

Total No. of Questions : 5]

SEAT No. :

PE5594

[Total No. of Pages : 2

[6521]-101

First Year M.A./M.Sc.

STATISTICS

ST 11 : Basics of Real Analysis and Calculus
(2019 Pattern) (Semester - I) (4 Credits)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt all the following questions.

[2 each]

- a) Define Compact set with an illustration.
- b) Evaluate $\int_a^b 2d(\sin^{-1}(x))$.
- c) If $x \neq 0, y \neq 0$ then show that $x \cdot y \neq 0; x, y \in \mathbb{R}$.
- d) Check whether the following set is countable set. Justify your answer.
 $A = \{1, 3, 5, \dots\}$.
- e) Define the Logarithmic Series.

Q2) Attempt any three of the following.

[5 each]

- a) Prove that $\left| \frac{1}{2} \right| = \sqrt{\pi}$.
- b) Prove that there is no rational number whose square is 2.
- c) Let A be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers of the form $-x$, where $x \in A$, then prove that $\inf A = -\sup(-A)$.
- d) State and prove finite intersection property of compact set.

P.T.O.

Q3) Attempt any three of the following. **[5 each]**

- a) A set is closed if and only if its complement is open.
- b) Show that the union of two countable sets is countable.
- c) Consider a function f defined on $[a, b]$. If f has a local maxima at a point $x \in (a, b)$, and $f'(x)$ exists, then show that $f'(x) = 0$. Also state analogous statement for local minima and prove it.
- d) Show by an example how the sequence $\{f_n\}$ oscillates finitely.

Q4) Attempt any three of the following. **[5 each]**

- a) Find the limit of the following.
 - i) $\lim_{n \rightarrow \infty} x^n$ for $|x| < 1$.
 - ii) $\lim_{n \rightarrow \infty} \frac{n^\alpha}{(1+p)^n}$ where $p > 0, \alpha \in \mathbb{R}$.
- b) Define metric space. Examine whether $d(x-y) = (x-y)^2$ for x and y in $\mathbb{R}$ is a metric or not.
- c) State and prove fundamental theorem of calculus.
- d) State and prove the relation between the Beta Function and the Gamma function.

Q5) Attempt any one of the following.

- a) Prove or disprove that following statements. **[5×3=15]**
 - i) Countable union of closed set is closed.
 - ii) Every convergent sequence is bounded sequence.
 - iii) If series $\sum a_n$ is convergent then series $\sum |a_n|$ is also convergent.
 - iv) Every neighbourhood is an open set.
 - v) Set of irrational numbers is uncountable.
- b)
 - i) State and prove Heine - Borel theorem. **[5]**
 - ii) State and prove root test for the convergence of series. Further examine whether $\sum \frac{(n+1)}{n2^n}$ converges or not. **[7]**
 - iii) Give an example of a set satisfying the condition in each of the following. **[3]**
 - 1) Set is neither closed nor open set.
 - 2) Bounded set which is closed set as well as open set.



Total No. of Questions : 5]

SEAT No. :

PE-5595

[Total No. of Pages : 3

[6521]-102

M.Sc. (Part - I)

STATISTICS

**ST-12 : Linear Algebra and Numerical Methods
(2019 Pattern) (Semester - I) (4 Credits)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all questions :

[5 × 2 = 10]

- a) If a matrix A is invertible then show that inverse of A is unique.
- b) Show that positive definite real symmetric matrix is non singular.
- c) Define Geometric and Algebraic multiplicity.
- d) Define Inner product space.
- e) Find the characteristic polynomial of $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$.

Q2) Attempt any three of the following :

[3 × 5 = 15]

- a) Let $V = \mathbb{R}^2$, with operations $(x_1, y_1) + (x_2, y_2) = (x_1, y_1 + y_2)$ and $k(x_1, y_1) = (kx_1, ky_1)$. Check whether V is vector space or not.
- b) Define linear dependent and linear independent set of vectors. Hence show that $S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ is linearly independent in $\mathbb{R}^3$.
- c) State and prove Cayley Hamilton theorem.
- d) Show that a real symmetric matrix A is positive definite if and only if there exist a non singular matrix Q such that $A = Q'A$.

P.T.O.

Q3) Attempt any three of the following :

[3 × 5 = 15]

- a) Find eigenspace corresponding to each eigenvalue of $A = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & -1 \\ 2 & 1 & 3 \end{bmatrix}$.
- b) Find the rank and nullity of the matrix $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$.
- c) Define Symmetric matrix and Skew Symmetric matrix with an illustration and Show that every square matrix A can be expressed as sum of Symmetric matrix and Skew Symmetric matrix.
- d) If $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_r$ are eigenvectors of A corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_r$ then show that $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_r\}$ is linearly independent.

Q4) Attempt any three of the following :

[3 × 5 = 15]

- a) Let S be a set with two or more vectors in a vector space V. Then show that S is linearly dependent if and only if at least one of the vectors in S is expressible as a linear combination of the rest of the vectors in S.
- b) If K is positive integer, λ is an eigenvalue of a matrix A and X is a corresponding eigenvector then show that λ^k is an eigenvalue of A^k .
- c) Define g-inverse and obtain two distinct g-inverses for the following matrix.

$$A = \begin{bmatrix} 5 & 2 & -1 \\ 3 & 2 & 3 \\ 3 & 2 & 3 \end{bmatrix}$$

- d) Solve following system of equation.

$$x_1 + 2x_2 - 3x_3 - 4x_4 = 0$$

$$x_1 + 3x_2 - x_3 - 2x_4 = 0$$

$$2x_1 + 5x_2 - 2x_3 - 4x_4 = 0$$

Q5) Attempt any one of the following :

[1 × 15 = 15]

- a) i) Define singular value decomposition of a real symmetric matrix.

Obtain the same for $A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$.

- ii) Solve the following equations by Newton Raphson method

$$X^2 + Y = 3 \text{ and } Y^2 + x = 3 \text{ when } X_0 = Y_0 = 1.3$$

[7+8]

- b) i) Let $S = \{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_r\}$ be a set of vectors in a vector space V. Then show that L(S) is a subspace of V. Hence show that L(S) is the smallest subspace of V containing S.

- ii) Define quadratic form. Hence find the maximum and minimum value of the quadratic form $Q = (X_1, X_2, X_3) = 3X_1^2 + 2X_2^2 + 3X_3^2 + 2X_1X_3$ subject to the constraint $X_1^2 + X_2^2 + X_3^2 = 1$.

[8+7]



Total No. of Questions : 5]

SEAT No. :

PE5596

[Total No. of Pages : 2

[6521]-103

First Year M.A./M.Sc.

STATISTICS

ST-13 : Probability Distributions

(2019 Pattern) (Semester - I) (4 Credits)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt all questions.

[5×2=10]

- a) Define the following terms.
 - i) Probability generating function
 - ii) Cumulative distribution function.
- b) Show that the mean of a symmetric distribution about zero is zero.
- c) If $X \sim U(0, 1)$, find the pdf of $Y = -2\ln(X)$.
- d) If $F(x)$ and $G(x)$ are distribution functions of two independent random variables then show that $F(x)^2G(x)^3$ is a distribution function.
- e) State Hogg and Craig Theorem.

Q2) Attempt any 3 questions out of 4 questions.

[3×5=15]

- a) Find the probability generating function of the random variable $X \sim B(n, p)$ and hence it's mean and variance.
- b) Let X is a random variable having pdf as,
$$f(x) = \begin{cases} 3x^2; & 0 < X < 1 \\ 0; & \text{Otherwise} \end{cases}$$
 Find its mean, variance and moment generating function of X .
- c) Let X be a random variable having $df \ f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$. Obtain the distribution of X^2 .
- d) Let $X \sim \text{Poisson}(\lambda)$. Show that $E[X(X-1)] = \lambda^2$ and deduce the variance of X .

P.T.O.

Q3) Attempt any 3 questions out of 4 questions. [3×5=15]

- a) Define bivariate exponential distribution. State the memory less property satisfied by this distribution.
- b) Derive probability density function of Dirichlet distribution as a bivariate beta distribution.
- c) If X and Y are independent normal random variables with means 0 and variances 1, find the pdf of $Z = X^2 + Y^2$. Identify the distribution of Z ?
- d) Let $X_1, X_2, \dots, X_n$ be a random sample from a uniform distribution $U(0, \theta)$. Derive the pdf of the maximum order statistic $X_{(n)}$.

