Total No. of Questions : 5]

SEAT No. :

P1618

[Total No. of Pages : 2

[5487] - 101
M.Sc. (Part - I) (Semester - I)
STATISTICS
ST - 11 : Mathematical Analysis
(2013 Pattern)

Time :3 hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of statistical tables and scientific calculator is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) Attempt any five questions from the following questions: **[5 × 2 = 10]**

a) Define ordered field of real numbers. Show that, if $x \neq 0$ then $x^2 > 0$.

b) State least upper bound property of a set of real numbers and show that least upper bound of a set if it exists is unique.

c) Is it true that finite union of finite set is finite? Justify your answer.

d) Examine whether, $d(x, y) = \left| \frac{x}{1+\sqrt{1+x^2}} - \frac{y}{1+\sqrt{1+y^2}} \right|$ is metric on real line.

e) Define the term neighborhood of a real number in metric space with one example.

f) Show that finite union of closed sets is closed.

g) Give one example of a set for the following situations:

i) Exactly one limit point

ii) Countably many limit points

Q2) Attempt any two questions from the following questions: **[2 × 5 = 10]**

a) State and prove the Archimedean principle of real numbers.

b) Show that a sequence $\{a_n\}$ of real numbers converges if and only if it is a Cauchy sequence.

c) Define a compact set. Show that every infinite subset E of a compact set K has a limit point in K.

P.T.O.

Q3) Attempt any two questions from the following questions: **[2 × 5 = 10]**

a) Suppose (X,d) is a metric space, Show that, $E \subset X$ is closed if and only if E^C is open.

b) State and prove Root test for the convergence of a series.

c) Show that if $a_n \to a$ and $b_n \to b$ then i) $a_n b_n \to ab$ ii) $|a_n| \to |a|$

Q4) Attempt any two questions from the following questions: **[2 × 5 = 10]**

a) Define power series. Find radius and interval of convergence for the following power series:

i) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n} x^n$ ii) $\sum_{n=0}^{\infty} \left(\frac{3}{5}\right)^{n} x^{2n}$

b) State and prove Bolzano– Weirstrass theorem.

c) State and prove Fundamental theorem of calculus.

Q5) Attempt any one question from the following questions:

a) i) Is it true that irrationals are dense in R? Justify your answer.

ii) Prove or disprove: The set of rational numbers is countable.

iii) Discuss the convergence or divergence of the following sequences.

A) $\left(1+\frac{1}{n}\right)^n$ B) $\left(\frac{1}{2^{n-1}}\right)$

[3+3+4]

b) i) Define Interior of a set and Closure of a set with one illustration each.

ii) If f: [a, b] $\to \mathbb{R}$ is continuous function then show that it attains its maximum and minimum in [a, b].

iii) Discuss the convergence or divergence of the following series.

A) $\sum \frac{(-1)^n}{n^2}$ B) $\sum \left(\frac{2n+7}{n^2+1}\right)$

[3+3+4]

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Total No. of Questions : 3]

SEAT No. :

P1619

[Total No. of Pages : 2

[5487] - 102
M.Sc. (Semester - I)
STATISTICS
ST - 12 : Integral Calculus and Statistical Computing
(2013 Pattern)

Time :1 Hour 30 Minutes] ***[Max. Marks :25***

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator and statistical table is allowed.*
4) *Symbols and abbreviations have their meaning.*

Q1) Attempt all the questions given below:

a) Define the terms with an illustration each: Norm of a partition, Upper Riemann sum. **[2]**

b) Describe the comparison test for testing the convergence of improper integral of type I **[1]**

c) Give an example of a function which is not Riemann integrable, with justification. **[1]**

d) State the situation when the use of boot strap method of estimation is recommened. **[1]**

Q2) Attempt any two of the following:

a) i) If $P_1 \subset P_2$ then show that $U(p_1,f) - L(p_1,f) \geq U(p_2,f) - L(p_2,f)$ State the significance of the result. **[3]**

ii) Evaluate $\int_1^3 \int_1^3 \frac{1}{\sqrt{x+y}}\, dydx$. using Simpsons one - third rule. **[2]**

b) Show that $\Gamma(m+n)B(m,n) = \Gamma(m)\Gamma(n)$ where $m > 0, n > 0$. **[5]**

c) i) Test the convergence of $\int_1^\infty \frac{x}{2x^3+3x^2-1}dx$ **[3]**

ii) Describe the linear congruential generator method of simulation of random numbers. **[2]**

P.T.O.

