any of the first three boxes?



Total No. of Questions : 4]

SEAT No. :

PE4404

[Total No. of Pages : 2

[6508]-7

M.A/M.Sc. (Part-I)

MATHEMATICS

**MTS - 506(D) - MJ : Lattice Theory
(2023 Credit Pattern) (Semester - I)**

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any THREE of the following. **[6]**

- a) Give an example of bounded lattice which is not completed.
- b) Define Hereditary subset.
- c) Does one-one meet-homomorphism implies join-homomorphism? Justify your answer.
- d) Draw the Hasse diagram of the lattice of all positive divisors of 12.

Q2) Attempt any TWO of the following. **[10]**

- a) Prove that, the absorption identities imply the idempotency of meet and join.
- b) Let L be a lattice and H, I be non-empty subsets of L . Prove that, I is an ideal if and only if following two conditions holds.
 - i) $a, b \in I$ implies that $a \vee b \in I$
 - ii) $a \in I, x \in L$ and $x \leq a$ imply that $x \in I$
- c) Let L be a lattice. If $\phi: L \rightarrow L_1$ is homomorphism of L onto L_1 and if θ is the congruence relation of L defined by $x \equiv y(\theta)$ if and only if $x\phi = y\phi$ then show that, $L/\theta \cong L_1$.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Show that following inequalities hold in any lattice L
- i) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$, for $x, y, z \in L$
 - ii) $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$, for $x, y, z \in L$
- b) Prove that every maximal chain C of finite distributive Lattice L is of length $|J(L)|$
- c) Let I and J be ideals of a distributive lattice L . if $I \wedge J$ and $I \vee J$ are principal ideals then show that I and J are principal ideals.

Q4) a) Attempt any one of the following.

[4]

- i) Define pseudo complement of an element in lattice and also give any two examples of pseudo complemented lattices.
- ii) Show that every distributive algebraic lattice is a pseudo complemented.

b) Attempt any one of the following.

[5]

- i) Prove that a modular lattice satisfies both the upper covering condition and the lower covering condition.
- ii) Let L be a modular lattice and $a, b \in L$ prove that,

$\phi_b : x \mapsto x \wedge b, x \in [a, a \vee b]$ is an isomorphism of $[a, a \vee b]$ and $[a \wedge b, b]$.



Total No. of Questions : 4]

SEAT No. :

PE4405

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[6508]-8

First Year M.A./M.Sc.

MATHEMATICS

**MTS 508 MJ : Research Methodology
(2023 Credit Pattern) (Semester - I) (NEP 2020)**

Time : 2Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *Attempt all questions.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following. **[6]**

- a) What is mean by Thesis.
- b) What is mean by Research?
- c) Explain the term patent.
- d) What is Literature review?
- e) What are mathematical tools used in Research methodology?

Q2) Attempt any Two of the following. **[10]**

- a) Write a short note on Scopus.
- b) Distinguish between Research methods and methodology.
- c) Explain significance of Research report writing.

Q3) Attempt any two of the following. **[10]**

- a) Describe in detail tools of data collection.
- b) Explain the characteristics of Research methodology.
- c) Explain the concepts of ISSN and ISBN and their importance.

Q4) a) Attempt any one of the following. **[5]**

- i) Write a short note on ZMATH.
- ii) Write a short note on Research proposal.

b) Attempt Any one of the following. **[4]**

- i) Explain h-index.
- ii) Write a difference between survey and experiment.



Total No. of Questions : 4]

SEAT No. :

[Total No. of Pages : 1

PE4406

[6508]-21

F.Y. M.A./M.Sc.

MATHEMATICS

MTS-551-MJ : Topology

(2023 Credit Pattern) (NEP 2020) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following: [6]

- a) Show that $\mathbb{Q}_+$ is countably infinite.
- b) Show that a subset of a countable set is countable.
- c) Define: Regular space.
- d) State Urysohn lemma.

Q2) Attempt any two of the following: [10]

- a) Prove that every finite point set in a Hausdorff space x is closed.
- b) Show that every order topology is Hausdorff.
- c) State and prove sequence lemma.

Q3) Attempt any two of the following: [10]

- a) Let x be locally path connected. Show that every connected open set in x is path connected.
- b) Show that if $f: x \rightarrow y$ is continuous, where x is compact and y is Hausdorff, then f is closed map.
- c) Prove that image of a compact space under a continuous map is compact.

Q4) Attempt any one of the following: [9]

- a) Prove that every regular space with a countable basis is normal.
- b) Show that a connected normal space having more than one point is uncountable.



Total No. of Questions : 7]

SEAT No. :

PE4407

[Total No. of Pages : 2

[6508]-22

F.Y.M.A./M.Sc.

MATHEMATICS

MTS - 553-MJ : Ring Theory

(2023 Credit Pattern) (NEP 2020) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any five of the following. **[10]**

- a) Let R is the ring of all continuous functions from the closed interval $[0, 1]$ to $\mathbb{R}$. Is R integral domain? Justify.
- b) Let R and S be rings and let $\phi: R \rightarrow S$ be homomorphism. Prove that the image of ϕ is subring of S .
- c) Prove that every field is a Euclidean Domain.
- d) Let R be a commutative ring and let $a, b \in R$ with $b \neq 0$. Then prove that b/a in a ring R if and only if $(a) \subseteq (b)$.
- e) Give an example of a principal ideal domain but not a Euclidean domain.
- f) Determine whether the polynomial $x^4 + 10x + 5$ in $\mathbb{Z}[x]$ is irreducible.
- g) Let $p(x, y, z) = 2x^2y - 3xy^3z + 4y^2z^5$ be polynomial in $\mathbb{Z}[x, y, z]$ Find the degree of p in the variable x with coefficients in $\mathbb{Z}[y, z]$.

Q2) Attempt any two of the following. **[10]**

- a) Let R and S be rings and $\phi: R \rightarrow S$ be an ring homomorphism. Prove that $R/\ker\phi$ is isomorphic to $\phi(R)$.
- b) Prove that the intersection of any nonempty collection of subrings of a ring is also a subring.
- c) Prove that every Euclidean domain is Principal Ideal Domain.

Q3) Attempt any two of the following. **[10]**

- a) Prove that, if F is field then $F[x]$ is Euclidean Domain.
- b) Prove that every nonzero prime ideal in a Principal Ideal Domain is a maximal ideal.
- c) Prove that a nonzero element in a Principal Ideal Domain is a prime if and only if it is irreducible.

P.T.O.

Q4) Attempt any two of the following. **[10]**

- a) Let a and b be two nonzero elements of the Unique factorization Domain R and suppose $a = up_1^{e_1} p_2^{e_2} \dots p_n^{e_n}$ and $b = vp_1^{f_1} p_2^{f_2} \dots p_n^{f_n}$ are Prime factorizations for a and b , where u and v are units, the primes $p_1, p_2, \dots, p_n$ are distinct and e_i and f_i are ≥ 0 . Then prove that the element $d = p_1^{\min\{e_1, f_1\}} \cdot p_2^{\min\{e_2, f_2\}} \dots p_n^{\min\{e_n, f_n\}}$ is a greatest common divisor of a and b .
- b) Prove that the Gaussian integers $\mathbb{Z}[i]$ are a Euclidean Domain with respect to the norm $N(a + bi) = a^2 + b^2$
- c) Let F be a field and let $p(x) \in F[x]$. Then prove that $p(x)$ has a factor of degree one if and only if there is an $\alpha \in F$ with $p(\alpha) = 0$.

Q5) Attempt any two of the following. **[10]**

- a) State and prove Gauss' Lemma.
- b) Prove that $\mathbb{Z}[x]$ is not principal Ideal Domain.
- c) Define a Boolean ring. Prove that every Boolean ring is commutative.

Q6) Attempt any two of the following. **[10]**

- a) Let $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 \in \mathbb{Z}[x]$ with $a_n \neq 0$. Prove that, if $\frac{r}{s} \in \mathbb{Q}$ with r and s are relatively prime integers and $\frac{r}{s}$ is a root of $p(x)$ then r/a_0 and s/a_n
- b) Prove that the prime number $p \in \mathbb{Z}$ divides an integer of the form $n^2 + 1$ if and only if p is either 2 or is an odd prime congruent to 1 modulo 4.
- c) Determine the greatest common divisor d of $a = 69$ and $b = 372$ and write d as a linear combination $ax + by$ of a and b .

Q7) Attempt any one of the following. **[10]**

- a) Prove that every principal Ideal domain is a Unique Factorization Domain.
- b) Let R is commutative prove that every maximal ideal of R is a prime ideal.



Total No. of Questions :7]

SEAT No. :

PE4408

[Total No. of Pages : 3

[6508]-23

First Year M.A./M.Sc.