Q4) Attempt any 3 questions out of 4 questions. [3×5=15]

- a) Define non - central Student's t-random variable and obtain its density function.
- b) Prove that, two quadratic forms $Q_1 = X'AX$ and $Q_2 = X'BX$ are said to be independent iff $AB = 0$.
- c) Let $X_1, X_2, \dots, X_n$ be a random sample from an exponential distribution with parameter λ . Show that $2\lambda \sum_{i=1}^n X_i$ follows a chi-square distribution with $2n$ degrees of freedom.
- d) Define the bivariate normal distribution and derive the expression for its correlation coefficient.

Q5) Attempt any 1 question out of 2 questions. [1×15=15]

- a) i) Suppose (X_1, X_2) follows jointly bivariate poisson variate with parameters $\lambda_1 = 7, \lambda_2 = 5$ and $\lambda_{12} = 4$. Compute marginal distribution of X_1 and X_2 and hence find 1) $E(X_1)$ and $E(X_2)$ 2) $var(X_1)$ and $var(X_2)$ 3) $corr(X_1, X_2)$ and iv) $P(X_1 = 4|X_2 = 6)$. [7]
ii) Let X_1, X_2 be independent exponential random variables with parameters λ_1 and λ_2 . If $U = X_1 + X_2$ and $V = \frac{X_1}{X_1 + X_2}$, then obtain the joint pdf of (U, V) where. Hence find the marginal distributions of U and V ? [8]
- b) i) State and prove Fisher Cochran's theorem. [8]
ii) Define order statistics and derive the pdf of the r^{th} order statistic in a random sample of size n from a continuous distribution. [7]



Total No. of Questions : 5]

SEAT No. :

PE5597

[Total No. of Pages : 2

[6521]-104

F.Y. M.A./M.Sc.

STATISTICS

ST-14 : Sampling Theory
(2019 Pattern) (Semester - I)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of scientific calculator and statistical table is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt each of the following: (2 Mark each)

- a) Distinguish between SRSWR and SRSWOR.
- b) Define Proportional allocation. Prove that under proportional allocation
$$n_i = \left(\frac{n}{N} \right) N_i.$$
- c) Write a note on two-phase sampling.
- d) Explain the Lahiri's Method.
- e) Why sample survey methods provided over census method.

Q2) Attempt any three of the following: (5 Mark each)

- a) Define Neyman's allocation. Under Neyman's allocation prove that $n_i \propto N_i S_i$. Also obtain an expression for sample size from i^{th} stratum.
- b) With usual notations prove that,

$$V(\bar{x}_{sys}) = \frac{N-1}{N} S^2 - \frac{k(n-1)}{N} S_{wsy}^2$$

- c) Derive the expression for expected value of the simple regression estimator and its mean square error.
- d) Write a note on sampling and non-sampling errors.

P.T.O.

Q3) Attempt any three of the following: (5 Mark each)

- Determine the sample sizes for a pre-specified value of width of confidence interval in SRSWR and SRSWOR for estimation the population mean.
- A simple random sample of n clusters each containing M units from N clusters is drawn. Show that the sample mean per element is an unbiased estimator of population mean. Also find its efficiency with respect to corresponding SRSWOR.
- Write a brief note on Yates-Grandy sampling estimate.
- Explain Hansen-Hurwitz technique for randomised response.

Q4) Attempt any three of the following: (5 Mark each)

- Under SRSWOR obtained an expression for variance of unbiased estimator of population mean.
- Prove that, under SRS the approximate value of Mean Square Error of $\hat{\bar{Y}}_R$ is, $MSE(\hat{\bar{Y}}_R) = V(\bar{y}) + R^2 V(\bar{x}) - 2R\rho\sigma_{\bar{x}}\sigma_{\bar{y}}$
Where, ρ is the correlation coefficient between $\bar{x}$ and $\bar{y}$.
- Define Jack-Knife ratio estimator. Show that it is unbiased estimator for population mean.
- Write a note on non-probability sampling methods.

Q5) Attempt any one of the following:

[8+7]

- Define the term post stratification with an illustration. Compare the variances of estimators of population mean under post stratification and stratified sampling with proportional allocation.
 - Explain Midzuno Sampling Design(MSD). In MSD prove that,

$$\pi_{ij} = \frac{n-1}{N-1} \left[\frac{N-n}{N-2} (p_i + p_j) + \frac{n-2}{N-2} \right].$$
- In two-stage sampling with equal second stage units. Find optimum value of the sample sizes to be used in the first and second stage of the sampling.
 - Explain modified systematic sampling. Obtain variance of sample mean when the population has linear trend under modified systematic sampling.



[6521]-201

First Year M.A./M.Sc.

STATISTICS

ST-21 : Probability Theory
(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right side indicate marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt each of the following.**[5×2=10]**

- a) Prove or disprove : $(a, b]$ is a Borel set.
- b) Write the economical definition of random variable.
- c) Show that $X_n \xrightarrow{P} 0$ if $E|X_n|^r \rightarrow 0, (r > 0)$.
- d) Define : (i) Almost sure convergence (ii) convergence in r^{th} mean.
- e) Suppose X and Y are simple random variables defined on a probability space $(\Omega, \mathcal{A}, P)$. If $X \geq Y$ on Ω then show that $E(X) \geq E(Y)$.

Q2) Attempt any three of the following.**[3×5=15]**

- a) Define probability measure. Prove that
 - i) Probability measure is subtractive
 - ii) Probability measure is finite sub-additive
- b) Let $\Omega = (a, b, c, d)$, $\mathcal{A} = \{\phi, \Omega, \{a, b\}, \{c, d\}\}$, $X(a) = X(b) = -1$, $X(c) = X(d) = 2$. Examine whether X is $\mathcal{A}$ -measurable.
- c) If $\{X_n, n \geq 1\}$ is sequence of random variables then show that $\overline{\lim} X_n$ and $\underline{\lim} X_n$ are also a random variables.
- d) Let $\mathbb{C}_1 = [(a, \infty); a \in \mathbb{R}]$ and $\mathbb{C}_2 = [(a, \infty); a \in \mathbb{R}]$. Then show that $\sigma(\mathbb{C}_1) = \sigma(\mathbb{C}_2)$.

P.T.O.

Q3) Attempt any three of the following:

[3×5=15]

- a) Let $F(x)$ be the distribution function (DF) of RV X , where

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \leq x < 1 \\ \frac{1}{2} & \text{if } 1 \leq x < 2 \\ \frac{1}{2} + \frac{(x-2)}{2} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3 \end{cases}$$

Obtain decomposition of DF $F(x)$.

- b) Examine the convergence of the sequence $(A_n; n \geq 1)$, where

$$A_n = \left[1 + \frac{1}{n}, 2 + \frac{1}{n} \right].$$

- c) Show that

i) If $X_n \xrightarrow{a.s.} X$ and $X_n \xrightarrow{a.s.} Y$ then $X = Y$ a.s.

ii) If $X_n \xrightarrow{a.s.} X$ and $Y_n \xrightarrow{a.s.} Y$ then $X_n + Y_n \xrightarrow{a.s.} X + Y$

- d) State and prove C_r -inequality.

Q4) Attempt any three of the following:

[3×5=15]

- a) Consider a function X defined as follows:

$$X(\omega) = \begin{cases} C_0 & \text{if } \omega \in A_0 \\ C_1 & \text{if } \omega \in A_1 \\ C_2 & \text{if } \omega \in A_2 \end{cases}$$

where $\{A_0, A_1, A_2\}$ forms partition of Ω . Find the σ -field induced by function X .

- b) i) Prove that Every σ -field is field.
ii) The intersection of an arbitrary number of σ -fields is a σ -field.

- c) Define distribution function (DF) of a random variable X. Let F and G be two DF's then examine (with reason) which of the following are DF's.
- $\frac{1}{8}F(x) + \frac{7}{8}G(x)$
 - $[F(x)]^{\frac{3}{2}}$
 - $2F(x) - G(x)$
 - $\frac{[F(x) + G(x)]^2}{4}$
- d) State and prove necessary and sufficient condition for convergence in probability.

Q5) Attempt any one of the following:

- Suppose X and Y are simple random variables defined on a probability space $(\Omega, \mathcal{A}, P)$. Then show that **[8]**
 - $E(X \pm Y) = E(X) \pm E(Y)$
 - $E(cX) = cE(X)$
 - If $X \geq 0$ on Ω then $E(X) \geq 0$
 - If $X \geq 0$ a.s. then $E(X) \geq 0$
 - State and prove continuity property of probability measure. **[7]**
- If X and Y are independent random variables then show that $E(XY) = E(X)E(Y)$. **[8]**
 - Define simple and elementary random variable. IF X_1 and X_2 are simple random variables then prove that, $X_1 + X_2$, $X_1 - X_2$, $X_1 * X_2$ and $\frac{X_1}{X_2}$ (provided it is defined) are also simple random variables. **[7]**



Total No. of Questions : 5]

SEAT No. :

PE-5492

[Total No. of Pages : 3

[6521]-202

First Year M.A./M.Sc.

