Total No. of Questions : 8]

SEAT No. :

P997

[Total No. of Pages : 2

[5428]-11

M.A./M.Sc. (Semester - I)

MATHEMATICS

MT 501 : Real Analysis

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If l^2 denote the set of all square summable sequences of complex numbers, then show that $\langle\{x_n\},\{y_n\}\rangle = \sum\ x_n \overline{y}_n$ is an inner product on l^2. **[6]**

b) Verify that C[a,b] with a supremum norm is a normed linear space. **[5]**

c) InC[0,1] with supremum norm, compute d (f,g) where f (x) = 1 and g (x)=x **[5]**

Q2) a) Consider a metric space M and A$\subseteq$M is compact. f: $\mathrm{M} \to \mathrm{R}$ is continuous then show that f (A) is compact. **[6]**

b) Show that C ([a,b], $\|.\|_2$) is not complete. **[5]**

c) Show that a closed unit ball in l^1 is not compact **[5]**

Q3) a) Suppose μ is a countably additive function defined on a ring R and A, A_1, A_2, ... $\in \mathrm{R}$ such that $\mathrm{A}_1 \subseteq \mathrm{A}_2 \subseteq$ .... with $\mathrm{A} = \bigcup_{n=1}^{\infty} \mathrm{A}_n$ then prove that $\lim_{n\to\infty} \mu(\mathrm{A}_n = \mu(\mathrm{A})$. **[6]**

b) Let m be a Labesgue measure defined on $\mathbb{R}^n$ and ε be a collection of all finite unions of disjoint intervals in $\mathbb{R}^n$. Prove that m is a measure on ε. **[5]**

P.T.O

c) Define a measurable set in $\mathbb{R}^n$ and show that a family of measurable sets forms a $\sigma-$ ring. **[5]**

Q4) a) Prove that the outer measure m^* is countably subadditive. **[6]**

b) Let M_F denote the collection of subsets A of $\mathbb{R}^n$ such that

$D(A_K, A) \to 0$ as $K \to \infty$ Where each A_K is the finite union of disjoint intervals. Prove that M_F is a ring. **[5]**

c) If f is a measurable function then show that |f| is also measurable. **[5]**

Q5) a) State and prove Lebesgue dominated convergence theorem. **[6]**

b) Prove that every real valued continuous function defined on $\mathbb{R}^n$ is measurable. **[5]**

c) For $1 \leq p < \infty$, define $L^b(\mu)$ and show that it is a linear space. **[5]**

Q6) a) State and prove Fatou's lemma. **[6]**

b) Prove that $L^\infty(\mu)$ is complete. **[5]**

c) Prove that step functions are dense in $L^p(\mu)$. **[5]**

Q7) a) Find Fourier series for $f(x) = x$. **[8]**

b) Apply Grahm-Smidt method to the functions $1, x, x^2$,............ and find first three Legendre Polynomials. **[8]**

Q8) a) State and prove Holder's inequality. **[8]**

b) State and prove Bessel's inequality. **[8]**

Total No. of Questions : 8]

SEAT No. :

P998

[Total No. of Pages : 3

[5428]-12
M.A/M.Sc. (Semester - I)
MATHEMATICS
MT 502 : Advanced Calculus
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove Mean Value theorem of derivatives of scalar fields. **[6]**

b) Let $V(r,t)=t^n e^{-r^2/4t}$. Find the value of the constant n such that V satisfies the following equation $\frac{\partial V}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)$. **[6]**

c) Let $\overline{f}:\mathbb{R}^n \rightarrow \mathbb{R}^n$ be a linear transformation. Prove that $\overline{f}$ is continuous at each point $\overline{a}$ in $\mathbb{R}^n$. **[4]**

Q2) a) State and prove chain rule for derivatives of scalar fields. **[6]**

b) Evaluate the directional derivative of f $(x,y,z) = 3x–5y+2$z at (2,2,1) in the direction of outword normal to the sphere $x^2+y^2+z^2 = 9$. **[5]**

c) Let $\overline{f}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $\overline{g}:\mathbb{R}^3 \rightarrow \mathbb{R}^2$ be two vector fields defined as follows;

$$\overline{f}(x,y)=e^{x+2y}\,\overline{i}+\sin(y+2x)\,\overline{j}$$

$$\overline{g}(u,v,w)=(u+2v^2+3w^3)\,\overline{i}+(2v-u^2)\,\overline{j}$$ **[5]**

i) Compute each of the Jacobian matrices Df (x,y) and Dg (u,v,w)

ii) Compute the Composition $\overline{h}(u,v,w)=\overline{f}[\overline{g}(u,v,w)]$