MATHEMATICS

MTS-554 MJ : Advanced Calculus

(2023 Credit Pattern) (Semester-II) (NEP-2020)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of single memory, non-programmable calculator is allowed.

Q1) Attempt any five of the following.

[10]

- a) Determine the set of point's (x, y) at which $f(x, y) = x^4 + y^4 - 4x^2y^2$ is continuous.
- b) Compute first order partial derivatives of the given scalar field $f(x, y) = x^2 + y^2 \sin(xy)$ defined on $\mathbb{R}^n$
- c) Define smooth path.
- d) Show that given vector field is not gradient $f(x, y) = y\bar{i} - x\bar{j}$
- e) Determine the jacobian determinant of cylindrical coordinates.
- f) Define area of a parametric surface.
- g) determine the curl of the vector field $\bar{F}(x, y, z) = (x^2 + yz)\bar{i} + (y^2 + xz)\bar{j} + (z^2 + xy)\bar{k}$

Q2) Attempt any two of the following.

[10]

- a) State and prove first fundamental theorem of calculus for line integral.
- b) Show that for the function $f(x, y) = \frac{x-y}{x+y}$, if $x + y \neq 0$ repeated limit exist But not equal.
- c) If a scalar field f is differentiable at a , then f is continuous at a .

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) If the derivative $f'(a + ty; y)$ exist, for each t in the interval $0 \leq t \leq l$. Then show that for some real θ in the open $0 < \theta < l$, we have
- $$f(a + y) - f(a) = f'(z; y) \text{ where } z = a + \theta y$$
- b) Calculate the line integral of the vector field $\vec{F}$ along the path described $\vec{F}(x, y, z) = (y^2 - z^2)\vec{i} + 2yz\vec{j} - x^2\vec{k}$ a long the path $\vec{\alpha}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}, 0 \leq t \leq l$.
- c) Evaluate the double integral $\iint_{\varphi} (x^3 + 3x^2y + y^3) dx dy$ where $\varphi = [0, 1] \times [0, 1]$

Q4) Attempt any two of the following.

[10]

- a) State and prove Green's theorem.
- b) Evaluate directional derivative of scalar field $f(x, y, z) = \left(\frac{x}{y}\right)$ at $(1, 1, 1)$ in the direction of $2\vec{i} + \vec{j} - \vec{k}$.
- c) Evaluate the line integral $\int_c (5 - xy - y^2) dx - (2xy - x^2) dy$ where c is the square with vertices $(0, 0), (1, 0), (1, 1), (0, 1)$ traversed counter clockwise

Q5) Attempt any two of the following.

[10]

- a) Prove that if $\vec{\alpha}$ & $\vec{\beta}$ be equivalent piecewise smooth path. Then
- $$\int_c \vec{f} \cdot d\vec{\alpha} = \int_c \vec{f} \cdot d\vec{\beta}, \text{ if } \vec{\alpha} \text{ \& } \vec{\beta} \text{ trace out } c \text{ in opposite direction and}$$
- $$\int_c \vec{f} \cdot d\vec{\alpha} = - \int_c \vec{f} \cdot d\vec{\beta}, \text{ if } \vec{\alpha} \text{ \& } \vec{\beta} \text{ trace out } c \text{ in same direction.}$$
- b) Find the fundamental vector product for the surface $\vec{r}(x, y) = x\vec{i} + y\vec{j} + f(x, y)\vec{k}$ what are the singular point's of this surface?
- c) Find the gradient vector of the function $f(x, y, z) = \log(x^2 + 2y^2 - 3z^2)$

Q6) Attempt any two of the following. **[10]**

- a) Compute the surface area of a hemisphere of radius a and center at origin, by using surface integral.
- b) Determine the curl and divergence for vector field $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$
- c) Find the magnitude of vector product

$$\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}, \text{ where } \vec{r}(u, v) = u\vec{i} + a \sin v \vec{j} + a \cos v \vec{k}$$

Q7) Attempt any two of the following. **[10]**

- a) Let V be a solid in 3-space bounded by an orientable closed surface S , and let $\vec{n}$ be the unit outer normal to S . If $\vec{F}$ is a continuously differentiable vector field defined on V , we have $\iiint_V (\text{div} \vec{F}) dx dy dz = \iint_S \vec{F} \cdot \vec{n} ds$.

- b) Determine the region S and interchange the order of integration

$$\int_0^1 \left[\int_{x^2}^x f(x, y) dy \right] dx$$

- c) Verify that the mixed partial derivatives $D_1(D_2 f)$ and $D_2(D_1 f)$ are equal for the function $f(x, y) = x^4 + y^4 - 4x^2 y^2$.



Total No. of Questions : 4]

SEAT No. :

PE4409

[Total No. of Pages : 3

[6508]-24

First Year M.A./M.Sc.

MATHEMATICS

MTS-556 (A)-MJ : Graph Theory

(2023 Credit Pattern) (NEP 2020) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[6]

- a) Prove that in any graph G with n vertices $v_1, v_2, \dots, v_n$ and ' e ' edges,

$$\sum_{i=1}^n d(v_i) = 2e.$$

- b) Draw the graph having the following matrix as it's adjacency matrix.

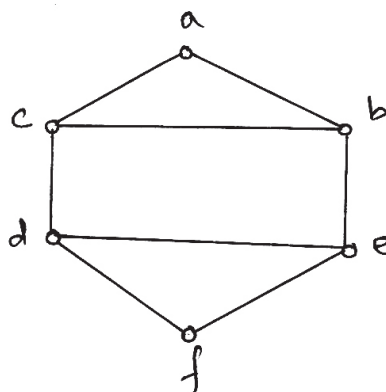
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- c) State true or false.

Every complete graph is a regular graph.

Justify your answer.

- d) Find the eccentricity of the vertices a, b, f and d for the following graph.

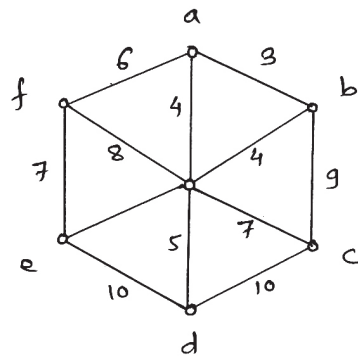


P.T.O.

Q2) Attempt any two of the following :

[10]

- a) Find a minimal spanning tree for the following connected weighted graph using Kruskal's algorithm.

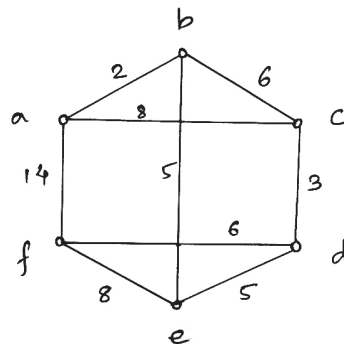


- b) Prove that for any graph G ,
 $\chi(G) \leq \Delta(G) + 1$
- c) Let G be a graph and u, v be any two vertices of G . Prove that every $u - v$ walk contains a $u - v$ path.

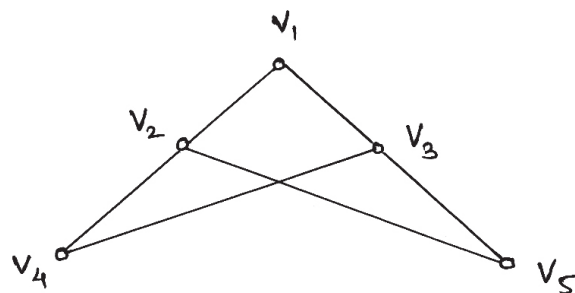
Q3) Attempt any two of the following.

[10]

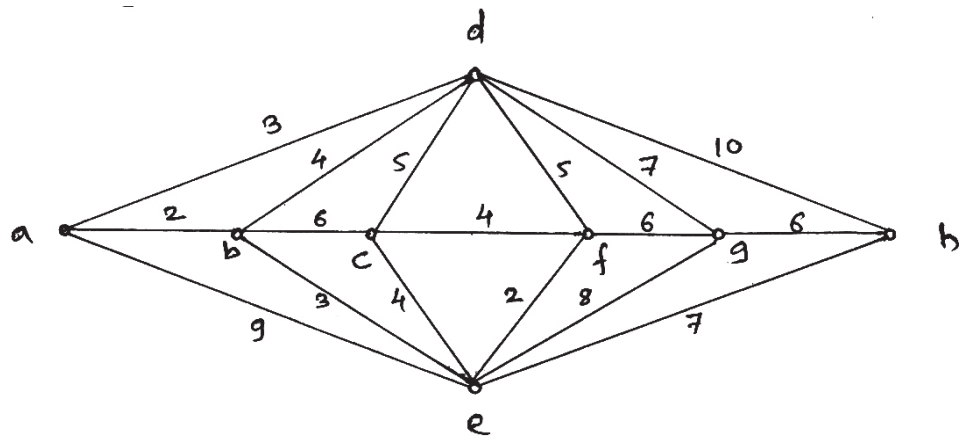
- a) Prove that a weakly connected digraph D with at least two vertices has a directed Eulerian trail if and only if D has two vertices u and v such that $\text{od}(u) = \text{id}(u) + 1$ and $\text{id}(v) = \text{od}(v) + 1$ and for all other vertices w of D , $\text{od}(w) = \text{id}(w)$. Furthermore, in this case the trail begins at u and ends at v .
- b) Use Dijkstra's Algorithm on the following connected weight graph to find lengths of shortest paths from the vertex a to each of the other vertices.