STATISTICS

ST-22 : Regression Analysis

(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt each of the questions.

[5×2=10]

- a) How condition number and condition index is useful in detection of multicollinearity?
- b) For multiple linear regression model, show that $SS_R(\beta) = y'Hy$.
- c) Derive an expression for the odds ratio for the regressor x_1 , in case of multiple logistic regression model.
- d) Give the inverse Gaussian link function for the generalized linear model.
- e) In polynomial regression model, test the significance of the highest order term.

Q2) Attempt any Three questions out of 4 questions:

[3×5=15]

- a) Obtain 100 (1 – α)% joint confidence region for both of the estimates of β_0 and β_1 of the centered version of the linear regression model.
- b) Derive the likelihood ratio test for testing $H_0 : R\beta = \underline{r}$.
- c) For the simple linear regression model, show that $Cov(\hat{\beta}_0, \hat{\beta}_1) = -\frac{x\sigma^2}{S_{xx}}$
and $Cov(\bar{y}, \hat{\beta}_1) = 0$.
- d) Suppose that a linear regression model with $k = 2$ regressors has been fit to $n = 25$ observations and $R^2 = 0.90$. Test the significance of regression at $\alpha = 0.05$. What is the smallest value of R^2 that would lead to the conclusion of a significant regression if $\alpha = 0.05$?

P.T.O.

Q3) Attempt any 3 questions out of 4 questions.

[3×5=15]

- a) What are outliers in regression models? Discuss any two techniques used in detecting the presence of outliers. If outlier is detected will discard it from data? Justify your answer.
- b) Under what conditions ridge regression is used for fitting the model to given data? Discuss the problem of selection of appropriate ridge estimators using graphical method.
- c) How various residual plots are useful in checking validity of assumptions made in analysis of experimental data.
- d) Discuss the fitting of polynomial regression of y on single regressor variable X using orthogonal polynomials. Hence, give the first three polynomials in X .

Q4) Attempt any 3 questions out of 4 questions.

[3×5=15]

- a) Explain the development of the PRESS statistic.
- b) Show that if $X'X$ is in correlation form, A is the diagonal matrix of eigenvalues of $X'X$, and T is the corresponding matrix of eigenvectors, then the variance inflation factors are the main diagonal elements of $TA^{-1}T'$.
- c) Show that the least squares estimate of β with the i^{th} observation deleted can be written in terms of the estimate based on all n points as

$$\hat{\beta}_{(i)} = \hat{\beta} - \frac{e_i}{1 - h_{ii}} (X'X)^{-1} x_i$$

- d) Discuss Score test in logistic regression model.

Q5) Attempt any 1 question out of 2 questions.

[1×15=15]

- a)
 - i) Explain the diagnostic measure of multicollinearity based on characteristic roots and proportion of variances. **[6]**
 - ii) Show that solution of normal equation actually minimizes the residual sum of squares. **[3]**
 - iii) Explain the terms: leverage points and influential points. Also, describe the importance of detecting influential observations. **[6]**

b) Suppose that the full model is $y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i, i = 1:n$, where x_{i1} and x_{i2} have been coded so that $S_{11} = S_{22} = 1$. Also, fit a subset model, $y_i = \beta_0 + \beta_1 x_{i1} + \varepsilon_i$.

i) Let $\hat{\beta}_1^*$ be the least squares estimate of β_1 from the full model.

Show that $\text{var}(\hat{\beta}_1^*) = \frac{\sigma^2}{(1 - r_{12}^2)}$ where r_{12} is the correlation between x_1 and x_2 . [6]

ii) Let $\hat{\beta}_1$ be the least squares estimate of β_1 from the subset model.

Show that $\text{var}(\hat{\beta}_1) = \sigma^2$. [3]

iii) Show that $E(\hat{\beta}_1) = \beta_1 + r_{12}\beta_2$. [3]

iv) Find the mean square error for the subset estimator $\hat{\beta}_1$. Compare $\text{var}(\hat{\beta}_1^*)$ and $MSE(\hat{\beta}_1)$. [3]



Total No. of Questions : 5]

SEAT No. :

PE-5493

[Total No. of Pages : 3

[6521]-203

First Year M.Sc.

STATISTICS

ST-23 : Statistical Inference-I

(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of Statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt all questions:

[5×2=10]

- a) Prove or disprove: MP test is unique.
- b) Let X_1, X_2 is a random sample from Bernoulli(p). Check whether $\psi(p) = p^3$ is estimable or not.
- c) State Rao-Blackwell theorem.
- d) Distinguish between estimator and estimate.
- e) State Bhattacharya bounds.

Q2) Attempt any three of the following:

[3×5=15]

- a) State and prove Neyman's factorization theorem for discrete case.
- b) Define one parameter exponential family. Show that Poisson distribution with mean λ is a member of exponential family. Give one example of a distribution which does not belong to exponential family.
- c) Define sufficiency. Let $X_1, X_2, \dots, X_n$ be i.i.d. geometric random variables with parameter θ . Find the sufficient statistic for θ .
- d) Let $X_1, X_2, \dots, X_n$ be i.i.d. $B(1, \theta)$. Define

$$T_1 = \begin{cases} 1, & X_1 = 1, X_2 = 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $T = \sum_{i=1}^n X_i$. Find the Rao-Blackwellize T_1 with respect to T .

P.T.O.

Q3) Attempt any three of the following: [3×5=15]

- State and prove Rao-Blackwell Lehmann-Scheffe' theorem.
- Explain likelihood equivalence. Let M be a statistic such that $M(\bar{x}) = M(\bar{y}) \Rightarrow \bar{x} \sim \bar{y}$ (likelihood equivalence). Then show that M is sufficient for θ .
- Define pitman family. Let $X_1, X_2, \dots, X_n$ be *i.i.d.* with pdf $f(x, \theta)$ which is a member of Pitman family. Let $S_\theta = (a(\theta), \infty)$ be support of $f(x, \theta)$. Obtain minimal sufficient statistic for θ .
- Define loss function. Let $X \sim B(n, \theta)$ and prior distribution of θ is $\beta_1(a, b)$. Find the Bayes estimator of θ under absolute error loss function.

Q4) Attempt any three of the following: [3×5=15]

- Prove that for distribution belonging to one parameter exponential family, UMP test can be obtained for one sided alternative.
- Define the term: Fisher Information. Compute it in case of $N(\mu, 1)$ and $N(0, \sigma^2)$ distribution.
- Let $X \sim N(\theta, 1), \theta \in \mathcal{R}$, show that for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta = \theta_1$, UMP does not exist.
- Define monotone likelihood ratio property of a distribution. Does Cauchy $(0, \theta)$ and Cauchy $(\theta, 1)$ satisfies MLR property?

Q5) Attempt any one of the following: [1×15=15]

- Give an example of statistic which is not sufficient and an example where a statistic is sufficient but not minimal sufficient. [4]
 - Obtain minimal sufficient statistics for $\bar{\theta} = (\mu, \sigma^2)$ where μ and σ^2 are parameters of normal distribution. [4]
 - Let $X_1, X_2, \dots, X_n$ be a random sample from

$$P(X = x) = \begin{cases} \theta(1 - \theta)^x, & \theta > 0; x = 0, 1, \dots \\ 0 & , \text{otherwise} \end{cases}$$

Show that $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ is UMVUE of $\frac{1 - \theta}{\theta}$. Examine whether it attains CRLB. [7]

- b) i) State and prove Neyman-Pearson Lemma. A sample of size 1 is taken from pdf $f(x) = \frac{2}{\theta^2}(\theta - x)$, where $0 \leq x < \theta$. Find an MP test of its size for testing $H_0: \theta = \theta_0$ Vs $H_1: \theta = \theta_1$ ($\theta_1 < \theta_0$). [9]
- ii) Write note on Shortest Expected Length Confidence Interval (SELCI). [6]



Total No. of Questions : 5]

SEAT No. :

PE-5494

[Total No. of Pages : 4

[6521]-204

First Year M.A./M.Sc.