P.T.O

Q3) a) Define the line integral of vector field. State and prove linearity and additive property for line integral. **[6]**

b) Evaluate the line integral of $\overline{f}(x,y) = (x^2 + y^2)\overline{i} + (x^2 - y^2)\overline{j}$, along the curve $y = 1 - |1 - x|$ from the points (0,0) to (2,0) **[5]**

c) State and prove second fundamental theorem of calculus for line integral. **[5]**

Q4) a) A particle of mass m moves along a curve under the action of a force field $\overline{f}$. If the speed of the particle at time t is v(t), its kinetic energy is defined to be $\frac{1}{2}mv^2(t)$. Prove that the change in kinetic energy in any time interval is equal to the work done by $\overline{f}$ during this time interval. **[6]**

b) Let $\overline{f}$ be a vector field continuous on an open connected set S in $\mathbb{R}^n$. If the line integral of $\overline{f}$ is zero around every piecewise smooth closed path in S, then prove that the line integral of $\overline{f}$ is independent of the path in S. **[5]**

c) Find the amount of work done by the force $\overline{f}(x,y) = (x^2 - y^2)\overline{i} + zxy\overline{j}$ in moving a particle (in a counter clockwise direction) once around the square bounded by the coordinate axes and the lines $x=a$ and $y=a$, $a>0$. **[5]**

Q5) a) State and prove Green's theorem for plane regions bounded by piecewise smooth Jordan curves. **[6]**

b) Let u and v be scalar fields having continuous first and second order partial derivatives in an open connected set S in the plane. Let R be a region on S bounded by a piecewise smooth Jordan curve C oriented counterclockwise. Show that

$$\oint_C uvdx + uvdy = \iint_R \left[v\left(\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right) + u\left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right)\right] dx\, dy. \quad \textbf{[5]}$$

c) Evaluate $\iint\limits_{s} e^{\frac{y-x}{y+x}}\, dxdy$, where S is the triangle bounded by the line $x+y=2$ and the two coordinate axes. **[5]**

Q6) a) Show that $\iint\limits_{s} f(xy)\,dx\,dy = \log 2 \int_{1}^{2} f(u)\,du$, where S is the region in the first quadrant bounded by the curves $xy=1$, $xy=2$, $y=x$, $y=4x$. **[6]**

b) Evaluate $\iiint\limits_{S} dxdydz$, where S is solid bounded by two concentric spheres of radii a and b, where $0 < a < b$ and the center is at the origin. **[5]**

c) State the transformation formula for n-fold integral and explain the terms involved in it. **[5]**

Q7) a) Let $\overline{F}(x, y, z) = (x^2 + yz)\overline{i} + (y^2 + xz)\overline{j} + (z^2 + xy)\overline{k}$.

Determine the Jacobian matric of $\overline{F}$. Also compute the curl and divergence of $\overline{F}$. **[6]**

b) Define and prove stokes theorem. **[8]**

c) Define a surface area of parametric surface. **[2]**

Q8) a) Define fundamental vector product. Show that fundamental vector product is normal to the surface. **[6]**

b) Let S denote the hemisphere $x^2+y^2+z^2=1$, $z \geq 0$, and let $\overline{F}(x, y, z) = x\overline{i} + y\overline{j}$. let $\overline{n}$ be the unit outword normal of S. Compute the value of the surface integral $\iint\limits_{s} \overline{F}.\overline{n}\, ds$. **[6]**

c) State : **[4]**

i) Divergence theorem

ii) Inverse function theorem

Total No. of Questions : 8]

SEAT No. :

P999

[Total No. of Pages : 3

[5428]-13
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 503 : Linear Algebra
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Answer any five.*
2) *Figures to the right indicate full marks.*
3) *Use of non programmable calculator is allowed.*

Q1) a) Let V be a finite dimensional vector space over K and let X and Y be finite subsets of V. If Y is linearly independent and V = <X> then show that $|Y| \leq |X|$. **[6]**

b) Let W_1 and W_2 be subspaces of vector space V. Show that $W_1 \cup W_2$ is subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. **[5]**

c) Find two dimensional subspace of $\mathbb{R}^4$ which do not contain vectors $[1, 3, 2, 5]^t$ and $[2, 4, 3, 1]^t$. **[5]**

Q2) a) Let V and V' be finite dimensional vector spaces over K. Then show that $V \cong V'$ if and only if dim V = dim V'. **[6]**

b) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be linear transformation defined as $T(x, y, z) = (2x + 2y - z, 2x + 5y - 2z, 4x + 4y - 2z)$. Find the Kernel and image of T. **[5]**

c) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be projection on the plane $3x + 2y - 2z = 0$ along the line passing through the origin and the point $[1, 1, 1]^t$. Find matrix representation of f with respect to standard bases. **[5]**

Q3) a) Give an example of 3×3 matrix which is not triangulable over $\mathbb{R}$, but is not diagonalizable over $\mathbb{C}$. **[6]**

b) Prove that a Jordan chain consists of linearly independent vectors. **[5]**

c) Write all possible Jordan canonical forms if the characteristic polynomial is $(x - 2)^4 (x - 3)^2$. **[5]**

P.T.O.