- c) Find an Eulerian trail in a graph given below using Fleury's Algorithm.



- Q4) a)** Attempt any one of the following. [4]
- Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.
 - Let G be a non-empty graph. Then prove that $\chi(G) = 2$ if and only if G is bipartite.
- b)** Attempt any one of the following. [5]
- Prove that a simple graph G is Hamiltonian if and only if its closure $C(G)$ is Hamiltonian.
 - Solve the Chinese Postman problem for the following graph.



Total No. of Questions : 4]

SEAT No. :

PE4410

[Total No. of Pages : 2

[6508]-25

First Year M.A./M.Sc.

MATHEMATICS

MTS-556 (B)-MJ : Dynamical Systems
(2023 Credit Pattern) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following :

[3×2=6]

- a) Find e^{At} if $A = \begin{pmatrix} 0 & -w \\ w & 0 \end{pmatrix}$
- b) Show that the solution of system $\dot{x} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} x$ are linearly independent.
- c) Find fixed point of differential equation $\dot{x} = x^2 - 9$.
- d) Find the polar form of the differential system.

$$\begin{aligned}\dot{x} &= y + x(1 - x^2 - y^2)(4 - x^2 - y^2) \\ \dot{y} &= -x + y(1 - x^2 - y^2)(4 - x^2 - y^2)\end{aligned}$$

Q2) Attempt any two of the following :

[2×5=10]

- a) Let x^* be an equilibrium point of $\dot{x} = F(x)$ where F is twice continuously differentiable function assume all the eigenvalues of $DF(x^*)$ has negative real part. Then show that x^* is asymptotically stable.
- b) Construct a Lyapunov function for $\dot{x} = -x^3$, $\dot{y} = -y^3$.
- c) Find the general solution of the non homogeneous system

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

P.T.O.

Q3) Attempt any two of the following. **[2×5=10]**

- a) Prove that any periodic orbit for the vander pol system is stable. Therefore there can be only one periodic orbit.
- b) Classify the origin for $\dot{x} = -x + y^2, \dot{y} = -2y$.

c) Find e^{At} if $A = \begin{bmatrix} c & 1 & 0 \\ 0 & c & 1 \\ 0 & 0 & c \end{bmatrix}$.

Q4) a) Attempt any one of the following. **[1×4=4]**

- i) Prove that the tent map T and quadratic map G have at least one period K orbit for each positive integer K .
- ii) Show that the system $\dot{x} = -(x + f(x)) + b$ has given Lypunov function.

b) Attempt any one of the following. **[1×5=5]**

- i) Consider the system $\dot{x} = -\nabla G(x)$, where $G(x) = G(x, y) = \frac{-x^2 + x^4 + y^2}{2}$

for all $x = (x, y) \in \mathbb{R}^2$, show that $(0, 0)$ is saddle and $(\pm 1, 0)$ are nodes of the system. Also sketch the phase portrait of the system.

- ii) Find the general solution of the system $\dot{x} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} x$ and sketch the phase portrait.



Total No. of Questions : 4]

SEAT No. :

PE4411

[Total No. of Pages : 2

[6508]-26

Frist Year M.A./M.Sc.

MATHEMATICS

MTS-556(C)-MJ : Coding Theory

(2023 Credit Pattern) (NEP-2020) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[3×2=6]

- a) Prove that there is no self-dual binary code of parameters $[10, 5, 4]$.
- b) Define the extended ternary Golay code.
- c) Define cyclic code and give example of it.
- d) Let $A = \{0, 1\}$ and $\bar{x} = 01010$, $\bar{y} = 01101$ find $d(\bar{x}, \bar{y})$.

Q2) Attempt any two of the following.

[2×5=10]

- a) Define reciprocal polynomial $h_R(x)$. Find $h_R(x)$ if $h(x) = 1 + 2x + 3x^5 + x^7 \in F_5[x]$.
- b) Prove that, if $x, y \in F_2^n$ then $wt(x+y) = wt(x) + wt(y) - 2wt(x*y)$
- c) Let $q = 2$, suppose $v = \langle s \rangle$, $s = \{0001, 0010, 0100\}$ then find v , dimension of v and number of bases for v .

P.T.O.

Q3) Attempt any two of the following.

[2×5=10]

- a) Prove that $A_2(5, 4) = 2$.
- b) Let $S = \{0100, 0101\} \subseteq F_2^4$. Verify that $\dim(\langle S \rangle) + \dim(S^\perp) = n$.
- c) Let $S = \{1020, 0201, 2001\}$. Find F_3 – linear span $\langle S \rangle$ and its orthogonal complement $S^\perp$.

Q4) Attempt any one of the following.

[1×9=9]

- a) For an integer $q > 1$ and integers n, d such that $1 \leq d \leq n$, show that

$$\frac{q^n}{\sum_{i=0}^{d-1} \binom{n}{i} (q-1)^i} \leq A_q(n, d)$$

- b) Let $c = \{0000, 1011, 0101, 1110\} \subseteq F_2^4$ be a linear code with parity check

matrix $H = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$ and coset leaders $\{0000, 0001, 0010, 1000\}$.

Decode following by using syndrome decoding.

i) $W = 1101$

ii) $W = 1111$



Total No. of Questions : 4]

SEAT No. :

PE4412

[Total No. of Pages : 3

[6508]-27

First Year M.A./M.Sc.

MATHEMATICS

MTS-556(D)-MJ : Operations Research

(2023 Credit Pattern) (Semester - II)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[6]

- a) Write the dual of the following LP problem

$$\text{Max} \quad z = 6x + 8y$$

$$\text{Subject to} \quad 5x + 2y \leq 20$$

$$x + 2y \leq 10$$

$$x, y \geq 0$$

- b) Construct the Lagrangian function of the following non - linear programming problem.

$$z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$$

$$\text{Subject to} \quad x_1 + x_2 + x_3 = 15$$

$$2x_1 - x_2 + 2x_3 = 20$$

$$x_1, x_2, x_3 \geq 0$$

- c) What are the basic differences between linear programming and Goal programming?
- d) Define Beale's method in quadratic programming.

P.T.O.

Q2) Attempt any two of the following.

[10]

- a) Prove using duality theory that the following LPP has a feasible but not optimal

$$\begin{aligned} \min \quad & z = x_1 - x_2 + x_3 \\ \text{subject to} \quad & x_1 - x_3 \geq 4 \\ & x_1 - x_2 + 2x_3 \geq 3 \\ & \text{and } x_1, x_2, x_3 \geq 0 \end{aligned}$$

- b) Solve the following problem by using the method of Lagrangian multiplier.

$$\begin{aligned} \min \quad & z = x_1^2 + x_2^2 + x_3^2 \\ \text{subject to the constraints} \\ & \text{i) } x_1 + x_2 + 3x_3 = 2 \\ & \text{ii) } 5x_1 + 2x_2 + x_3 = 5 \\ & \text{and } x_1, x_2 \geq 0 \end{aligned}$$

- c) Explain wolfe's modified simplex method in quadratic programming.

Q3) Attempt any two of the following.

[10]

- a) Using the modified simplex method to solve the following goal programming problem.

$$\begin{aligned} \text{minimize} \quad & z = p_1 d_1^- + p_2 (2d_2^- + d_3^-) + p_3 d_1^+ \\ \text{Depending on the constraints} \\ & x_1 + x_2 + d_1^- - d_1^+ = 400 \\ & x_1 + d_2^- = 240 \\ & x_2 + d_3^- = 300 \\ & \text{and } x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0 \end{aligned}$$

- b) Solve the following LPP using simplex method

$$\begin{aligned} \text{Max.} \quad & z = 3x_1 + 2x_2 + 5x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 \leq 430 \\ & 3x_1 + 2x_3 \leq 200 \\ & x_1 + 4x_2 \leq 420 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

- c) Explain the concept of Branch and Bound method with an example.