STATISTICS

ST-24 : Multivariate Analysis
(2019 Pattern) (Semester - II)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt each of the questions:

[5×2=10]

- a) Define the following:
 - i) Sample mean vector
 - ii) Correlation matrix
- b) Define Multivariate Normal Distribution (MVND). Write m.g.f. of MVND.

c) If $\underline{X} \sim N_3(0, \Sigma)$, where $\underline{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{bmatrix}$. Find the value

of ρ for which $X_1 + X_2 + X_3$ and $X_1 - X_2 - X_3$ are independent.

d) Define Mahalanobis D^2 statistic. Give the relation between Mahalanobis D^2 statistic and Hotelling T^2 statistic.

e) Suppose the random vector $\underline{X}$ of two components X_1 & X_2 having variance-covariance matrix $\Sigma = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$. Obtain the proportion of variation explained by first principal component.

P.T.O.

Q2) Attempt any three of the following:

[3×5=15]

- a) The following data matrix contains data on test sources with

X_1 = Score on first test

X_2 = Score on second test

X_3 = total score on the two test

$$X = \begin{bmatrix} 12 & 18 & 14 & 20 & 16 \\ 17 & 20 & 16 & 18 & 19 \\ 29 & 38 & 30 & 38 & 35 \end{bmatrix}$$

Obtain mean vector and sample covariance matrix.

- b) Define the Hotelling- T^2 statistic. If $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$ and $C'\underline{X}$ is a non-singular transformation of $\underline{X}$ then show that Hotelling- T^2 based on $\underline{X}$ and $C'\underline{X}$ are same.
- c) Write a note on One Way MANOVA.
- d) Derive the region of classification to classify an observation into one of the two Normal populations when $\Sigma_1 \neq \Sigma_2$.

Q3) Attempt any three of the following:

[3×5=15]

- a) Let $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$. If A be any $q \times p$ matrix then prove that

$A\underline{X} \sim Nq(A\underline{\mu}, A\underline{\Sigma}A')$. If $\underline{X} \sim N_3(\underline{\mu}, \underline{\Sigma})$ where

$$\underline{\mu} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \text{ and } \underline{\Sigma} = \begin{bmatrix} 25 & -2 & 4 \\ -2 & 4 & 1 \\ 4 & 1 & 9 \end{bmatrix}$$

Obtain the distribution of $\begin{bmatrix} X_1 - X_2 \\ X_1 + 2X_2 - X_3 \end{bmatrix}$

- b) Write a note on:
- i) Complete Linkage Method
- ii) Average Linkage Method

- c) Let $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$
- Obtain the characteristic function of $\underline{X}$.
 - Find the distribution of $(\underline{a}', \underline{X} + b)$ where b is known constant and $\underline{a}$ is $P \times 1$ vector of constants.
- d) Write a note on scree plot.

Q4) Attempt any three of the following: **[3×5=15]**

- State and prove a necessary and sufficient condition for the two multivariate normal vectors to be independent.
- If $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$ and $\underline{X}$ is partition as $\begin{bmatrix} \underline{X}^{(1)} \\ \underline{X}^{(2)} \end{bmatrix}$. Find the conditional distribution of $\underline{X}^{(2)}$ given $\underline{X}^{(1)} = \underline{x}^{(1)}$.
- Derive the density of Wishart distribution in general case after assuming the density in canonical case.
- Suppose the random vector $\underline{X}$ of three components X_1, X_2 & X_3 having variance-covariance matrix $\underline{\Sigma} = \begin{bmatrix} 13 & -4 & 2 \\ -4 & 13 & -2 \\ 2 & -2 & 10 \end{bmatrix}$. Find the first two population principal components. How much population variance explained by first two PC out of total variation.

Q5) Attempt any one of the following:

- Let $\underline{X} \sim N_4(\underline{\mu}, \underline{\Sigma})$ with $\underline{\mu} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ and $\underline{\Sigma} = \begin{bmatrix} 2 & 1 & 1 & 1 \\ & 2 & 1 & 1 \\ & & 2 & 1 \\ & & & 3 \end{bmatrix}$, then find the conditional distribution of $\begin{bmatrix} X_1 \\ X_3 \end{bmatrix} / \begin{bmatrix} X_2 \\ X_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. **[8]**
 - Derive the characteristic function of Wishart distribution. **[7]**

b) i) If $D \sim W_p(n, \Sigma)$, then show that [8]

A. $\left| \frac{D}{\Sigma} \right|$ is distributed as the product of p independent X^2 variates with $n(n-1) \dots n-(p-1)$ d.f. respectively.

B. $\frac{\sigma^{pp}}{d^{pp}} \sim X^2_{n-(p-1)}$, where d^{pp} and σ^{pp} are the last elements of D^{-1} and Σ^{-1} respectively.

ii) Derive the expression to obtain first canonical correlation and first pair of canonical variables. [7]



Total No. of Questions : 5]

SEAT No. :

PE-5598

[Total No. of Pages : 3

[6521]-301

M.Sc.

STATISTICS

ST - 31 : Applied Stochastic Processes
(2019 Pattern) (Semester - III) (4 Credits)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt all questions :

[5 × 2 = 10]

- a) State Chapman-Kolmogorov equations.
- b) Define the following terms.
 - i) Communicating Class of Markov Chain
 - ii) Closed Class of Markov Chain
- c) Define Renewal Process with real life example.
- d) State any two properties of Brownian motion.
- e) Give the Transition Probability Matrix (TPM) for random walk with absorbing barriers and birth-death chain.

Q2) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) Define Poisson Process and find probability distribution of interarrival time in Poisson process.
- b) Prove that the inter-arrival times in the Poisson process are exponentially independently identically distributed random variables. Hence obtain the distribution function of time of the n^{th} arrival.
- c) Define ergodic Markov chain and prove for an irreducible Markov chain there exist unique stationary distribution.
- d) If a Markov chain X_0, X_1, X_2 has the Transition Probability Matrix (TPM)

with state space $\{0, 1, 2\}$ is $P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$, then check whether

given TPM is doubly stochastic or not? Also, check whether it is irreducible.

P.T.O.

Q3) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) Define a renewal process. Also, state the elementary renewal theorem with its application.
- b) Define Birth and death process and obtain its stationary distribution.
- c) Define Doubly Stochastic Matrix. Show that a markov chain having doubly stochastic matrix as its one step TPM has uniform distribution.
- d) Let a markov chain X_0, X_1, X_2 has the TPM with state space $\{0,1,2\}$ is

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix} \text{ and } P(X = i) = 1/3, i = 1, 2, 3. \text{ Find}$$

- i) $P(X_3 = 1 | X_0 = 1)$
- ii) $P(X_3 = 1, X_1 = 1, X_0 = 2)$

Q4) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) Give the two definitions of Poisson process and establish their equivalence.
- b) Prove or disprove: Mean recurrence time is a class property.
- c) Show that the state j of Markov chain is transient iff $\sum_{n=1}^{\infty} P_{jj}^{(n)} < \infty$, where $P_{jj}^{(n)}$ is the probability of transition from j to j in n -steps. Hence or otherwise show that all states of a finite Markov chain can not be transient.
- d) Describe the renewal reward process with an example and if $E(R_i) < \infty$ and $E(X_i) < \infty$ then as $t \rightarrow \infty$ with probability one show that
$$\frac{R(t)}{t} \rightarrow \frac{E(R_i)}{E(X_i)} \text{ and } \frac{E(R(t))}{t} \rightarrow \frac{E(R_i)}{E(X_i)}.$$

Q5) Attempt any 1 question out of 2 questions.

[1 × 15 = 15]

- a) i) State and prove addition property of Poisson Process. **[7]**
- ii) Suppose $\{N(t) \geq 0\}$ is a Poisson process. Given that $N(t) = n$, prove that n arrival times $T_1, T_2, \dots, T_n$ have the same distribution as the order statistic corresponding to n independent random variables uniformly distributed on $(0, t)$. **[4]**
- iii) Suppose $\{N(t) \geq 0\}$ is renewal process and $m(t)$ be renewal function then show that the renewal process satisfies the equation $m(t) = F(t) + \int_0^t m(t-x) f(x) dx$, where $F(t)$ and $f(x)$ be the respective distribution function and density function of inter-arrival time. **[4]**
- b) i) For $S = \{1, 2, n\}$ for $n > 10$, TPM is given by $P = \{P_{ij}\}$. Let $P_{ij} > 0$, if $|i - j|$ is even and $P_{ij} = 0$, if $|i - j|$ is odd. Check whether given TPM is irreducible or not and State which states are recurrent. **[8]**
- ii) Describe gambler's ruin problem. If X_n is the capital of a Gambler at n^{th} trial, show that $\{X_n, n > 0\}$ is a Markov chain. Classify the states of Markov chain as persistent or transient. **[7]**



Total No. of Questions : 5]

SEAT No. :

PE-5599

[Total No. of Pages : 3

[6521]-302
M.Sc.
STATISTICS
ST-32(A) : Bayesian Inference
(2019 Pattern) (Semester - III) (4 Credits)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all questions.