Q4) a) Reduce the following matrix to its triangular form : **[6]**

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

b) Verify that 1 is the eigen value of the following matrix A and find its geometric multiplicity. **[5]**

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & -2 & 1 \end{bmatrix}$$

c) Prove that eigen vectors corresponding to distinct eigen values of linear operator are linearly independent. **[5]**

Q5) a) Let V be an inner product space over $\mathbb{F}$ and let us v $\in$ V. Then show that

i) $\|u+v\|^2 + \|u-v\|^2 = 2\|u\|^2 + 2\|v\|^2$

ii) $\|\lambda u\| = |\lambda| \|u\|$ for all $\lambda \in \mathbb{F}$. **[6]**

b) Prove that in a finite dimensional inner product space, complete orthonormal set is a basis. **[5]**

c) Let W be a subspace of finite dimensional inner product space V; then show that $V = W \oplus W^{+}$. **[5]**

Q6) a) Use Gram-Schmidt orthonormalization process to obtain an orthonormal basis spanned by the following vectors in the standard inner product space.

$[2, 1, 2]^t$, $[4, 1, 0]^t$, $[3, 1, -1]^t$. **[8]**

b) Find unitary matrix U such that U* AU is upper triangular, where **[8]**

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Q7) a) Let $A = \begin{bmatrix} 5 & 0 & 4i \\ 0 & 1 & -4i \\ -4i & 4i & 3 \end{bmatrix} \in C^{3\times 3}$ **[8]**

i) Verify that 3 is an eigenvalue of A.

ii) Find the matrix U such that U* AU = diag (3, 9, –3)

b) Let S and T be self adjoint operators on an inner product space V then show that **[8]**

i) S + T is self adjoint

ii) T^{-1} is self adjoint if T is invertible.

Q8) a) State and prove Cayley - Hamilton theorem. **[8]**

b) Find orthogonal projection of vector x onto subspace W of $\mathbb{F}^4$ if **[8]**

$$x = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 5 \end{bmatrix} \text{ and } W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \middle/ \begin{matrix} 2x_1 + x_2 + x_3 - 2x_4 = 0 \\ x_1 + 2x_2 + x_3 - 3x_4 = 0 \\ x_1 + x_2 + 2x_3 + x_4 = 0 \end{matrix} \right\}$$

Total No. of Questions : 8]

SEAT No. :

P1000

[Total No. of Pages : 3

[5428]-14

M.A/M.Sc. (Semester - I)

MATHEMATICS

MT 504 : Number Theory

(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If p is an odd prime then prove that $\left(\frac{a}{p}\right) \equiv a^{(p-1)/2} (\mathrm{mod}\, p)$. **[6]**

b) If a is selected at random from 1,2,3, -------,14 and b is selected at random from 1,2,3, --------,15. What is the probability that $ax \equiv b$ (mod 15) has exactly one solution? **[6]**

c) State and prove divisibility test for 11. **[4]**

Q2) a) Let $f(x)$ be a fixed polynomial with integral coefficients and for any positive integer m let N(m) denote the number of solutions of the congruence $f(x) \equiv 0$ (mod m). If $m=m_1m_2$ where $(m_1, m_2) = 1$ then prove that $N(m) = N(m_1)\ N(m_2)$. **[6]**

b) Prove that if p and q are distinct primes of the form 4k+3 and if $x^2 \equiv p (\mathrm{mod}\, q)$ has no solution then $x^2 \equiv q (\mathrm{mod}\, p)$ has two solutions. **[6]**

c) Let p be a prime and let (a,p) = (b,p) = 1 prove that $x^2 \equiv a (\mathrm{mod}\, p)$ and $x^2 \equiv b (\mathrm{mod}\, p)$ are not solvable then $x^2 \equiv a^b (\mathrm{mod}\, p)$ is solvable. **[4]**

P.T.O

Q3) a) For every positive integer n prove that **[6]**

i) $\sigma(n) = \prod_{p^{\alpha} \| n} \left(\frac{p^{\alpha+1} - 1}{p - 1} \right)$.

ii) $\sum_{d/n} \phi(d) = n$.

b) State and prove Möbius inversion formula. **[6]**

c) Prove that $11 + 2\sqrt{6}$ is a prime in $Q\left(\sqrt{6}\right)$. **[4]**

Q4) a) Let q be a prime factor of a^2+b^2. If $q \equiv 3 (\text{mod} 4)$ then prove that $q|a$ and $q|b$. **[6]**

b) Let f, g and h be arithmetic functions such that $h(n) = \sum_{d|n} f(d) g\left(\frac{n}{d}\right)$ for all n. Show that if f and g are multiplicative then h is also multiplicative. **[5]**

c) Prove that $Q\left(\sqrt{5}\right)$ has the unique factorization property. **[5]**