Q4) Attempt any one of the following.

[9]

- a) Find an optimum integer solution to the following LPP

$$\text{Max } z = x_1 + 2x_2$$

Subject to the constraints,

$$2x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$2x_1 \leq 11$$

$$x_1, x_2 \geq 0 \text{ and } x_1, x_2 \text{ are integers}$$

- b) Apply wolfe's method to solve quadratic programming problem.

$$\text{Max } z = 2x_1 + 3x_2 - 2x_1^2$$

$$\text{Subject to } x_1 + 4x_2 \leq 4$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$



Total No. of Questions : 4]

SEAT No. :

PE4413

[Total No. of Pages : 2

[6508]-31

S.Y.M.A./M.Sc.

MATHEMATICS

MTS-601MJ : Complex Analysis
(2023 Credit Pattern) (Semester-III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[6]

- a) Show that z is purely real number if and only if $z = \bar{z}$
- b) Find the radius of convergence of $\sum_{n=0}^{\infty} a^n z^n, a \in \mathbb{C}$.
- c) Evaluate $\int_v \frac{dz}{(z-a)}, v(t) = a + re^{it}, 0 \leq t \leq 2\pi$.
- d) Evaluate cross ratio $(2, 1-i, 1, 1+i)$.

Q2) Attempt any two of the following.

[10]

- a) Let f and g be analytic on G and Ω respectively and suppose $f(G) \subset \Omega$ then show that $g \circ f$ is analytic on G and $(g \circ f)'(z) = g'(f(z)) \cdot f'(z)$ for all z in G
- b) Show that for $a > 1$ $\int_0^{\pi} \frac{d\theta}{a + \cos \theta} = \frac{\pi}{\sqrt{a^2 - 1}}$
- c) Prove that $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta, n \in \mathbb{Z}$.

P.T.O.

Q3) Attempt any two of the following.

[10]

a) Let G be a region and Let $f : G \rightarrow \mathbb{C}$ be a continuous function such that

$\int_T f = 0$ for every triangular path T in G then show that f is analytic in G .

b) Evaluate the integral $\int_{\gamma} \frac{\sin z}{z^3} dz, \gamma(t) = e^{it} \quad 0 \leq t \leq 2\pi$.

c) Show that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$

Q4) a) Attempt any one of the following.

[5]

i) Show that $u(x, y) = \log(x^2 + y^2)^{1/2}$ is Harmonic function on $G = \mathbb{C} - \{0\}$

ii) Let G be an open set and $f : G \rightarrow \mathbb{C}$ be a differentiable function then show that f is analytic on G .

b) Attempt any one of the following.

[4]

i) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$

ii) If f is bounded entire function then show that f is constant.



Total No. of Questions : 7]

SEAT No. :

PE4414

[Total No. of Pages : 2

[6508]-32
S.Y. M.A./M.Sc.
MATHEMATICS
MTS-603-MJ : Field Theory
(2023 Credit Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any five of the following. **[10]**

- a) Prove or disprove “Every integral domain is a field”
- b) Show that $x^2 + x + 1$ is irreducible polynomial over $\mathbb{Z}_2$.
- c) Is $x^2 - 2$ separable over $\mathbb{Q}$? Justify.
- d) Define n^{th} cyclotomic polynomials.
- e) Define
 - i) Galois extension of a field F .
 - ii) Abelian extension of a field F .
- f) Determine the minimal polynomial over $\mathbb{Q}$ for the element $\sqrt{2} + \sqrt{3}$.
- g) Show that any automorphism of a field k fixes. It's prime subfield.

Q2) Attempt any two of the following. **[10]**

- a) If F is a field and $P(x) \in F[x]$ is an irreducible polynomial then prove that there is an extension of F in which $P(x)$ has a root.
- b) Prove that if $[F(\alpha) : F]$ is odd then $F(\alpha) = F(\alpha^2)$.
- c) Show that $\left[\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q} \right] = 4$.

Q3) Attempt any two of the following. **[10]**

- a) If α is a rational root of monic polynomial in $\mathbb{Z}[x]$ then prove that α is an integer.
- b) Let F be a field and $f(x) \in F[x]$. Then prove that there exist an extension K of F which is splitting field for $f(x)$ over F .
- c) If F be any field then prove that $F = \bar{F}$ iff F is algebraically closed.

P.T.O.

Q4) Attempt any two of the following. **[10]**

- a) Show that a polynomial $f(x)$ has multiple root α iff α is also root of $D_x(f(x))$.
- b) If K is algebraic over F and L is algebraic over K then prove that L is algebraic over F .
- c) Show that doubling the cube is impossible using straight edge and compass.

Q5) Attempt any two of the following. **[10]**

- a) Show that cyclotomic polynomial $\Phi(x)$ is a monic polynomial in $\mathbb{Z}[x]$ in of degree $\phi(n)$.
- b) Show that the degree of extension of the splitting field for $x^3 - 2 \in \mathbb{Q}[x]$ is 6.
- c) Find all automorphisms of $\mathbb{Q}(\sqrt{2})$ over $\mathbb{Q}$.

Q6) Attempt any two of the following. **[10]**

- a) Show that any quadratic extension K of any field F of characteristic not equal to 2 is Galois.
- b) State fundamental theorem of Galois theory.
- c) If K/F is any finite extension then prove that $|\text{Aut}(K/F)| \leq [K:F]$.
Further prove that $|\text{Aut}(K/F)| = [K:F]$ iff F is fixed field of $\text{Aut}(K/F)$.

Q7) Attempt any two of the following. **[10]**

- a) Show that an irreducible polynomial $x^4 + 1$ in $\mathbb{Z}[x]$ is reducible over $\mathbb{Z}_p$.
- b) Show that Galois group of $x^{p^n} - x$ over $\mathbb{F}_p$ is cyclic group of order n .
- c) Show that a Galois extension of a Galois extension is not necessary Galois.



Total No. of Questions : 7]

SEAT No. :

PE4415

[Total No. of Pages : 2

[6508]-33

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-604-MJ : Differential Geometry
(2023 Credit Pattern) (Semester - III)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following.

[10]

- a) Define an integral curve.
- b) What is height of the level set?
- c) Explain the difference between a surface and a hypersurface.
- d) Give two examples of oriented 2-surface in $\mathbb{R}^3$.
- e) What is the range of the Gauss map?
- f) If $\alpha(t):I \rightarrow \mathbb{R}^2$ is a parametrized curve defined by $\alpha(t)=(t,t^2)$. Find $\dot{\alpha}(t), \ddot{\alpha}(t)$.
- g) Define Weingarten map.

Q2) Attempt any two of the following.

[10]

- a) Explain why an integral curve cannot cross itself as does the parametrized curve.
- b) Show that the graph of any function $f:\mathbb{R}^n \rightarrow \mathbb{R}$ is a level set for some function $G:\mathbb{R}^{n+1} \rightarrow \mathbb{R}$.
- c) Sketch the level sets for the function $f(x_1, x_2) = x_1^2 + x_2^2$ with $c = 0, 1, 4$.

Q3) Attempt any two of the following.

[10]

- a) Define complete vector field and check whether the vector field $x(x_1, x_2) = (x_1, x_2, 1, 0)$ is complete or not.
- b) Show that the velocity vector field along a parametrized curve $\alpha(t)$ is an n-surface s is parallel if and only if $\alpha(t)$ is a geodesic.
- c) Show that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to $f^{-1}(c)$ at 'p'.

P.T.O.

Q4) Attempt any two of the following. **[10]**

- a) Show that Möbius band is an unorientable 2-surface.
- b) Find the global parametrization of the curve $(x_1 - a)^2 + (x_2 - b)^2 = r^2$.
- c) State and prove Lagrange's Multiplier theorem.

Q5) Attempt any two of the following. **[10]**

- a) Show that the unit sphere $x_1^2 + x_2^2 + \dots + x_{n+1}^2 = 1$ is connected for $n > 1$.
- b) Explain how level sets were used in Geography.
- c) Explain the difference between local and global parametrization of the curve C .

Q6) Attempt any two of the following. **[10]**

- a) State and prove any two properties of Levi - Civita parallel.
- b) If C is connected oriented plane curve and $\beta : I \rightarrow C$ is a unit speed global parametrization of C . Then show that β is either one - to - one or periodic.
- c) If $\alpha(t) = (x(t), y(t)) (t \in I)$ is a local parametrization of the oriented plane curve C . Then show that.

$$k \circ \alpha = (x'y'' - y'x'') / (x''^2 + y''^2)^{3/2}$$

Q7) Attempt any two of the following. **[10]**

- a) If s is an oriented 2-surface in $\mathbb{R}^3$ and $p \in s$. Then show that, for each $u, v \in S_p$, $L_p(u) \times L_p(v) = k_{(p)} u \times v$.
- b) Define Geodesics. Hence show that, geodesics have constant speed.
- c) If s is an n -surface in $\mathbb{R}^{n+1}$ with $p, q \in s$ and let α is a piecewise smooth parametrized curve from p to q . Then show that the parallel transport $p_\alpha : s_p \rightarrow s_q$ along α is vector space isomorphism which preserves dot products.