Q3) Attempt any two of the following:

a) i) Evaluate $\int_1^4 x^2 d(x^2)$. State the results you have used. **[2]**

ii) Derive the Newton - Raphson iterative formula to solve two bivariate simultaneous transcendental equations f(x,y) = 0 and g(x,y) = 0. **[3]**

b) i) Minimise $f(x,y) = x^2 - xy + 3y^2$ using steepest descent method with initial solution (1,2) carry out two iterations. **[3]**

ii) Describe the procedure to estimate coefficient of correlation using jack - knife method. **[2]**

c) i) Show that bounded and continuous function on [a, b] is Riemann - stieltjes integrable. **[2]**

ii) Use Newton's interpolation formula to find f(12,17) given the following table of the values of f(x,y) **[3]**

y \ x	0	10	20
10	11	13	17
15	13	16	21
20	12	21	27

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Total No. of Questions : 5]

SEAT No. :

P1620

[Total No. of Pages : 3

[5487] - 103
M.Sc. (Semester - I)
STATISTICS
ST - 13 : Linear Algebra
(2013 Pattern) (4 Credits)

Time :3 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Use of statistical tables and scientific calculator is allowed.***
4) ***Symbols and abbreviations have their usual meaning.***

Q1) Attempt each of the following.

a) In each of the following cases, choose the correct alternative. **[1Each] [2]**

i) Let C (R) be the vector space of all continuous real functions over R. Then which of the following sets is linearly independent in C (R)?

A) $\{1, \cos 2x, \sin^2 x\}$

B) $\{1, e^x, e^{2x}\}$

C) $\{1, x^2 + 2x + 1, x^2, (x-2)^2\}$

D) $\{1, \log(1 + |x|), \log(1 + |x|)^2\}$

ii) Suppose A and B similar matrices. Then which one of the following is false?

A) If A is self - adjoint then B is also self - adjoint.

B) If A is non - singular then so is B.

C) Determinant of A is same as the determinant of B.

D) Trace of A is same as the trace of B.

b) State whether each of the following statement is True or False. Justify?**[2]** **[1Each]**

i) If A and B are invertible matrices then A+B is invertible.

ii) Algebraic multiplicity is always greater than geometric multiplicity of the same matrix.

c) Let $V = R^3$ be the vector space of ordered triples of real number. Determine whether the following is $W = \{(x,y,z) / x + y + z = 1\}$ subspace of $V = R^3$ **[2]**

P.T.O.

d) Let A be a matrix of order m × n and B be a matrix of order n × m. Show that (AB)' = B'A' **[2]**

e) Show that positive definite real symmetric matrix is non singular. **[2]**

Q2) Attempt any two of the following:

a) Define Determinant of a n × n matrix, n ≥1. Consider a matrix A = (a_{ij})

where $(a_{ij}) = \begin{cases} a & if \quad i = j = 1,..........n \\ b & if \quad i \neq j = 1,..........n \end{cases}$

Show that det (A) = $[a + (n - 1)b]\,(a - b)^{n-1}$ **[1+4]**

b) Define linear dependent and linear independent set of vectors. Let $\vec{u}_1 = (1,\ 2,\ -3), \vec{u}_2 = (1, -3, 2),\ \vec{u}_3 = (2, -1,\ 5)$ be a vectors in R^3. Show that the set B = $[\vec{u}_1, \vec{u}_2,\ \vec{u}_3]$ is a basis for R^3. **[2+3]**

c) i) Define eigen value and eigen vector of a square matrix. If λ is an eigen value of matrix A then show that λ^k is an eigen value of A^k, where k is positive integer.