[5 × 2 = 10]

- a) What is the posterior distribution? State the relationship between prior, posterior and likelihood, given by Bayes.
- b) Write the steps to obtain Bayes estimator.
- c) State the properties of large sample Bayes estimator.
- d) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$, $\sigma^2 > 0$ known. Is $\bar{X}$ Bayes under squared error for some choice of prior distribution?
- e) Define improper prior with an illustration.

Q2) Attempt any THREE of the following :

[3 × 5 = 15]

- a) Explain the term maximum aposterior (MAP) estimator. Let X be a random variable having geometric distribution with parameter p . The prior distribution of p is given by $\pi(p) = \begin{cases} 2p & \text{if } 0 \leq p \leq 1 \\ 0 & \text{otherwise} \end{cases}$. Find the MAP estimate of p .
- b) Define quadratic loss function and obtain the Bayes estimator under it.

P.T.O.

- c) Suppose $\underline{X} = (X_1, X_2, \dots, X_{10})$ is a random sample of size 10 from $N(\mu, 4)$ with $\bar{X} = 2.1$. Suppose we assume a normal prior $N(100, 4)$ for μ . Find the Bayesian estimator of μ under absolute error loss.
- d) Let X and Y be jointly normal and $X \sim N(0, 1)$, $Y \sim N(1, 4)$ and $\rho(X, Y) = 12$. Find a 95% credible interval for X , given $Y = 2$ is observed.

Q3) Attempt any THREE of the following:

[3 × 5 = 15]

- a) A blood test is considered for determining the sugar level of person with diabetes two hours after he had his breakfast. It is of interest to see if medication has controlled his blood sugar level. Assume that test result X has $N(\theta, 100)$ distribution, where θ is the true level. In the appropriate population (diabetic but under this treatment), θ is distributed according to $N(100, 900)$. Suppose we want to test $H_0: \theta \leq 130$ against $H_1: \theta > 130$. If a blood test shows a sugar level of 130, what can be concluded?
- b) Write note on EM algorithm.
- c) Define maximum aposterior estimator. Let $(X_1 = 2, X_2 = 3, X_3 = 1)$ be a i.i.d. observations from $Binomial(5, p)$ and let the prior distribution of $p \sim U(0, 1)$. Find the maximum aposterior estimator of p based on the given data.
- d) Write short note on informative prior and non-informative prior.

Q4) Attempt any THREE of the following :

[3 × 5 = 15]

- a) Explain Bayes rule. A bag I contains 4 white and 6 black balls while other bag contains 4 white and 3 black balls. One ball is chosen at a random from one of the bags and it is found to be black. Find the probability that it was drawn from bag I. (Assuming bag is chosen randomly).
- b) Discuss conjugate class of priors with an example.
- c) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, σ^2 known. Suppose $\mu \sim N(\mu_0, \sigma_0^2)$. Obtain $100(1 - \alpha)\%$ highest posterior density (HPD) confidence interval for μ .
- d) Define the Jeffrey's prior. Let $X \sim Binomial(n, p)$, find the Jeffrey's prior of p .

Q5) Attempt any ONE of the following:

[1 × 15 = 15]

- a) i) Let $X_1, X_2, \dots, X_n$ be a random sample from $Poisson(\theta)$. The prior distribution of θ has $Gamma(\alpha = 2, \beta = 5)$. Obtain posterior distribution of θ . For $n = 8$ and $\sum_{i=1}^8 x_i = 5$, determine Bayesian estimate for θ under squared error loss and absolute error loss. **[8]**
- ii) Explain the terms: Decision function, Loss function and Bayes risk. **[7]**
- b) i) Let $Y|\theta \sim N(\theta, 4)$ and prior distribution of θ is $N(0, 4)$. Suppose random sample $Y_1, Y_2, \dots, Y_{25}$ drawn from $N(\theta, 4)$. Find 95% credible interval for θ given that we have observed $\bar{Y} = \frac{1}{25} \sum_{i=1}^{25} Y_i = 0.56$ **[9]**
- ii) Distinguish between frequentist approach and Bayesian approach for hypothesis testing. **[6]**



Total No. of Questions : 5]

SEAT No. :

PE-5600

[Total No. of Pages : 3

[6521]-303

M.Sc.

STATISTICS

**ST-32-B : Statistical Quality Control
(2019 Pattern) (Semester - III) (4 Credits)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the condidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all questions :

[5 × 2 = 10]

- a) Explain the significance of process capability indices C_p and C_{pk} in measuring process performance.
- b) Describe how a shift in process mean can be detected using a Shewhart control chart.
- c) List any two advantages and two disadvantages of acceptance sampling.
- d) For an $\bar{X}$ chart with $n = 5$, $\sigma = 2$ and $L = 3$, calculate the probability of a Type I error.
- e) Explain the use of a c-chart for monitoring low defect counts and provide an example from manufacturing.

Q2) Attempt any 3 questions out of 4 questions :

[3 × 5 = 15]

- a) Write a note on a double sampling acceptance plan and its advantages over single sampling.
- b) Compare Shewhart charts with CUSUM V-mask procedure in terms of sensitivity to small and large shifts.
- c) Describe practical situations where a moving range chart is preferable over $\bar{X}$ –R charts.
- d) Explain the role of Average Run Length (ARL) in comparing different types of control charts.

P.T.O.

Q3) Attempt any 3 questions out of 4 questions : **[3 × 5 = 15]**

- a) Compare attribute and variable control charts with examples of their application.
- b) Explain the effect of process non-normality on the estimation of capability indices C_p and C_{pk} .
- c) Describe the working of an EWMA control chart for monitoring process mean and explain its advantages over Shewhart charts.
- d) List and explain atleast four sensitizing rules used with Shewhart control charts.

Q4) Attempt any 3 questions out of 4 questions : **[3 × 5 = 15]**

- a) Explain the concept of multivariate control charts and discuss when they are needed.
- b) Construct a cause-and-effect diagram for a factory accident involving a chemical spill and outline possible root causes.
- c) Obtain a 95% confidence interval for C_p and C_{pk} for a production process and explain how to test if the process is capable.
- d) Explain the construction and working of a tabular CUSUM chart for monitoring process mean, including the selection of reference value and decision interval.

Q5) Attempt any 1 question out of 2 questions : **[1 × 15 = 15]**

- a) i) A process produces machine components. Sample of $n = 4$ items each are taken from a process at regular intervals. After 30 sample we have $\sum_{i=1}^{30} \bar{X}_i = 600$ and $\sum_{i=1}^{30} R_i = 90$. Compute the control limits for the $\bar{X}$ and R control chart.
 - If the specification limits are 19 ± 3 . What is your conclusion regarding the ability of process to produce items conforming to specifications?
 - If the mean shifts to 20, calculate the probability of scrap and rework. **[8]**
- ii) Write a note on synthetic control chart. **[4]**
- iii) Write a short note on AOQ curve and its significance in sampling plans. **[3]**

- b) i) Discuss non-parametric control charts based on sign tests, including the probability of the chart statistic and $ARL(0)$. [4]
- ii) Describe of MIL STD sampling plan. [4]
- iii) Explain the construction and working of non-parametric control chart based on sign test. State the probability of chart statistic and find the expression for $ARL(0)$. [7]



Total No. of Questions : 5]

SEAT No. :

PE5601

[Total No. of Pages : 4

[6521]-304

S.Y. M.A./M.Sc.

STATISTICS

**ST - 33 : Design and Analysis of Experiments
(2019 Pattern) (Semester - III) (4 Credits)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the following :

[5×2=10]

- a) What are the assumption of ANOVA.
- b) Check whether following block design is connected :

Block - I Block - II Block - III

A	C	A
B	D	B

- c) Define the following :
 - i) Total confounding
 - ii) Partial confounding
- d) Write the hypothesis and ANOVA table of One-way ANOVA.
- e) In BIBD prove that : $\lambda(v - 1) = r(k - 1)$

P.T.O.

Q2) Attempt any three of the following :

[3×5=15]

- a) Explain the following Tests :
 - i) Levene's Test
 - ii) Bartlett's Test
- b) An teacher sets three question papers and distribute them to randomly to their students. After evaluation of answer book following marks are obtained to students :

Test	Test Marks
Test - I	63, 64, 95, 64, 60, 85
Test - II	58, 56, 51, 84, 77
Test - III	100, 79, 82, 80, 74, 97

The teacher would like to know whether the three sets are equally difficult by testing equality of test marks. Analyze the data using Kruskal - Wallis Test.