Total No. of Questions : 4]

SEAT No. :

PE4416

[Total No. of Pages : 2

[6508]-34

Second Year M.A./M.Sc.

MATHEMATICS

MTS - 611 (A) MJ : Mathematical Statistics
(2023 Credit Pattern) (Semester - III)

Time : 2Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any three of the following.

[6]

- a) Define simple hypothesis and statistical hypothesis.
- b) State Bayes Theorem.
- c) Define correlation coefficient.
- d) Prove that the probability of an impossible event is zero.

Q2) Attempt any Two of the following.

[10]

- a) State and prove law of addition of probabilities and prove that ,
 $P(\bar{A}) = 1 - P(A)$.
- b) A random variable X has the following probability distribution:

x	0	1	2	3	4	5	6	7
$\phi(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

- i) Find k
- ii) Evaluate $p(x < 6)$, $p(x \geq 6)$

If $p(x \leq c) > \frac{1}{2}$, find minimum value of c.

- c) Define independent event. Show that, if A and B are independent events, then A and $\bar{B}$ are also independent events.

P.T.O.

Q3) Attempt any two of the following.

[10]

a) Given the frequency function:

$$f(x, \theta) = \frac{1}{\theta}, 0 \leq x \leq \theta$$
$$= 0, \text{ elsewhere}$$

and that you are testing the null hypothesis $H_0: \theta=1$ against $H_1: \theta=2$, by means of a single observed value of x . What would be the sizes of the type-I and type-II errors, if you choose the interval

- i) $0.5 \leq x$
- ii) $1 \leq x \leq 1.5$ as the critical regions?

b) A card is drawn from a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

c) Show that the moment generating function of a gamma random variable X is $\left(\frac{\lambda}{\lambda - t}\right)^\alpha$.

Q4) a) Attempt any one of the following.

[5]

- i) If two dice are thrown, what is the probability that the sum is
 - 1) greater than 8 and
 - 2) neither 7 nor 11?
- ii) State and prove Bayes Theorem.

b) Attempt any one of the following.

[4]

- i) Show that the moment generating function of a normal random variable is $\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$ where μ and σ^2 are the parameters.
- ii) Explain the concepts of type-I and type-II errors with examples.



Total No. of Questions : 4]

SEAT No. :

[Total No. of Pages : 2

PE4417

[6508]-35

Second Year M.A./M.Sc.

MATHEMATICS

**MTS-611(B) MJ : Algebraic Topology
(2023 Credit Pattern) (CBCS) (Semester - III)**

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following.

[6]

- a) Define a covering map.
- b) State the general lifting lemma.
- c) State the Borsuk-Ulam Theorem for S^2 .
- d) Give an example of a non-identity covering map from S^1 onto S^1 .

Q2) Attempt any two of the following.

[10]

- a) Given spaces X and Y , let $[X, Y]$ denote the set of homotopy classes of maps of X into Y . Show that if Y is path connected, then the set $[I, Y]$ has a single element.
- b) Find a star convex set that is not convex.
- c) Let $q : X \rightarrow Y$ and $r : Y \rightarrow Z$ be covering maps. Let $p = r \circ q$. Show that if $r^{-1}(z)$ is finite for each $z \in Z$, then p is a covering map.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Let $f : X \rightarrow Y$ be continuous and let $f(x_0) = y_0$. If f is a homotopy equivalence, prove that $f_* : \pi_1(X, x_0) \rightarrow \pi_1(Y, y_0)$ is an isomorphism.
- b) If $G = G_1 * G_2$, show that

$$\frac{G}{[G, G]} \cong \left(\frac{G_1}{[G_1, G_1]} \right) \oplus \left(\frac{G_2}{[G_2, G_2]} \right).$$

- c) Prove that the fundamental group of the torus is a free abelian group of rank 2.

Q4) a) Attempt any one of the following.

[4]

- i) Prove that the fundamental group of the torus has a presentation consisting of two generators α, β and a single relation $\alpha\beta\alpha^{-1}\beta^{-1}$.
- ii) State the Seifert-van Kampen theorem.

b) Attempt any one of the following.

[5]

- i) Let $p : E \rightarrow B$ and $p' : E' \rightarrow B$ be covering maps. Let $p(e_0) = p'(e'_0) = b_0$. Prove that, there is an equivalence $h : E \rightarrow E'$ such that $h(e_0) = e'_0$ if and only if the groups $H_0 = p_*(\pi_1(E, e_0))$ and $H'_0 = p'_*(\pi_1(E', e'_0))$ are equal. Also, prove that if h exists, it is unique.
- ii) Show that if $n > 1$, every continuous map $f : S^n \rightarrow S^1$ is nullhomotopic.



Total No. of Questions : 4]

SEAT No. :

[Total No. of Pages : 2

PE4418

[6508]-36

S.Y. M.A./M.Sc.

MATHEMATICS

**MTS-611(C) MJ : Integral Transforms and Special Functions
(2023 Credit Pattern) (Semester - III)**

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following.

[6]

- a) Find $\mathcal{L}\{te^{-3t}\}$.
- b) With usual notations, find $C_n\{t\}$.
- c) Prove that $\Gamma(1) = 1$.
- d) Prove that $B(x+1, y) = \frac{x}{x+y} B(x, y)$.

Q2) Attempt any two of the following.

[10]

- a) Using the convolution method, determine:

i) $\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)}\right\}$

ii) $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s-2)}\right\}$

- b) Solve the following initial value problem:

$$y''(t) + y(t) = \delta(t - \pi) \cos t, y(0) = 1, y'(0) = 0$$

- c) Prove that:

i) $J'_n(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$

ii) $J'_n(x) = \frac{n}{x} J_n(x) - J_{n+1}(x)$

PTO.

Q3) Attempt any two of the following.

[10]

a) Show that:

i) $E_{\alpha}\{t f(t)\} = i F'_{\alpha}(\alpha)$

ii) $E_{\alpha}\{t^2 f(t)\} = -F''_{\alpha}(\alpha)$

iii) $E_{\alpha}\{t^n f(t)\} = i^n F^n_{\alpha}(\alpha)$

b) Prove that:

$$K_n(x) = \frac{\pi}{2} i^{n+1} \{J_n(ix) + i Y_n(ix)\} = \frac{\pi}{2} i^{n+1} H_n^{(1)}(ix).$$

c) Prove that $\Gamma(x)\Gamma(1-x) = \frac{x}{\sin \pi x}$.

Q4) a) Attempt any one of the following.

[4]

i) Solve for $y(t)$ in the integro-differential equation & check the solution.

$$\int_0^t y(\tau) d\tau - y'(t) = 1, \quad y(0) = 1$$

ii) Prove that:

$$I_n(x) = \sum_{r=0}^{\infty} \frac{1}{r! \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{(2r+n)}$$

b) Attempt any one of the following.

[5]

i) Solve the following boundary value problem:

$$y_{xx} = 4y_{tt}, \quad t > 0, x > 0$$

$$y(x, 0) = 0, \quad x > 0$$

$$y_t(x, 0) = -1, \quad x > 0$$

$$y(0, t) = t, \quad x > 0$$

$$\lim_{x \rightarrow \infty} y(x, t) \text{ exist for a fixed } t > 0.$$

ii) Show that $K_n(x)$ satisfies the differential equation:

$$\frac{d^4 y}{dx^4} + \frac{2}{x} \frac{d^3 y}{dx^3} - \frac{2n^2 + 1}{x^2} \frac{d^2 y}{dx^2} + \frac{2n^2 + 1}{x^3} \frac{dy}{dx} + \left(\frac{n^4 - 4n^2}{x^4} - 1 \right) y = 0.$$



Total No. of Questions : 4]

SEAT No. :

[Total No. of Pages : 2

PE4419

[6508]-37

Second Year M.A./M.Sc.

MATHEMATICS

MTS-611(D) MJ : Mechanics

(2023 Credit Pattern) (Semester - III)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following.

[6]

- a) Define : Degrees of freedom.
- b) Explain the term Angular Momentum.
- c) State Hamilton's principle for Non conservative system.
- d) Write any two proportion of poission Bracket.

Q2) Attempt any two of the following.