ii) If matrix A and matrix B are similar then prove that eigen value of matrix A and B are same. **[3+2]**

Q3) Attempt any two of the following.

a) Define g-inverse and obtain two distinct g-inverses for the following matrix. **[1+2+2]**

$$A = \begin{pmatrix} 5 & 2 & -1 \\ 3 & 2 & 3 \\ 3 & 2 & 3 \end{pmatrix}$$

b) Define rank of matrix and prove that.

i) *rank (AB)*= min *(rank (A), rank (B)).*

ii) *rank (A+B)≤ rank (A)+ rank (B).* **[1+2+2]**

c) Determine a non singular linear transformation that reduces the quadratic form $2x^2 + 2y^2 + 2z - 2xy - 2yz - 2xz$ to canonical form. **[5]**

Q4) Attempt any two of the following.

a) i) Define Moore Penrose generalized inverse. If A is symmetric idempotent matrix then show that A itself is its Moore penrose generalized inverse.

ii) Prove that g – inverse of orthogonal matrix is unique. **[1+2+2]**

b) Consider the system of equation Ax = 0, where A is m × n matrix with rank (A) = r. Show that there are n - r linearly independent solution to this system. **[5]**

c) Show that a real symmetric matrix is positive definite if all its eigen value are positive. **[5]**

Q5) Attempt any one of the following.

a) i) Explain Gram - Schmidt orthogonalization process. Use this process to find orthonormal basis for the vector spanned by the following vectors.

$$\vec{u}_1 = (1,\ 1,1),\ \vec{u}_2 = (-1,\ 1,\ 0),\ \vec{u}_3 = (1,\ 2,\ 1)$$

ii) Define a quadratic form. Describe its usual classification. Examine the nature of the following quadratic form $Q(x,y,z) = x^2+y^2 + z^2 - xy - yz - xz$. **[6+4]**

b) i) Define Spectral decomposition of real symmetric matrix. Obtain the same for the matrix. $A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

ii) Show that characteristic vector associated with distinct characteristic roots of a symmetric matrix are orthogonal. **[6+4]**

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Total No. of Questions : 3]

SEAT No. :

P1621

[Total No. of Pages : 2

[5487] - 104
M.Sc. (Semester - I)
STATISTICS
ST - 14 : Probability Distributions - I
(2013 Pattern) (2 Credits)

Time :1½ Hours] *[Max. Marks :25*

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of scientific calculator and statistical table is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) a) Choose the correct alternative for each of the following. **[1each]**

i) Let X be random variable with p.d.f.

$$f(x) = \frac{\theta}{x^{\theta+1}} \text{ if } x \geq 1, \quad \theta > 0$$

E (x) exists if

a) $\theta = 1$ b) $\theta > 1$

c) $\theta < 1$ d) $\theta > e$

ii) If $X \to B(n, \frac{1}{2})$ and P_x (s) is the PGF of X then

a) $P_x(s) = P_{n-x}(s)$

b) $P_x(s) = P_x(1/s)$

c) $P_x(s) = P_x(-s)$

d) $P_x(s) = S^n P_x(-s)$

iii) Let ϕ (.) be the distribution function of N (0, 1) then which of the following is true?

a) $\phi(x) = \phi(-x)$

b) $\phi(x) = 1 - \phi(x)$

c) $\phi(0) = ½$

d) for $a > 0$, $\phi(x+a) = \phi(x) + \phi(-x) - \phi(-x-a)$

b) If F (x) is a distribution function of r.v.x then show that $\frac{F(x)}{2-F(x)}$ is a distribution function. **[2]**

P.T.O.