Q5) a) Let a, b and c be integers with not both a and b equal to 0 and let g=g.c.d (a,b). If $g \times c$ then prove that the equation ax+by=c has no solution in integers and if g|c then this equation has infinitely many solutions. **[6]**

b) Find all primes p such that $\left(\frac{10}{p}\right)$=1. **[5]**

c) Show that the product of three consecutive integers is divisible by 504 if the middle one is a cube. **[5]**

Q6) a) If P and Q are odd and positive and if (P,Q)=1 then prove that $\left(\frac{P}{Q}\right)\left(\frac{Q}{P}\right) = (-1)^{\{(p-1)/2\}\{(Q-1)/2\}}$. **[6]**

b) Find all solutions of equation $5x+3y=52$. **[5]**

c) Prove that the number of positive irreducible fractions ≤ 1 with denominator $\leq n$ is $\phi(1)+\phi(2)+...............+\phi(n)$. **[5]**

Q7) a) Prove that an algebraic number ξ satisfies a unique irreducible monic polynomial equation g(x)=0 over Q and every polynomial equation over Q satisfied by ξ is divisible by $g(x)$. **[8]**

b) Show that $\sum_{j=1}^{\infty} 10^{-j!}$ is a transcendental number. **[5]**

c) Find the minimal polynomial of each of the following. **[3]**

i) 7

ii) $\left(1+\sqrt[3]{7}\right)/2$

Q8) a) If (a, m)=1 then prove that $a^{\phi(m)} \equiv 1(\bmod m)$. **[6]**

b) Find all integers that satisfy simultaneously

$x \equiv 2(\bmod 3), x \equiv 3(\bmod 5), x \equiv 5(\bmod 2)$. **[5]**

c) Find formula for the highest exponent e of the prime p such that p^e divides the product of the first n odd numbers. **[5]**

Total No. of Questions : 8]

SEAT No. :

P1001

[Total No. of Pages : 2

[5428]-15

M.A/M.Sc. (Semester - I)

MATHEMATICS

MT - 505 : Ordinary Differential Equations

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If $y_1 = \cos x$ is one solution of $y''+y=0$, find other solution. **[4]**

b) State and prove principle of superposition. **[6]**

c) Use the method of reduction of order to solve the differential equation $y''=y'e^y$ with initial conditions $y(0)=0$ and $y'(0)=1$. **[6]**

Q2) a) Explain the method of variation of parameters. **[8]**

b) Find the normal form of equation $y''+p(n)y'+y=0$. **[4]**

c) Solve $x^2y''+2xy'+3y=0$. **[4]**

Q3) a) Use the method of Frobenius series to solve the differential equation $2x^2y'' + x(2x+1)y'-y=0$. **[6]**

b) Show that the series $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots\ldots\ldots\ldots.$ **[4]**

Satisfies the differential equation $y''=-y$

c) Solve the system of linear equation

$$\frac{dx}{dt} = 3x - 4y$$
$$\frac{dy}{dt} = x - y \quad .$$
[6]

P.T.O

Q4) a) Show that the solution of the initial value problem $\frac{dy}{dx} = F(x, y)$ with $y(x_0) = y_0$, may not be unique although $F(x, y)$ is continuous. **[8]**

b) Discuss the method of undetermined coefficients to find particular solution of the non-homogeneous differential equation $y''+py'+qy=R(x)$, where p,q are constants. **[8]**

Q5) a) State and prove picard's theorem. **[6]**

b) Find the solution of $y''+2y'+5y=e^{-x} \sec 2x$ **[6]**

c) Explain the terms **[4]**

i) Regular singular point

ii) Irregular singular point

Q6) a) Prove that the Function $E(x,y)=ax^2 + bxy+cy^2$ is positive definite if and only if $a>0$ and $b^2-4ac<0$. **[8]**

b) Let $u(x)$ be any non-trivial solution of $u''+q(x)u=0$. where $q(x)>0$ for all $x>0$ If $\int_1^\infty q(x)dx = \infty$ then prove that $u(x)$ has infinitely many zeros on the positive x-axis. **[8]**

Q7) a) Write Bessel's equation and show that for p=0 and indicial equation of Bessel's equation has only one root. **[6]**

b) Prove that $y=(1+x)^p$ is the solution of $(1+x)y''=py$ with $y(0) = 1$. **[6]**

c) Are the functions $\phi_1(x) = x^3$ and $\phi_2(x) = x^2|x|$ defined on the interval $[-1, 1]$ linearly independent? Justify. **[4]**