[10]

- a) Prove that if force acting on a particle is conservative then the total energy is conserved.
- b) Set up the Lagrangian and Lagranges equation of motion for simple pendulum.
- c) Show that geodesic on right circular cylinder is a helix.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Find the minimum of the functional $F(y(x)) = \int_0^1 \left(\frac{1}{2} y'^2 + y y' + y' + y \right) dx$ if the values of end points not given.
- b) Show that the Hamilton's principle $\delta \int_{t_0}^{t_1} L dt = 0$ also holds for the non conservative system.
- c) Prove that the transformation equations between two sets of co-ordinates given by $P = 2(1 + q^{1/2} \cos p) q^{1/2} \sin p$, $Q = \log(1 + q^{1/2} \cos p)$ are canonical transformation.

Q4) a) Attempt any one of the following.

[4]

- i) Apply Hamilton - Jacobi method to determine the motion of a body falling vertically in a uniform gravitational field.
- ii) Show that the transformation defined by $q = \sqrt{2P} \sin Q$, $p = \sqrt{2P} \cos Q$ is canonical by poisson bracket.

b) Attempt any one of the following.

[5]

- i) Find plane curve of fixed perimeter and maximum area.
- ii) Show that the shortest distance between two points in a plane is a straight line.



Total No. of Questions : 4]

SEAT No. :

PE4420

[Total No. of Pages : 2

[6508]-41

Second Year M.A./M.Sc.

MATHEMATICS

MTS - 651 MJ : Functional Analysis
(2023 Credit Pattern) (Semester - IV)

Time : 2Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any Three of the following. [6]

- a) Let $X = \mathbb{K}^n$ and $x = (x_1, x_2, \dots, x_n) \in \mathbb{K}^n$ and $\|x\|_1 = \sum_{i=1}^n |x_i|$ be the norm on $\mathbb{K}^n$. Show that $(X, \|\cdot\|_1)$ is normed linear space.
- b) Define
 - i) Normal operator on a Hilbert space
 - ii) Hermitian operator on a Hilbert space
- c) Prove that eigen values of a self-adjoint operator $T \in BL(H)$ are real.
- d) State Hahn- Banach Theorem.

Q2) Attempt any Two of the following. [10]

- a) Let C be closed convex subset of a Hilbert space H . Show that there exists a unique $a \in C$ such that $\|a\| \leq \|x\|$ for all $x \in C$.
- b) Let X be $\mathbb{K}^n$ and for $1 \leq p \leq \infty$ let $\|x\|_p = \left(\sum_{i=1}^n |x_i|^p \right)^{1/p}$ and for $p = \infty$ let $\|x\|_\infty = \max \{|x_i| : 1 \leq i \leq n\}$, for $p > 1$, let q be such that $\frac{1}{p} + \frac{1}{q} = 1$ then derive the Holder's inequality
$$\sum_{i=1}^n |a_i| \cdot |b_i| \leq \|a\|_p \|b\|_q$$
- c) Prove That, The trigonometric system
$$\mathbb{B} = \left\{ \frac{1}{\sqrt{2\pi}}, \frac{\cos(mx)}{\sqrt{\pi}}, \frac{\sin(mx)}{\sqrt{\pi}} : m, n \in \mathbb{N} \right\}$$
 is an orthonormal set for $L^2([-\pi, \pi], d\lambda)$.

P.T.O.

Q3) Attempt any Two of the following.

[10]

- a) Show that $(B(X, \mathbb{K}), \| \cdot \|_\infty)$ is a Banach space.
- b) Let X and Y be Banach spaces. Let $T: X \rightarrow Y$ be a bijective continuous linear map prove that $T^{-1}: Y \rightarrow X$ is continuous.
- c) Let H be finite dimensional and $T: H \rightarrow H$ be self-adjoint prove that one of the numbers $\| T \|$ is an eigen value of T .

Q4) a) Attempt any one of the following.

[5]

- i) Show that, The normed linear space $(\ell', \| \cdot \|_1)$ is complete.
- ii) Prove that any Hilbert space H has an Hilbert basis.

b) Attempt any one of the following.

[4]

- i) Show that, two norms $\| \cdot \|_1$ and $\| \cdot \|_2$ on X are equivalent if and only if there exist constants C_1 and C_2 such that $C_1 \| x \|_1 \leq \| x \|_2 \leq C_2 \| x \|_1$ for all $x \in X$.
- ii) Let H be a complex Hilbert space Then prove That
 - 1) $(\alpha A + \beta B)^* = \bar{\alpha} A^* + \bar{\beta} B^*$ for $\alpha, \beta \in \mathbb{C}$ and $A, B \in BL(H)$
 - 2) $(AB)^* = B^* A^*$ for $A, B \in BL(H)$



Total No. of Questions : 7]

SEAT No. :

PE4421

[6508]-42

[Total No. of Pages : 2

Second Year M.A./M.Sc.

MATHEMATICS

MTS - 653-MJ : Partial Differential Equations and Boundary Value Problems

(2023 Credit Pattern) (NEP 2020) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicates full marks.

Q1) Attempt any five of the following. [10]

- a) Give an example of second order non-linear PDE.
- b) Explain the difference between integral curve and integral surface.
- c) Write 2-dimensional heat equation in Polar Form.
- d) What is the meaning of Particular Integral?
- e) When we say that, "System of linear PDE is compatible"?
- f) What is the order of the PDE $u_t = u_{xxx} + \sin x$?
- g) Explain the term : Canonical form of linear PDE.

Q2) Attempt any two of the following. [10]

- a) Show that the solution of the linear PDE $P_p + Q_q = R$ can be written in the form $F(u,v) = 0$. Where, F is an arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ Form a solution of the equation $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
- b) Find the solution of $(D^3 - 3D^2D^1 + 4D^{1^3})u = 0$.
- c) Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity.

Q3) Attempt any two of the following. [10]

- a) Solve : $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$.
- b) Reduce the equation $(1+x^2)u_{xx} + (1+y^2)u_{yy} + xu_x + yu_y = 0$ to canonical form.
- c) Solve $u_{xx} + u_{yy} = 0$ using variable separable method.

P.T.O.

Q4) Attempt any two of the following. **[10]**

- Solve the Tricomi equation $u_{xx} + xu_{yy} = 0$ For $x < 0$ to a canonical form.
- Define Dirac Delta function $\delta(t)$ and hence show that
$$\int_{-\infty}^{\infty} \delta(t-a) f(t) dt = f(a).$$
- Explain Charpit's method for finding the complete integral of a non linear PDE of first order of the form $f(x, y, z, p, q) = 0$.

Q5) Attempt any two of the following. **[10]**

- Find the complete integral of $x^2 p^2 + y^2 q^2 = 4$ using Charpit's method.
- Show that the PDEs $xp - yq = x$ and $x^2 p + q = xz$ are compatible and hence find their solution.
- Derive the Diffusion equation of second order linear partial differential Equation.

Q6) Attempt any two of the following. **[10]**

- Show that a linear PDE of the type $\sum_i \sum_j G_{ij} x^i y^j \frac{\partial^{i+j} u}{\partial x^i \partial y^j} = f(x, y)$ can be reduced to a one with constant coefficients by the substitution $\xi = \log x, \eta = \log y$.
- Reduce the equation $u_{xx} - 2 \sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$ to a canonical form.
- Solve $u_{tt} - 2u_{xx} = f(x, t)$ subject to the initial condition $u(x, 0) = \eta(x)$ and $u_t(x, 0) = v(x)$.

Q7) Attempt any two of the following. **[10]**

- Solve : $\nabla^2 u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$, $0 \leq r \leq a, 0 \leq \theta \leq 2\pi$
 $BC: u(a, \theta) = f(\theta)$, $0 \leq \theta \leq 2\pi$
 using variable separable form. Where, $f(\theta)$ is a continuous function on the boundary of circle R.
- Define wave function and hence show that it is periodic solution of the wave equation.
- Solve the boundary value problem $u_x = 4u_y$ with $u(0, y) = 8e^{-3y}$ by the method of separation of variables.



Total No. of Questions :7]

SEAT No. :

PE4422

[Total No. of Pages : 3

[6508]-43

M.A./M.Sc.-II

MATHEMATICS

MTS-654 MJ : Measure Theory and Integration
(2023 Credit Pattern) (Semester-IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.

Q1) Attempt any five of the following.

[10]

- a) Show that, if $m^*(A)=0$ then $m^*(A \cup B)=m^*(B)$ for any set B.
- b) Show that every non-empty open set has positive outer measure.
- c) Show that, If F is measurable and $m(E) < \infty$ and $A \leq F \leq B$ then
$$Am(E) \leq \int_E F dx \leq B m(E),$$
 where A, B are constants.
- d) Define Lebesgue integrable function.
- e) Show that, $L^\infty(X, \mu)$ is a vector space over the real number.
- f) Show that , $\text{ess sup } F = - \text{ess inf } (-F)$
- g) Give an example where the strict inequality occurs in the Fatau's lemma.