Q2) Attempt any two of the following:

a) i) If S^{th} order raw moment (μ'_s) exists then show that r^{th} order raw moment also exist for r < s. **[3]**

ii) If $X \rightarrow U(-\theta,\theta)$ Find probability density function of $Y = \frac{1}{x^2}$. **[2]**

b) Express the following distribution function as a mixture of discrete and continuous distribution functions. **[5]**

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2 + \frac{1}{5} & \text{if } 0 \le x < ½ \\ x & \text{if } ½ \le x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

c) Suppose $X \rightarrow B(n, p)B$ and $P \rightarrow U(0,1)$ find the marginal distribution of X. **[5]**

Q3) Attempt any two of the following.

a) Let X and Y be independent and identically distributed U(0,1). Using the method of convolution find the probability distribution of X + Y. **[5]**

b) Let (X,Y) be a bivariate r.v. with joint p.d.f. **[5]**

$$f(x, y) = 21x^2y^3 \quad ; \text{if } 0 < x < y < 1$$
$$= 0 \quad ; \text{ otherwise}$$

i) Find the marginal probability distributions of X and Y

ii) Find the conditional probability distribution of X given Y = y

iii) Find $E(X \mid Y = y)$

c) Show that Var (X) = Var [E(X|Y = y)]+ E [Var (X|Y = y)]. **[5]**

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Total No. of Questions : 4]

SEAT No. :

P1622

[Total No. of Pages : 2

[5487] - 105
M.Sc. (Semester - I)
STATISTICS
ST - 15 : Probability Distributions - II
(2013 Pattern) (3 Credits)

Time :2 Hours 15 Minutes] ***[Max. Marks :38***

Instructions to the candidates:

1) *All questions are compulsory.*
2) *Figures to the right indicate full marks.*
3) *Use of statistical tables and scientific calculator is allowed.*
4) *Symbols and abbreviations have their usual meaning.*

Q1) a) Define bivariate Poisson distribution. Obtain its joint moment generating function. **[2]**

b) Let (X, Y) follow bivariate normal distribution with parameters $(0,0,\sigma_1^2,\sigma_2^2,\rho)$. Find the conditional distribution of Y given $X = x$. **[2]**

c) Prove or disprove the following statements. Justify your answer.

i) The family of distributions,

$\{N(\mu,\sigma^2),\mu\in\{-1,0,1\},\sigma>0\}$ is a two - parameter exponential family. **[2]**

ii) Suppose X and Y are independent exponential random variable with mean θ. Then min $\{X, Y\}$ has exponential distribution. **[2]**

Q2) Attempt any TWO of the following questions.

a) Suppose U, V, W are three independent random variables, each distributed as U (0,1). Let X = max (U, V, W) and Y = min (U, V, W). Find the conditional density and then conditional expectation of X given $Y = y$.**[5]**

b) Describe the procedure of Wilcoxon signed rank test. State the null hypothesis and obtain the distribution of the statistic under the null hypothesis. **[5]**

c) Define multi parameter exponential family of distributions. Give an example of it. **[2+3]**

P.T.O.

Q3) Attempt any TWO of the following questions.

a) State and prove Fisher-Cochran theorem. **[5]**

b) Define non-central F distribution with m,n degrees of freedom and non-centrality parameter, δ. Obtain its mean. **[2+3]**

c) Define location-scale family of distributions. Give an example of a probability distribution belonging to the locations- scale family and a probability distribution not belonging to the family. **[1+2+2]**

Q4) Attempt any ONE of the following questions.

a) Let $Y_1,...., Y_n$ be the order statistics corresponding to a random sample of size n from the pdf

$$f(x;\alpha,\beta)=\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \quad \exp\{-\left(\frac{x}{\beta}\right)^{\alpha}\}\ x>0;\alpha,\beta>0$$

i) Obtain the cdf and pdf of Y_1. **[3+3]**

ii) Compute $E(Y_1)$ **[4]**

b) i) Let $X_1, X_2, ..., X_n$, be independent N (0, 1) random variables. Show that $\underline{X}'A\underline{X}$ and $\underline{X}'B\underline{X}$ are independently distributed if and only if AB is a null matrix. **[7]**

ii) Suppose X has $N(0,1)$ and Y given $X = x$ follows $N(x + 1,1)$. Find the marginal distribution of Y. **[3]**