- c) What is BIBD. Give statistical analysis of BIBD.
- d) What is Simplex Lattice Design. Explain a [4, 2] Simplex Lattice Design.

Q3) Attempt any three of the following :

[3×5=15]

- a) Explain Yates method to compute factorial effect totals in a 3^2 factorial experiment.
- b) Construct one-quarter fraction of the 2^7 design with highest possible resolution. Write down its alias structure.
- c) Give a statistical analysis of 2^3 factorial experiment in which 4 replicates are used and ABC is totally confounded.
- d) Write a note on Central Composite Design (CCD).

Q4) Attempt any three of the following :

[3×5=15]

- a) Explain following method for comparing pairs of treatment means :
- DMRT Test
 - Newman Keuls Test
- b) Obtain parameters of the following PBIBD.

Blocks	Treatments			
1	1	2	5	10
2	1	3	6	9
3	1	4	7	8
4	1	8	9	10
5	2	6	7	10
6	3	4	5	10
7	2	3	7	8
8	2	4	6	9
9	3	5	7	9
10	4	5	6	8

- c) Give the statistical analysis of 2^3 factorial experiment through RBD.
- d) Confound ABC^2 in 3^3 factorial experiment in 3 blocks.

Q5) Attempt any one of the following :

- a) i) Given below are the incidence matrices N_A and N_B of design A and B : **[8]**

$$N_A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 2 & 2 \end{bmatrix}, N_B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$$

Check the orthogonality of design A and B.

- ii) Show that RBD is connected block design.

[7]

- b) i) Explain PBIBD(m). Give the statistical analysis of PBIBD(2). [8]
- ii) Explain Friedman Test. A group of 6 different rats with their swim speed on different days as a result of a change in the water temperature given in the following table : [7]

	Water temperature			
Rats	20°C	24°C	27°C	32°C
1	39	38	34	33
2	29	25	20	20
3	36	37	29	24
4	25	29	18	19
5	21	27	24	22
6	34	33	30	31

The question is whether or not changing the water temperature of affects the swimming speed of rats. Use Friedman test for analysis.

x x x

Total No. of Questions : 5]

SEAT No. :

[Total No. of Pages : 2

PE5602

[6521]-305

S.Y. M.A./M.Sc.

STATISTICS

ST-34 : Machine Learning

(2019 Pattern) (Semester - III) (4 Credits)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all questions.

[5×2=10]

- a) What are the main types of Machine Learning?
- b) Which are the main layers in artificial neural network?
- c) What is machine learning generative model?
- d) Write down the different types of nodes in decision tree.
- e) Define unsupervised machine learning with a suitable example.

Q2) Attempt any 3 questions out of 4 questions.

[3×5=15]

- a) What are the applications of Machine learning?
- b) What are the methods to determine number of clusters explain any one in brief?
- c) Explain the term market basket analysis?
- d) Given a set of 5 objects {A, B, C, D, E} and their distance matrix, create clusters using Agglomerative Hierarchical Clustering.

	A	B	C	D	E
A	0	2	6	10	9
B		0	5	9	8
C			0	4	5
D				0	3
E					0

P.T.O.

Q3) Attempt any 3 questions out of 4 questions. [3×5=15]

- a) What is support vector machine and how does it work?
- b) Explain advantages of artificial Neural Network.
- c) What is the Classification task in Machine Learning, and how does the Bayes rule relate to its optimality?
- d) Describe the difference between linear and non-linear discriminant functions in classification.

Q4) Attempt any 3 questions out of 4 questions. [3×5=15]

- a) What is Machine learning and explain reinforcement learning.
- b) What is the relationship between data mining and machine learning?
- c) Explain how does backpropagation work?
- d) What is decision tree explain how does it work?

Q5) Attempt any 1 question out of 2 questions. [15]

- a) i) Compare K- Nearest Neighbors (KNN) and Support Vector Machine (SVM). [8]
ii) What do you understand about information gain? Also explain the mathematical formulation associated with it. [7]
- b) i) What are activation functions in Machine Learning? [6]
ii) The following table gives data set about stolen vehicles Using Naïve Bayes classifier classify the new data (Color=Red, Type= SUV, Origin= Domestic). [9]

Color	Type	Origin	Stolen
Red	Sports	Domestic	Yes
Red	Sports	Domestic	No
Red	Sports	Domestic	Yes
Yellow	Sports	Domestic	No
Yellow	Sports	Imported	Yes
Yellow	SUV	Imported	No
Yellow	SUV	Imported	Yes
Yellow	SUV	Domestic	No
Red	SUV	Imported	No
Red	Sports	Imported	Yes



Total No. of Questions : 5]

SEAT No.:

PE-5495

[Total No. of Pages : 2

[6521]-401

S.Y. M.A./M.Sc.

STATISTICS

**ST-41 : Asymptotic Inference
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figure to the right indicate marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the following.

[5×2=10]

- a) Define BAN estimator with illustration.
- b) Give Cramm's regularity conditions.
- c) What is the relation between CAN estimator and MLE in case of one parameter exponential family?
- d) Define joint consistency and marginal consistency.
- e) State one parameter Canonical Form.

Q2) Attempt any three of the following.

[3×5=15]

- a) Explain method of percentile to obtain CAN estimator. Also give CAN estimator of θ when random sample of size n is drawn from $(U(\theta, \theta + 1))$.
- b) Let $X_1, X_2, \dots, X_n$ is random sample from $\text{Exp}(\theta_1)$ and $Y_1, Y_2, \dots, Y_n$ is random sample from $\text{Exp}(\theta_2)$ and X_i and Y_j are independent. Find consistent estimator for $P(X < Y)$.
- c) Explain in detail locally most powerful test with example.
- d) Discuss Super-efficient estimator with illustration.

P.T.O.

Q3) Attempt any three of the following:

[3×5=15]

- a) Explain likelihood ratio test and Wald test in briefly.
- b) Let $X_1, X_2, \dots, X_n$ is random sample from Binomial $(1, \theta)$. What is the asymptotic distribution of $\bar{x}(1 - \bar{x})$ at $\theta = \frac{1}{2}$? Justify it.
- c) Let $X_1, X_2, \dots, X_n$ is random sample from $U(\theta - 1, \theta + 1)$. Find maximum likelihood estimate of θ .
- d) Under crammer regularity conditions show that likelihood equation estimator is CAN for θ .

Q4) Attempt any three of the following:

[3×5=15]

- a) Consider density function $f(x, \theta) = \theta x^{\theta} e^{-x^{\theta}}; x > 0$ and $\theta > 0$. Obtain CAN estimator for θ based on percentile method. Also obtain its asymptotic variance.
- b) Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\theta, 1)$. Obtain $100(1 - \alpha)\%$ shortest expected length confidence interval for θ .
- c) Let $X_1, X_2, \dots, X_n$ is random sample from Cauchy (μ, λ) . Find consistent estimator for (μ, λ) .
- d) Discuss Variance Stabilizing Transformation (VST) for construction of $100(1 - \alpha)\%$ confidence interval.

Q5) Attempt any one of the following:

[1×15=15]

- a)
 - i) Write a note on Newton Raphson Method to obtain MLE with one example.
 - ii) Explain Bartlett's test for homogeneity of variance.
- b)
 - i) Prove consistency property of MLE of parameter of distribution belongs to one parameter exponential family.
 - ii) Explain method of percentile to obtain CAN estimator. Also find CAN estimator of θ when random sample of size n is drawn from $(U(\theta, \theta + 1))$



Total No. of Questions : 5]

SEAT No. :

PE-5496

[Total No. of Pages : 2

[6521]-402

S.Y. M.A./M.Sc.

STATISTICS

**ST-42(A) : Econometrics and Time Series Analysis
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt each of the questions.

[5×2=10]

- a) Define the following.
 - i) MA (1) process
 - ii) AR(1) process
- b) Define SARIMA $(p, d, q) \times (P, D, Q)_s$ model.
- c) Explain the general approach to time series modelling.
- d) Define PACF.
- e) Define AIC and AICc.