Q2) Attempt any two of the following.

[10]

- a) State and prove Lindelöf's theorem.
- b) Prove that, outer measure of an interval equals to it's length.
- c) Prove that, the class $\mathcal{M}$ is a δ algebra.

P.T.O.

Q3) Attempt any Two of the following. [10]

- Prove that every interval is measurable.
- If $m^*(E) < \infty$ then prove that, E is measurable if and only if $\forall \epsilon > 0, \exists$ disjoint finite intervals $I_1, I_2, \dots, I_n$ such that, $m^*\left(E \Delta \bigcup_{i=1}^n I_i\right) < \epsilon$. Where the intervals I_i may be open, closed or half-open.
- Show that, the following statements regarding the set E are equivalent.
 - E is measurable
 - $\forall \epsilon > 0, \exists O$, an open set, $O \supseteq E$ such that, $m^*(O - E) \leq \epsilon$.
 - $\exists G$, a G_δ -sets, $G \supseteq E$ such that, $m^*(G - E) = 0$.

Q4) Attempt any two of the following. [10]

- State and prove Fatau's Lemma.
- Let $\{F_n\}$ be a sequence of measurable function such that, $|F_n| \leq g$, where g is integrable, and let $\lim F_n = F$ almost every- where then show that, F is integrable and $\lim \int F_n dx = \int F dx$
- If ϕ is measurable simple function then show that,
 - $\int_E \phi dx = \sum_{i=1}^n a_i m(A_i \cap E)$, for any measurable set E .
 - $\int_{A \cup B} \phi dx = \int_A \phi dx + \int_B \phi dx$, for any disjoint measurable set A and B .
 - $\int a \phi dx = a \int \phi dx$, if $a > 0$

Q5) Attempt any two of the following. [10]

- Show that, $\lim_{n \rightarrow \infty} \int_a^\infty \frac{n^2 x e^{-n^2 x^2}}{1 + x^2} dx$, for $a > 0$ and for $a = 0$
- Prove that, function $F \in B \vee [a, b]$ if and only if F is difference of two finite valued monotone increasing function on $[a, b]$ where a and b are finite.
- If $f \in L[a, b]$ then show that,
 - $F(x) = \int_a^x f(t) dt$ is continuous function on $[a, b]$
 - $F \in B \vee [a, b]$

Q6) Attempt any two of the following. **[10]**

a) Let $[a, b]$ be a finite interval and Let $F \in L^1[a, b]$ with indefinite integral F , then show that, $F' = F$ almost everywhere in $[a, b]$.

b) Let $1 < p < \infty, 1 < q < \infty, \frac{1}{p} + \frac{1}{q} = 1$ and let $f \in L^p(\mu), g \in L^q(\mu)$ then show that, $f \cdot g \in L^1(\mu)$ and

$$\int |f \cdot g| d\mu \leq \left(\int |f|^p d\mu \right)^{1/p} \cdot \left(\int |g|^q d\mu \right)^{1/q}$$

c) Let $[x, \delta, \mu]$ be a measure space with $\mu(x) = 1$. If Ψ is a convex on (a, b) where $-\infty < a < b < \infty$, and F is a measurable function such that, $a < f(x) < b$, for all x , then show that, $\Psi\left(\int F d\mu\right) \leq \int \Psi \circ f d\mu$

Q7) Attempt any two of the following. **[10]**

a) If a sequence of measurable function converges in measure, then show that the limit function is unique almost everywhere.

b) Let $\{F_n\}$ be a sequence of measurable function such that, $|F_n| < g$, an integrable function, and let $F_n \rightarrow F$ in measure, where F is measurable then show that,

i) f is integrable

ii) $\lim \int F_n d\mu = \int F d\mu$ and $\lim \int |F_n - F| d\mu = 0$

c) Let $\mu(x)$ be finite. If $\rho(F) = \int \frac{|F|}{1+|F|} d\mu$ and

$\rho(f, g) = \rho(f - g)$, then show that, ρ is metric space of measurable functions and finite valued almost everywhere provided that, function equal almost everywhere are identified. Also show that, convergence in this metric is equivalent to convergence in measure, the metric space defined is complete.



Total No. of Questions : 4]

SEAT No. :

PE4423

[Total No. of Pages : 2

[6508]-44

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-661(A) MJ : Commutative Algebra

(2023 Credit Pattern) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following :

[6]

- a) Let x be a nilpotent element of a ring A . Show that $1 + x$ is a unit in A .
- b) Define a valuation ring.
- c) Give an example of a ring which satisfies A.C.C. but not D.C.C. on ideals.
- d) Define Artin ring.

Q2) Attempt any two of the following :

[10]

- a) Let A be a commutative ring $\neq 0$ with an identity element and let $\mathfrak{R}$ be the Jacobson radical of A . Prove that $x \in \mathfrak{R} \Leftrightarrow 1 - xy$ is a unit in A for all $y \in A$.
- b) Let M be a finitely generated A -module and $\mathfrak{a}$ an ideal of A contained in the Jacobson radical $\mathfrak{R}$ of A . Then prove that $\mathfrak{a}M = M$ implies $M = 0$.
- c) Let A be a commutative ring $\neq 0$ with an identity element. In the ring $A[x]$, the ring of polynomials in an indeterminate x , is the Jacobson radical equal to the nilradical? Justify your answer.

P.T.O.

Q3) Attempt any two of the following.

[10]

- a) Let A be a local ring, and let M and N be finitely generated A -modules. Prove that if $M \otimes_A N = 0$, then $M = 0$ or $N = 0$.
- b) Let $A \rightarrow B$ be a ring homomorphism and let p be a prime ideal of A . Then prove that p is the contraction of a prime ideal of B if and only if $p^{ec} = p$.
- c) Let S be a multiplicatively closed subset of A , and let q be a p -primary ideal.
 - i) If $S \cap p \neq \emptyset$, then prove that $S^{-1}q = S^{-1}A$.
 - ii) If $S \cap p \neq \emptyset$, then prove that $S^{-1}q$ is $S^{-1}p$ -primary and its contraction in A is q .

Q4) a) Attempt any one of the following.

[4]

- i) Let M be an A module. If every nonempty set of finitely generated submodules of M has a maximal element, then prove that M is Noetherian.
- ii) Prove that in a Noetherian ring, every irreducible ideal is primary.

b) Attempt any one of the following.

[5]

- i) Prove that an Artin ring A has only a finite number of maximal ideals.
- ii) Prove that M is a Noetherian A -module if and only if every submodule of M is finitely generated.



Total No. of Questions : 4]

SEAT No. :

PE4424

[Total No. of Pages : 2

[6508]-45

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-661 (B) MJ : Financial Mathematics
(2023 Credit Pattern) (NEP 2020) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Symbols have their usual meanings.

Q1) Attempt any three of the following. **[6]**

- a) How much would you pay today to receive \$1,000 after 3 months (91 days) if you require a return of 2%?
- b) Explain self-financing investment strategy.
- c) Prove that the variance σ_V^2 of a portfolio cannot exceed the greater of the variances σ_1^2 and σ_2^2 of the components, $\sigma_V^2 \leq \max\{\sigma_1^2, \sigma_2^2\}$ if short sales are not allowed.
- d) Define :
 - i) Portfolio Management
 - ii) Risk Management

Q2) Attempt any two of the following. **[10]**

- a) A sum of \$10,000 paid into a bank account for three months (91 days) to attract simple interest will produce \$10,300 at the end of the term. Find the interest rate r and the return on this investment.
- b) Suppose that the time step is taken to be three months, $\tau = 1/4$, and the quarterly returns $K(1), K(2), K(3), K(4)$ are independent and identically distributed. Find the expected quarterly return $E(K(1))$ and the expected annual return $E(K(0, 4))$ if the expected return $E(K(0, 3))$ over three quarters is 12%.
- c) Given that the stock price $S(n)$ has become known at time n , prove that the risk-neutral conditional expectation of $S(n+1)$ will be $E_*(S(n+1) | S(n)) = S(n) (1 + r)$

P.T.O.