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Total No. of Questions : 5]

SEAT No. :

P1623

[Total No. of Pages : 3

[5487] - 106
M.Sc. (Semester - I)
STATISTICS
ST - 16 : Sampling Theory
(2013 Pattern) *(4 Credits)*

Time :3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***All questions are compulsory.***
2) ***Figures to the right indicate full marks.***
3) ***Use of statistical tables and scientific calculator is allowed.***
4) ***Symbols and abbreviations have their usual meaning.***

Q1) a) Choose the correct alternative for each of the following: **[1each]**

i) The technique of Collapsed strata is used when

a) n > 2 b) n ≥ 2

c) n ≤ 2 d) n < 2

ii) Supposing that, in cluster sampling S_W^2 represents the variance within the clusters and S_b^2 between clusters. What is relation between S_W^2 and S_b^2?

a) $S_W^2 = S_b^2$ b) $S_W^2 \geq S_b^2$

c) $S_W^2 \leq S_b^2$ d) $S_W^2 < S_b^2$

b) In each of the following, state whether the given statement is true or false. Justify your answer. **[1each]**

i) In SRSWR design sample mean based on distinct observations is a better estimator of population mean than sample mean based on repeated observations.

ii) Sampling error decreases with increase in sample size.

c) Define the following terms with an illustration. **[1each]**

i) Sampling design.

ii) Sampling unit.

P.T.O.

d) i) Explain the method of stratified sampling. Also state two real life situations where stratification can be used. **[2]**

ii) State regression estimator for sample mean. Also explain the situation where regression estimator is used. **[2]**

Q2) Attempt any two of the following. **[5each]**

a) Suggest an unbiased estimator of population mean using sample under SRSWOR. Obtain its variance. Obtain the expression for minimum sample size (n) required in SRSWOR design when the population mean is to be estimated with desired accuracy (d).

b) Define a probability proportional to size (PPS) sampling design. Obtain an unbiased estimator of population mean in this design. Find its variance. Examine whether SRSWR is particular case of this design.

c) Show that under Midzuno scheme of sampling, the Yates - Grundy estimate of variance of Horvitz- Thompson (HT) estimator is never negative.

Q3) Attempt any two of the following. **[5each]**

a) Define ordered and unordered sample. For population of size N and sample of size 2, state the probability of drawing an ordered sample and unordered sample using SRSWOR.

b) Explain the procedure of deep stratification. State the estimator of population mean using this procedure. Also obtain its expected value.

c) In cluster sampling with clusters of unequal sizes, define three estimators of population mean. Also compare them.

Q4) Attempt any two of the following. **[5each]**

a) Explain the method of two stage sampling. Define an unbiased estimator of population mean with equal number of first stage and second stage units. Derive the expression for its variance. Show that two stage sampling is generalization of stratified sampling.

b) Describe any two practical situations in which systematic sampling can be used. Propose an estimator of population mean in this method of sampling. Also derive the expression for its variance in terms of intra class correlation coefficient.

c) Explain Hansen Hurwitz technique for randomized response.

Q5) Attempt any one of the following.

a) i) Obtain the expression for mean squared error (MSE) in case of simple ratio estimator when SRSWOR is used. Describe the ratio estimator of population mean in stratified sampling using separate strata and combined strata. Which estimator is better? Why?

ii) Write short note on 'Circular Systematic Sampling'

[7+3]

b) Calculate inclusion probabilities π_i and π_{ij} for the following artificial population of four units with x as the size variable when two units are selected without replacement with PPS at each step. Also compute the variance of the Horvitz-Thompson (HT) estimator of population total. **[10]**

Unit number	y_i	x_i
1	3	1
2	7	3
3	9	4
4	25	12

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