Q2) Attempt any Three of the following:

[3×5=15]

- a) Explain the following tests for testing the estimated noise sequence:
 - i) The Portmanteau test
 - ii) The turning point test
- b) Define ARIMA (p, d, q) process. Show that ARIMA(0, 1, 1) is a random walk model.
- c) Explain the Durbin-Levinson Algorithm.
- d) Determine which of the following processes are causal and which of them are invertible.
 - i) $X_t - 0.75X_{t-1} + 0.5625X_{t-2} = Z_t + 1.25Z_{t-1}$, where $\{Z_t\} \sim WN(0, \sigma^2)$
 - ii) $X_t + 0.6X_{t-1} = Z_t + 1.2Z_{t-1}$, where $\{Z_t\} \sim WN(0, \sigma^2)$

P.T.O.

Q3) Attempt any Three of the following:

[3×5=15]

- a) Write a note on exponential-Smoothing method.
- b) Define sample ACVF and Sample ACF. Obtain it for the following data.

t	1	2	3	4
X_t	2	6	5	8

- c) Obtain stationary solution of ARMA(1, 1) process. At what condition process is invertible and non-invertible.
- d) Let $\{\epsilon_t\}$ be a stationary process with $E(\epsilon_t) = 0$. Let a and b be constants.
 $X_t = (a + bt)S_t + \epsilon_t$.

Where S_t is seasonal component with period $d = 12$. Find the difference operator that reduces $\{X_t\}$ to be a stationary process.

Q4) Attempt any Three of the following:

[3×5=15]

- a) Obtain ACVF of causal ARMA(1, 1) process.
- b) Calculate ACVF of the following stationary time series.
 - i) $Y_t = \mu + Z_t + \theta_1 Z_{t-1} + \theta_{12} Z_{t-12}$, where $\{Z_t\} \sim WN(0, \sigma^2)$
 - ii) $X_t = Z_1 + \frac{1}{\theta} z_{t-1}$, where $\{Z\} \sim WN(0, \sigma^2 \theta^2)$.
- c) Discuss the simultaneous equation models : endogenous and exogenous models.
- d) Explain unit root test.

Q5) Attempt any One of the following:

- a)
 - i) Estimate the parameters of stationary AR(p) process using Yule-Walker equations. **[8]**
 - ii) What is order selection problem in fitting AR model to the given time series data $x_1, x_2, \dots, x_n$. Write brief note on the FPE criteria for order selection. **[7]**
- b)
 - i) Obtain one step best linear predictor of AR(1) process. Also obtain its mean square error. **[8]**
 - ii) Define ARMA(1, 1) process. Obtain the sequence $\{\Psi_j\}$ and $\{\pi_j\}$. **[7]**



Total No. of Questions : 5]

SEAT No. :

PE-5497

[Total No. of Pages : 3

[6521]-403

S.Y. M.A./M.Sc.

STATISTICS

ST42(B) : Operation Research

(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of statistical tables and scientific calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt all questions.

[5×2=10]

- a) State the assumptions of LPP.
- b) Give any two applications of queuing theory.
- c) Define artificial variable and slack variable in Linear Programming Problem (LPP).
- d) What is non-linear programming problem?
- e) Write any two applications of simulation.

Q2) Attempt any Three of the following:

[3×5=15]

- a) Prove that a collection of all feasible solution (if exist) of LPP constitute a convex set.
- b) Define pure Integer Programming Problem (IPP). Derive Gomory's cut for it.
- c) What is Quadratic Programming Problem (QPP). Explain Beale's method of solving.
- d) Obtain the value of given problem by using dynamic programming:
$$\text{Max } Z = y_1 \cdot y_2 \cdot y_3$$

subject to the constraint $y_1 + y_2 + y_3 = 15$

and $y_1, y_2, y_3 \geq 0$.

P.T.O.

Q3) Attempt any Three of the following:

[3×5=15]

- a) Explain the Hungarian method of solving maximization assignment problem.
- b) Define a transshipment problem. Discuss method of solving transshipment problem.
- c) Describe the general formation of non-linear programming problem.
- d) The mathematical model for daily data of two products of manufacturing firm has the quadratic form as given below:

$$\text{Max } Z = 12x + 21y + 2xy - 2x^2 - 2y^2$$

Subject to constraints

$$8 - y \geq 0$$

$$10 - x - y \geq 0$$

$$x, y \geq 0$$

Find the solution to the problem by Beale's method when $\lambda_1 = 0$ & $\lambda_2 \neq 0$.

Q4) Attempt any Three of the following:

[3×5=15]

- a) Define a Transportation Problem (TP). Prove that the number of basic variables in TP are at most $m + n - 1$.
- b) Explain the concept of queuing theory. State the meaning of M/M/I : FCFS/ ∞/∞ of queuing theory.
- c) Explain the maximal flow model of networking.
- d) Write a note on stochastic dynamic programming.

Q5) Attempt any One of the following:

- a) i) Explain the following terms of network models with suitable diagrams: **[8]**
 - A) Merge event
 - B) Dummy activity
 - C) Activity on node
 - D) Activity on arrow
- ii) State and prove reduction theorem of assignment problem. **[7]**

- b) i) Explain the concept of local and global optimum for nonlinear programming problem. [5]
- ii) What is queue discipline? State queue discipline for service given to the customers in case of static and dynamic way. [6]
- iii) Write the dual of the transportation problem. [4]



Total No. of Questions : 5]

SEAT No. :

PE-5498

[Total No. of Pages : 2

[6521]-404

S.Y. M.A./M.Sc.

STATISTICS

ST-43(A) : Survival Analysis
(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) All questions are compulsory.
- 2) Figures to the right indicate full marks.
- 3) Use of Statistical tables and calculator is allowed.
- 4) Symbols and abbreviations have their usual meaning.

Q1) Attempt the following:

[10]

- a) Discuss left random censoring with an illustration.
- b) Suppose T is random variable having Weibull distribution with scale parameter 4 and shape parameter 0.5, find hazard rate for random variable T .
- c) Show that empirical survival function is an consistent as well as unbiased estimator of survival function.
- d) If $T \rightarrow \text{Exp}(\lambda = 4)$, then find mean residual life function of T .
- e) Define the following terms:
 - i) NBU class of life distribution
 - ii) Kernel function of degree r .

Q2) Attempt the following: (any three)

[15]

- a) Discuss in brief undesigned censoring and right random censoring with an illustration.
- b) Define spacing and find distribution of spacing.
- c) Describe the procedure to obtain maximum likelihood estimator of parameters of Weibull distribution in case of complete data.
- d) Obtain maximum likelihood estimator of only one parameter of Proportional hazard rate family in case of right random censored data.

P.T.O.

Q3) Attempt the following: (any three) [15]

- a) Prove that no-ageing is characterized by exponential equilibrium distribution function.
- b) Prove the following implications
 - i) IFRA $\Rightarrow$ NBU
 - ii) NBUE $\Rightarrow$ HNBUE
- c) Obtain the Greenwood's formula for the estimator of variance of actuarial estimator of survival function.
- d) Define total time on test (TTT) transform. Hence obtain maximum value of TTT transform.

Q4) Attempt the following: (any three) [15]

- a) Discuss any two methods of testing exponentiality of data graphically.
- b) If T has Weibull distribution with scale parameter λ and shape parameter γ then show that $Y = T^\gamma$ has exponential distribution with parameter λ .
- c) Suppose 20 items from an exponential distribution are put on life test and observed for 150 hours. During the period 15 items fail with the following life times, measured in hours. 3, 19, 23, 26, 27, 37, 38, 41, 45, 58, 84, 90, 99, 109, 138.
Test the hypothesis $H_0 : \mu = 65$ against $H_1 : \mu > 65$ at 2.5% level of significance.
- d) Discuss the graphical test for testing exponentiality of data against IFR class of life distribution.

Q5) Attempt the following: (Any one) [15]

- a) i) State and prove Cauchy functional equation. [6]
ii) Explain the Gehan's test for testing whether two samples come from the populations having same distribution in presence of right random censored data. [9]
- b) i) Explain redistribution to right algorithm to obtain estimator of the survival function with the help of following data. [7]
9, 13, 13+, 18, 23, 28+, 31, 34, 45+, 48, 161+
ii) Explain Deshpande's test for testing exponentiality against IFRA class of life distribution. [8]



Total No. of Questions : 5]

SEAT No. :

PE-5499

[Total No. of Pages : 3

[6521]-405

S.Y. M.A./M.Sc.

STATISTICS

ST-43(B) : CATEGORICAL DATA ANALYSIS

(2019 Pattern) (Semester - IV)

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistics tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all of the following questions:

[5×2=10]

- a) Distinguish between 'interval scale' and 'ratio scale' of measurements.
- b) State any two properties of Yule's Q coefficient.
- c) State the formule of Pearson residual and standardized residual for GLM model.
- d) Explain 'Baseline Category Logits' for nominal response variables.
- e) Define Odds Ratio with interpretation in case control study.