Q3) Attempt any two the following. **[10]**

- a) Consider two scenarios, ω_1 with probability $\frac{1}{4}$ and ω_2 with probability $\frac{3}{4}$. Suppose that the return on a certain security is $K_1(\omega_1) = -2\%$ in the first scenario and $K_1(\omega_2) = 8\%$ in the second scenario. If the return on another security is $K_2(\omega_1) = -4\%$ in the first scenario, find the return $K_2(\omega_2)$ in the other scenario such that the two securities have the same risk.
- b) Consider a market with a risk-free asset such that $A(0) = 100$, $A(1) = 110$, $A(2) = 121$ dollars and a risky asset, the price of which can follow three possible scenarios,

Scenario	$S(0)$	$S(1)$	$S(2)$
ω_1	100	120	144
ω_2	100	120	96
ω_3	100	90	96

Is there an arbitrage opportunity if i) There are no restrictions on short selling, and ii) no short selling of the risky asset is allowed?

- c) Prove that the return K_V on a portfolio consisting of two securities is the weighted average $K_V = w_1 K_1 + w_2 K_2$, where w_1 and w_2 are the weights and K_1 and K_2 the returns on the two components.

Q4) A) Attempt any one of the following. **[4]**

- a) Prove that the future value $V(t)$ increases if any one of the parameters m , t , r or P increases, the others remaining unchanged.
- b) Suppose that \$32, \$28 and x and the possible value of $S(2)$. Find x , assuming that stock prices follow a binomial tree. Can you complete the tree? Can this be done uniquely.

B) Attempt any one of the following. **[5]**

- a) Compute the risk $\text{Var}(K_1)$, $\text{Var}(K_2)$ and $\text{Var}(K_3)$ in each of the following three investment projects, where the returns K_1 , K_2 and K_3 depend on the market scenario :

Scenario	Probability	Return K_1	Return K_2	Return K_3
ω_1	0.25	12%	11%	2%
ω_2	0.75	12%	13%	22%

Which of these is the most risky and the least risky project?

- b) Prove that the variance of the return on a portfolio is given by

$$\text{Var}(K_V) = w_1^2 \text{Var}(K_1) + w_2^2 \text{Var}(K_2) + 2w_1 w_2 \text{Cov}(K_1, K_2).$$



Total No. of Questions : 4]

PE4425

SEAT No. :

[Total No. of Pages : 2

[6508]-46

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-661 (C) MJ : Algebraic Curves

(2023 Credit Pattern) (Semester - IV) (CBCS)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*

Q1) Attempt any three of the following.

[6]

- a) Show that the set $\{(\cos t, \sin t) \in \mathbb{A}^2(\mathbb{R}) \mid t \in \mathbb{R}\}$ is algebraic set.
- b) State the Hilbert Basis Theorem and explain its importance.
- c) Explain the concept of an affine variety.
- d) State Bézout's theorem for curves in the projective plane.

Q2) Attempt any two of the following.

[10]

- a) If a field L is ring-finite over a subfield K , then prove that L is module finite (and hence algebraic) over K .
- b) Let $I \subseteq J$ be ideals in a ring R . Show that there is a natural ring homomorphism from R/I onto R/J .
- c) Find the intersection number of the curves $A : Y - X^2$ and $B : Y^2 - X^3$ at the point $P = (0, 0)$.

P.T.O.

Q3) Attempt any two the following.

[10]

- a) Give an example of a countable collection of algebraic sets whose union is not algebraic.
- b) Describe the relationship between coordinate rings and affine varieties. Illustrate with an example.
- c) Discuss the concept of discrete valuation rings. Give an example and explain their significance.

Q4) a) Attempt any one of the following.

[4]

- i) Find the multiple points and their multiplicities for the projective curve $F(X, Y, Z) = XY^4 + YZ^4 + XZ^4$.
- ii) Prove that $P=(0, 0)$ is the only multiple point on the curve $C : Y^2 = X^3 + X^2$.

b) Attemp any one of the following.

[5]

- i) Show that every nonsingular projective plane curve is irreducible.
- ii) Show that an ideal $I \subset k[X_1, \dots, X_{n+1}]$ is homogeneous if and only if it is generated by a (finite) set of forms.



[6508]-47

S.Y. M.A./M.Sc.

MATHEMATICS

MTS-661 (D) MJ : Optimization Techniques
(2023 Credit Pattern) (NEP - 2020) (Semester - IV)

Time : 2 Hours]

[Max. Marks : 35

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of non - program mable, scientific calculator is allowed.

Q1) Attempt any three of the following.

[6]

- a) Find Hessian matrix at point $\left(\frac{4}{3}, \frac{5}{3}\right)$ of the following function

$$f(X) = x_1 + 2x_2 + x_1x_2 - x_1^2 - x_2^2$$

- b) Find the minimax and maximin value of the following game.

	I	II	III	IV
I	-5	2	1	20
II	5	5	4	6
III	4	-2	0	-5

- c) Explain the terms.

- i) Carrying Cost
- ii) Ordering Cost

- d) Draw the network diagram for the following relationship.

Activity	(1-2)	(1-3)	(2-3)	(2-4)	(3-4)	(4-5)
Preceding Activity	-	-	(1-2)	(1-2)	(1-3),(2-3)	(2-4), (3-4)

P.T.O.

Q2) Attempt any two of the following.

[10]

- a) Two players A and B without showing each other, put on a table a coin, with head or tail up. A wins Rs. 8 when both the coins show head and Rs. 1 when both are tails. B wins Rs. 3 when the coins do not match. Given the choice of being matching player (A) or non-matching player (B), which one would you choose and what would be your strategy?
- b) Derive the formula

$$Q^* = \sqrt{\frac{2DC_0}{C_h} \left(\frac{p}{p-d} \right)}$$

for economic order quantity Q^*

Where C_0 = ordering cost or set-up cost

C_h = holding cost per unit per time

d = usage rate per unit of time

p = production rate per unit of time, with $p > d$

- c) A project has following data :

Activity	Times in months
(1-2)	2
(1-3)	2
(1-4)	1
(2-5)	4
(3-6)	8
(3-7)	5
(4-6)	3
(5-8)	1
(6-9)	5
(7-8)	4
(8-9)	3

- i) Draw an arrow diagram for this project.
- ii) Indicate the critical path.

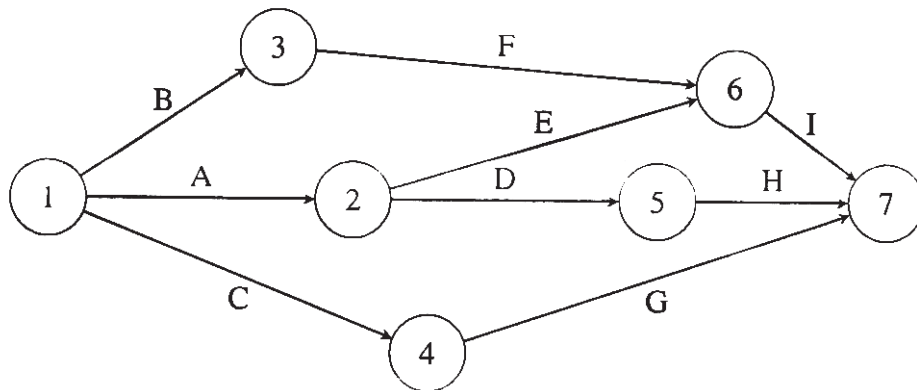
Q3) Attempt any two the following.

[10]

- a) Solve the following game by dominance principle.

		Player B				
		I	II	III	IV	V
Player A	I	3	5	4	9	6
	II	5	6	3	7	8
	III	8	7	9	8	7
	III	4	2	8	5	3

- b) A project is represented by the network shown and has the following data:



Task	A	B	C	D	E	F	G	H	I
t_o	5	18	26	16	15	6	7	7	3
t_p	10	22	40	20	25	12	12	9	5
t_m	8	20	33	18	20	9	10	8	4

Determine the following.

- Expected task time and their variance.
 - Critical path,
- c) Find the dimensions of a rectangular parallelepiped with largest volume whose sides are parallel to the coordinates planes, to be inscribed in the ellipsoid.

$$g(x, y, z) \equiv \left(\frac{x^2}{a^2} \right) + \left(\frac{y^2}{b^2} \right) + \left(\frac{z^2}{c^2} \right) - 1 = 0$$

Q4) a) Attempt any one of the following. **[4]**

- i) Find the maximum or minimum of the function.

$$f(x) = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 56$$

- ii) The annual requirements for a particular raw material are 2,000 units, costing Rs.1/- each to manufacture. The ordering cost is Rs.10/- per order, and the carrying cost is 16% of the average inventory value. Find the Economic Order Quantity (EOQ) and the total inventory cost per annum.

b) Attempt any one of the following. **[5]**

- i) Solve the following 2×4 game graphically.

Player A	Player B			
	B1	B2	B3	B4
A1	2	2	3	-2
A2	4	3	2	6

- ii) Explain the four types of floats used in network analysis.