Q8) a) Explain the method of successive approximations. **[8]**

b) Solve the differential equation $y''-2y'-2y= 4x^2$ by the method of variation of parameters. **[8]**

Total No. of Questions : 8]

SEAT No. :

P1002

[Total No. of Pages : 2

[5428]-21
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT - 601 : General Topology
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define countably infinite set. Show that $\mathbb{Z}_+ \times \mathbb{Z}_+$ is countably infinite. **[6]**

b) Let X be a set. Let $\tau_f = \{U \subseteq X \mid X \setminus U \text{ either is finite or is all of } X\}$. Prove that τ_f is a topology on X. **[5]**

c) Let $\{\tau_\alpha\}$ be a family of topologies on X. Show that there is a unique smallest topology on X containing all the collections τ_α and a unique largest topology on X contained in all τ_α. **[5]**

Q2) a) Let B and B' be bases for the topologies τ and τ', respectively, on X. Then prove that the following statements are equivalent : **[8]**

i) τ' is finer than τ.

ii) for each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subset B$.

b) Define the order topology. Prove that the standard topology on $\mathbb{R}$ is the order topology derived from the usual order on $\mathbb{R}$. **[4]**

c) Let Y be a subspace of X. If U is open in Y and Y is open in X, then prove that U is open in X. **[4]**

Q3) a) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$. **[8]**

P.T.O

b) Let A be a subset of the topological space X; let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$. **[5]**

c) Prove that every simply ordered set is Hausdorff space in the order topology. **[3]**

Q4) a) Let f : $A \to X \times Y$ be given by the equation f (a) = (f_1 (a), f_2 (a)). Then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. **[8]**

b) Show that the subspace (a,b) of $\mathbb{R}$ is homeomorphic with (0,1) and the subspace [a,b] of $\mathbb{R}$ is homeomorphic with [0,1]. **[4]**

c) State and prove the pasting lemma. **[4]**

Q5) a) Let f : $X \to Y$. If the function f is continuous, then prove that for every convergent sequence $x_n \to x$ in X, the sequence f (x_n) converges to f (x) and the converse holds if X is metrizable. **[8]**

b) Let $g : X \to Z$ be a surjective continuous map. Let $X^* = \left\{g^{-1}\{z\} \middle| z \in Z\right\}$ Give X^*, the quotient topology. Prove that the map g induces a bijective continuous map f : $X^* \to Z$ which is homeomorphism if and only if g is a quotinet map. **[5]**

c) Show that if A is closed in Y and Y is closed in X, then A is closed in X. **[3]**

Q6) a) Define box and product topology. State and prove the comparison of the box and product topologies. **[8]**

b) State and prove the sequence lemma. **[5]**

c) If the sets C and D forms a separation of X and if Y is a connected subset of X, then prove that Y lies entirely within either C or D. **[3]**

Q7) a) Prove that every compact subspace of a Hausdorff space is closed. **[8]**

b) Prove that every compact Hausdorff space is normal. **[8]**

Q8) a) State and prove the Tychonoff theorem. **[12]**

b) State : **[4]**

i) The urysohn lemma

ii) The urysohn metrization theorem.

Total No. of Questions : 8]

SEAT No. :

P1003

[Total No. of Pages : 3

[5428]-22

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 602 : Differential Geometry

(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Answer any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State Inverse Function Theorem. **[2]**

b) Let U be an open set in $\mathbb{R}^{n+1}$ and $P \in U$, X be smooth vector field on U. Let $\alpha : I \to U$ be the maximal integral curve of X through P. Show that, if $\beta : \tilde{I} \to U$ is any integral curve of X with $\beta(t_o) = P$ for some $t_o \in \tilde{I}$ then $\beta(-t) = \alpha(t - t_o)$, $\forall t \in \tilde{I}$. **[6]**

c) State and prove Lagrange's Multiplier Theorem. **[8]**

Q2) a) Show that the Möbius band is an unorientable 2-surface . **[6]**

b) Explain why an integral curve cannot cross itself as it does for parameterized curve. **[6]**

c) Define the terms :

i) level sets

ii) graph of a function with an example. **[4]**

Q3) a) Let a,b,c $\in \mathbb{R}$ be such that ac - $b^2 > 0$ then show that the maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $ax_1^2 + 2bx_1x_2 + cx_2^2 = 1$ are of the form $1/\lambda_1$, $1/\lambda_2$. Where λ_1, λ_2 are eigen values of the matrix $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$. **[8]**

b) Let C be a connected oriented plane curve and $\beta : I \to C$ be unit speed parameterization of C. Prove that β is one to one or periodic. Also prove that β is periodic if and only if C is compact. **[8]**