Q2) Attempt any three questions from the following questions:

[5×3=15]

- a) Explain General Linear Model (GLM) and provide any two model performance measures for GLM.
- b) Explain multinomial Logistic Regression.
- c) Explain Negative Binomial Regression.
- d) Explain Conditional Logistic Regression.

P.T.O.

Q3) Attempt any three questions from the following questions: [5×3=15]

- a) Explain Ordinal Logistic Regression.
- b) Explain any four real life applications of binary logistic model.
- c) Explain log linear analysis and state its use.
- d) Explain Fisher exact test for 2×2 contingency table.

Q4) Attempt any three questions from the following questions: [5×3=15]

- a) x = mother's alcohol consumption and Y = whether a baby has sex organ malformation. With scores (0, 0.5, 1.5, 4.0, 7.0) for alcohol consumption, ML fitting of the linear probability model has the output:

Parameter	Estimate	Standard error	Likelihood ratio 95% confidence limits	
Intercept	0.00255	0.0003	0.0019	0.0032
Alcohol	0.00109	0.0007	-0.0001	0.0027

- i) State the prediction equation and interpret the intercept and slope.
- ii) Use the model fit to estimate the
 - 1) Probabilities of malformation for alcohol levels 0 and 7.0
 - 2) Relative risk comparing those levels.
- b) What is categorical data? Explain the types of categorical data.
- c) Explain the Hierarchical Model.
- d) Explain the Components of GLM Model in details.

Q5) Attempt any one questions from the following questions. [15×1=15]

- a) A study used logistic regression to determine characteristics associated with Y = whether a cancer patient achieved remission (1 = yes). The most important explanatory variable was a labeling index (LI) that measures

proliferative activity of cells after a patient receives an injection of tritiated thymidine. It represents the percentage of cells that are “labeled”. Software reports for a logistic regression model using LI to predict $\pi = P(Y = 1)$

Parameter	Estimate	Standard error	Likelihood ratio 95% confidence limits		Chi-Square
Intercept	-3.7771	1.3786	-6.9946	-1.4097	7.51
li	0.1449	0.0593	0.0425	0.2846	5.96

LR Statistic						
Source			DF	Chi-Square	Pr > ChiSq	
li			1	8.30	0.0040	
Obs	li	remiss	n	pi_hat	lower	upper
1	8	0	2	0.06797	0.01121	0.31925
2	10	0	2	0.08879	0.01809	0.34010

- i) Conduct a Wald test for the LI effect interpret.
 - ii) Construct a Wald confidence interval for the odds ratio corresponding to a 1-unit increase in LI. Interpret.
 - iii) Conduct a likelihood-ratio test for the LI effect. Interpret.
 - iv) Construct the likelihood-ratio confidence interval for the odds ratio, Interpret.
- b) Explain model for matched pairs also explain the types of matched pair.



Total No. of Questions : 5]

SEAT No. :

PE-5500

[Total No. of Pages : 3

[6521]-406

S.Y. M.A./M.Sc.

STATISTICS

**ST-44 (A): Computer Intensive Statistical Methods
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical table and scientific calculator is allowed.*
- 4) *Symbol and abbreviations have their usual meaning.*

Q1) Attempt all questions.

[5 × 2 = 10]

- a) Define the quantile function with illustration.
- b) Suppose a random sample of size n is available from Half-normal distribution with pdf $f(x) = \sqrt{\frac{2}{\pi}} \exp\left\{-\frac{x^2}{2}\right\}$, $x > 0$. State the most appropriate way to generate random sample from $N(0, 1)$ using it?
- c) What are the three types of missing data?
- d) How many distinct jackknife samples can be drawn from a sample of size n assuming that all n observations are distinct? Justify your answer.
- e) LOOCV stands for ...

Q2) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) Describe the procedure to construct a confidence interval for the median of exponential distribution via parametric bootstrap.
- b) Suppose that we obtain a bootstrap sample from a set of n observations. What is the probability that the j^{th} observation is not in the bootstrap sample?
- c) In which circumstances we need independence M-H algorithm? What is $\alpha(x, y)$ for it? Derive the acceptance rate of it.
- d) What are the three types of missing data? Explain in detail.

P.T.O.

Q3) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) R code and its output are shown below:

```
n = 1000
```

```
lambda = 0.14
```

```
u = runif(n,0,1).
```

```
Qstar = [u^{lambda} - (1-u)^{lambda}] / lambda \\
```

Output

```
summary(Qstar)
```

Min. 1st Qu. Median Mean 3rd Qu. Max.

−4.3443 − 0.95578 0.05778 0.02398 1.06468 4.59969

```
var(Qstar)
```

2.114796

```
Shapiro-test(Qstar)
```

Shapiro-Wilk normality test

data: Qstar

W = 0.99786, p-value = 0.7017

Describe what is being done in the R code.

- b) Consider a Metropolis-Hasting (MH) algorithm where you propose x_{new} from a $N(x_{old}, 1)$ distribution (where x_{old} is the current sample) and accept

with probability $\min \left(1, \frac{2 + x_{old}^2}{2 + x_{new}^2} \right)$. What is the target distribution?

- c) Explain the procedure to estimate the value of π by Monte Carlo simulation.
- d) What is parametric bootstrap? How is it different from the non parametric bootstrap?

Q4) Attempt any 3 questions out of 4 questions.

[3 × 5 = 15]

- a) Explain the differences between the histogram density estimator and the naive density estimator. What are the pros and cons of each of them?
- b) Explain the concept of complete and incomplete data in the context of EM algorithm. Is the mapping between incomplete and complete data one to one, many to one or one to many? Provide an illustration to justify your answer.
- c) Explain the k-nearest neighbour estimation procedure of estimating a density function in the univariate setup.
- d) Describe the procedure for boosting in a classification tree.

Q5) Attempt any 1 question out of 2 questions.

[1 × 15 = 15]

- a) What is Gibbs sampling? Explain its working in hierarchical models.
- b) Derive the formulae for the estimates of missing proportions and other parameters in a finite mixture model via EM algorithm.



Total No. of Questions : 5]

SEAT No. :

PE-5501

[Total No. of Pages : 2

[6521]-407

S.Y. M.A./M.Sc.

STATISTICS

**ST-44(B) : Analysis of Clinical Trials
(2019 Pattern) (Semester - IV)**

Time : 3 Hours]

[Max. Marks : 70

Instructions to the candidates:

- 1) *All questions are compulsory.*
- 2) *Figures to the right indicate full marks.*
- 3) *Use of statistical tables and scientific calculator is allowed.*
- 4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt all questions:

[5×2=10]

- a) Differentiate between inter-subject and intra-subject variability.
- b) What is meaning of blinding?
- c) State the goals of clinical trials.
- d) Write a note on combination trials.
- e) Explain the difference between washout period and run in period.

Q2) Attempt any 3 questions out of 4 questions:

[3×5=15]

- a) Explain the treatment IND and withdraw and termination of IND.
- b) Discuss testing procedure for carryover, direct drug and period effect in standard 2×2 crossover design using analysis of variance.
- c)
 - i) What are the primary and secondary pharmacokinetic parameters.
 - ii) What are the routes of administration of the drug?
- d) What is Interim analysis and safety report in clinical trial?

Q3) Attempt any 3 questions out of 4 questions.

[3×5=15]

- a) Explain the patient selection procedure involved in Clinical Trials.
- b) Explain the method of permuted block randomization with example and its advantages over complete randomization.

P.T.O.

- c) Write a note on:
 - i) Target population
 - ii) Patient selection
- d) Explain the following:
 - i) Complete
 - ii) Permuted Block
 - iii) Adaptive Randomization

Q4) Attempt any 3 questions out of 4 questions: [3×5=15]

- a) Describe the following designs with the appropriate layouts:
 - i) Balance Incomplete block design
 - ii) William's design and Balaam's design
- b) Define the following terms:
 - i) Intersubject variability
 - ii) Treatment IND
- c) What are the differences between analysis of multicenter trial and meta-analysis?
- d) Define standard titration design? List all of them.

Q5) Attempt any 1 question out of 2 questions: [1×15=15]

- a) i) List out the kinds of uncertainty and biases involved in clinical trials. **[8]**
- ii) What are dose response trials? What are their objectives? What are their hypotheses of interest? **[7]**
- b) i) Classify the Clinical Trials depending upon their functioning. Explain their respective functions in brief. **[9]**
- ii) Discuss: Bioavailability and bioequivalence studies. **[6]**