P.T.O

Q4) a) Let $S \subset \mathbb{R}^{n+1}$ be connected n-surface in $\mathbb{R}^{n+1}$. Then there exist exactly two smooth unit normal vector field N_1 and N_2 on S with $N_2(p) = -N_1(p)$, $\forall p \in S$. **[6]**

b) Show that the unit n-surface sphere $x_1^2 + x_2^2 + ... + x_{n+1}^2 = 1$ is connected for n > 1. **[4]**

c) Show that gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p. **[6]**

Q5) a) Let $\alpha(t) = (x(t), y(t)), t \in I$ be local parametrization of the oriented plane curve C. Then show that $k o\alpha(t) = (x'y'' - y'x'') / (x'^2 + y'^2)^{3/2}$. **[8]**

b) Let f: $U \to \mathbb{R}$ be smooth function and $\alpha : I \to U$ be an integral curve of $\triangle$F then show that $\frac{d}{dt}(fo\alpha)(t) = \|\nabla f(\alpha(t))\|^2, \forall t \in I$ **[8]**

Q6) a) Let S be an oriented n-surface in $\mathbb{R}^{n+1}$ and $v_\in$ Sp be unit vector and $p \in S$. Then show that there exist an open set $V \subseteq \mathbb{R}^{n+1}$ containing 'p' such that sn N(v) $\cap$V is a plane curve. **[8]**

b) Let $\phi : V \to \mathbb{R}^{n+1}$ be a parameterized n-surface in $\mathbb{R}^{n+1}$ and $p \in U$ then show that there exist an open set $U_1 \subset U$ about p such that $\phi(U_1)$ is an n-surface. **[6]**

c) Define the terms:

i) Gauss map

ii) Weingarten map. **[2]**

Q7) a) Show that if S is connected n-surface in $\mathbb{R}^{n+1}$ and $g : s \to \mathbb{R}$ is smooth and takes on only the +1 and –1 then, g is constant. **[6]**

b) Show that if $\alpha : I \to \mathbb{R}^{n+1}$ is a parameterized curve with constant speed then $\dot{\alpha}(t) \perp \ddot{\alpha}(t)$, $\forall t \in I$ **[6]**

c) Define curvature of a surface with an example. **[4]**

Q8) a) Determine whether the vector field $X(x_1, x_2) = (x_1, x_2, 1 + x^2, \forall)$ is complete or not. **[6]**

b) Show that if X and Y are two parallel vector fields along $\alpha(t)$ then X.Y is constant along $\alpha(t)$. **[6]**

c) Let X be smooth vector field along the parameterized curve $\alpha : I \to \mathbb{R}^{n+1}$ and *f* be smooth function along $\alpha(t)$. Then prove that $(\dot{fX}) = f'X + f\dot{X}$. **[4]**

Total No. of Questions : 8]

SEAT No. :

P1004

[Total No. of Pages : 2

[5428]-23

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 603 : Groups and Rings

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that set of 2×2 matrices with determinant 1 having entries from $\mathbb{Q}$ forms a group under matrix multiplication. Is this group Abelian? Justify. **[5]**

b) Find the inverse of the element $A=\begin{bmatrix}12 & 3\\ 3 & 5\end{bmatrix}$ in $GL(2,\mathbb{Z}_{13})$. **[5]**

c) Give an example of an infinite group whose every element is of finite order. Justify the answer. **[6]**

Q2) a) Suppose that G is a group that has exactly one non trivial proper subgroup. Prove that G is isomorphic to $\mathbb{Z}_{p^2}$ where p is prime. **[5]**

b) Give examples of two non isomorphic groups of order 8 with justification. **[5]**

c) Suppose that H is a proper subgroup of $\mathbb{Z}$ under addition and H contains 12 and 15. Determine H. **[6]**

Q3) a) Let $H=\{a+b\omega \mid a,b\in\mathbb{Q},\omega^2+\omega+1=0\}$. Prove or disprove that H is a subgroup of $\mathbb{C}^*$ under multiplication. **[5]**

b) Define a subgroup. Let H be finite subset of a group G. Then prove that H is a subgroup of G if and only if H is closed under the operation of G. **[5]**

c) State and prove the Cayley's theorem. **[6]**

P.T.O

Q4) a) Find the inverse and the order of each of the following permutations in S_{13}

i) (1 13 9) (11 3 10) (4 6 5 12)

ii) (2 11 3 1 4) (7 12 5) (6 8 9) **[5]**

b) Prove that $SL(2,\mathbb{R})$ is a normal subgroup of $GL(2,\mathbb{R})$. **[5]**

c) State and prove the Lagrange's theorem for finite groups. Is the converse of the theorem true? Justify. **[6]**

Q5) a) State and prove the orbit stabilizer theorem. **[5]**

b) For any integer $n>2$, show that there are atleast two elements in $U(n)$, the group of units modulo n that satisfy $x^2=1$. **[5]**

c) If $\tau=(7\ 11\ 4)(6\ 5\ 10), \rho=(6\ 7\ 4\ 3\ 5\ 9)(8\ 2\ 10)\in S_{11}$

Then find $\tau^{-1}\rho\tau$ and $\rho^{-1}\tau\rho$. **[6]**

Q6) a) Let G and H be finite Abelian groups. Then prove that $G\oplus H$ is cyclic if and only if $|G|$ and $|H|$ are relatively prime. **[5]**

b) Determine all the homomorphisms from $\mathbb{Z}_{25}$ to $\mathbb{Z}_{35}$. **[5]**

c) Find all the non isomorphic Abelian groups of order 3430. **[6]**

Q7) a) Let G be a group. Prove that the mapping $\phi(g)=g^{-1}\forall g\in G$ is an automorphism if and only if G is Abelian. **[5]**

b) Prove that for any group G, $G/Z(G)$ is isomorphic to group of inner automorphisms of G, $Inn(G)$. **[5]**

c) If H is a subgroup of a finite group G and $|H|$ is a power of a prime p, Then prove that H is contained in some Sylow p-subgroup of G. **[6]**

Q8) a) Let G be a group with $2p$ elements where p is an odd prime. Then prove that G is isomorphic to $\mathbb{Z}_{2p}$ or D_p. **[5]**

b) Prove that a non cyclic group of order 21 has 14 elements of order 3. **[5]**

c) Prove that the group of order 36 is not simple. **[6]**

Total No. of Questions : 8]

SEAT No. :

P1005

[Total No. of Pages : 3

[5428]-24

M.A./M.Sc.

MATHEMATICS

MT-604: Complex Analysis

(2008 Pattern) (Semester - II)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) If $\sum a_n z^n$ is given power series with radius of convergence R, Then prove that R = $\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$ if this limit exists. **[6]**

b) Define Möbius transformation. Prove that every Möbius transformation maps circles of $\mathbb{C}_\infty$ onto circles $\mathbb{C}_\infty$. **[6]**

c) Evaluate the cross ratio (2 + 3i, 0, 1, ∞). **[4]**

Q2) a) Let f be analytic in the disk B($a;$ R) and suppose that γ is a closed rectifiable curve in B($a;$ R). Prove that $\int_\gamma f = 0$. **[6]**

b) If G is open and connected and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G, then show that f is constant. **[5]**

c) Let the degrees of the polynomials. **[5]**

$$p(z) = a_0 + a_1 z + a_2 z^2 + \ldots.. + a_n z^n \quad a_n \neq 0$$
$$q(z) = b_0 + b_1 z + b_2 z^2 + \ldots.. + b_m z^m \quad b_m \neq 0$$

be such that $m \geq n + 2$. Show that if all the zeros of $q(z)$ are interior to the simple closed contour C then $\int_C \frac{p(z)}{q(z)} dz = 0$.

P.T.O.

Q3) a) State and prove the Fundamental Theorem of Algebra. **[6]**

b) Let G be an open disk in $\mathbb{C}$ and $u: G \to \mathbb{R}$ is harmonic function then prove that u has a harmonic conjugate. **[5]**

c) Let G be a region and let f and g be analytic functions on G such that $f(z)g(z) = 0, \forall z \in G$. Show that either $f \equiv 0$ or $g \equiv 0$. **[5]**

Q4) a) State and prove the Liouville's Theorem. **[6]**

b) Let γ be a closed rectifiable curve in $\mathbb{C}$. Prove that $\eta(\gamma;a)$ is constant for a belonging to a component of $G = \mathbb{C} - \{\gamma\}$. Also, prove that $\eta(\gamma;a) = 0$ for a belonging to unbounded component of G. **[6]**

c) Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the transformation $w = \exp z$, and label corresponding portions of the boundaries. **[4]**

Q5) a) Let G be an open subset of the plane and f be an analytic function. If γ is a closed rectifiable curve in G such that $\eta(\gamma; w) = 0$ for all w in $\mathbb{C} - G$ then for $a \in G - \{\gamma\}$ prove that $\eta(\gamma : a) f(a) = \frac{1}{2\pi i} \int \frac{f(z)}{z - a} dz.$ **[8]**

b) Evaluate the integral $\int_\gamma \frac{\cos z}{z^3} dz, \quad \gamma(t) = e^{it}, \; 0 \leq t \leq 2\pi$. **[4]**

c) If $f(z)$ and $\overline{f(z)}$ are both analytic in a domain D, show that f is constant in D. **[4]**

Q6) a) State and prove Morera's Theorem. **[6]**

b) State and prove Chauchy's Residue Theorem. **[5]**

c) If f has an isolated singularity at a then prove that the point $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z - a) f(z) = 0$. **[5]**

Q7) a) Evaluate improper integral $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)}dx$ by using residues. **[6]**

b) Let G be a region and $f : \mathrm{G} \to \mathbb{C}$ be analytic function. Prove that $f \equiv 0$ if and only if there exist a in G such that $f^{(n)}(a) \geq 0,\ \forall n \geq 0.$ **[5]**

c) If G is a region with $a \in \mathrm{G}$ and if f is analytic on G – a with a pole at $z = a$ then prove that there is analytic function $g : \mathrm{G} \to \mathbb{C}$ and a positive integer m such that $f(z) = \frac{g(z)}{(z-a)^m}$. **[5]**

Q8) a) State and prove Casorati-Weierstrass Theorem. **[6]**

b) Let a function f be continuous on a closed bounded region R, and let it be analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R, prove that $|f(z)|$ has a minimum value m in R which occurs on the boundary of R. **[5]**

c) Represent a function $f(z) = \frac{1}{z(1+z^2)}$ by its Laurent series in its domain $1 < |z| < \infty$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1006

[Total No. of Pages : 2

[5428]-25

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 605 : Partial Differential Equations

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the arbitrary function F from the equation $x+y+z=F(x^2+y^2+z^2)$. and find the corresponding partial differential equation. **[6]**

b) Find the general solution of $x(y-z)\,p+y\,(z-x)\,q=z\,(x-y)$. **[5]**

c) Explain the method of solving the following first order partial differential equation.

i) f(p,q)=0

ii) g(x,p)=h(y,q) **[5]**

Q2) a) Find the general solution of

$(y^2+yz)dx+(z^2+xz)dy+(y^2-xy)dz=0.$ **[6]**

b) Show that the equations $p^2+q^2-1=0$ *and* $(p^2+q^2)\,x-pz=0$ are compatible. Also find their common solution. **[6]**

c) Find the general integral of $y^2p-xyq=x(z-2y)$. **[4]**

Q3) a) Prove that the pfaffian differential equation

$$\vec{X}.\overrightarrow{dr} = P(x,y,z)dx + Q(x,y,z)\,dy + R(x,y,z)\,dz = 0$$

is integrable if and only if $\overrightarrow{X}.\overrightarrow{curl X} = 0$ **[6]**

P.T.O

b) Find the complete integral of $px+qy=pq$ by Charpit's method. **[6]**

c) Find the integral of pfaffian differential equation
$y(1+z^2)dx-x(1+z^2)dy+(x^2+y^2)dz=0$. **[4]**

Q4) a) Find the general integral surface of the D.E. $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ and the particular solution through curve C: $xz=a^2, y=0$. **[6]**

b) Verify that the pfaffian differential equation is integrable and find the corresponding integral of $z(z+y^2)dx+z(z+x^2)dy-xy(x+y)dz=0$. **[6]**

c) Find the complete integral of $xU_x + yU_{y=}U_z^2$ by Jacobi's method **[4]**

Q5) a) Find by the method of characteristics, the integral surface of pq=z which passes through curve C:$x_0=0, y_0=s, z_0=s^2$. **[8]**

b) Prove that the solution of the Neumann problem is unique up to the addition of a constant. **[4]**

c) Reduce the equation $U_{xx}+2U_{xy}+17U_{yy=0}$ to canonical form and solve it. **[4]**

Q6) a) State and prove Harnack's theorem. **[8]**

b) Find the solution of the Heat-equation in an infinite rod which is defined

$$as\ U_t = kU_{xx}, -\infty < x < \infty, t > 0$$
$$U(x,0) = f(x), -\infty < x < \infty$$ **[8]**

Q7) a) If U(x,y) is harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$. Then U attains it's maximum on the boundary B of D. **[8]**

b) State and prove Kelvin's inversion theorem. **[6]**

c) Classify the following equation into hyperbolic, parabolic or elliptic type
$u_{xx}+2u_{yz}+cosxu_z-ey^2u=coshz$ **[2]**

Q8) a) State Dirichlet's problem for rectangle and find it, solution. **[8]**

b) Use Duhamel's principle and solve the non homogeneous wave equation

$$u_{tt} - c^2u_{xx} = F(x,t), -\infty < x < \infty, t > 0$$

with conditions $\text{u}(x,0) = u_{t(x,0)} = 0 - \infty < x < \infty$. **[8]**

Total No. of Questions : 4]

SEAT No. :

P1007

[Total No. of Pages : 1

[5428]-26
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT-606 : O.O.P Using C++
(2008 Pattern)

Time :2 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Question No. 1 is compulsory.***
2) ***Attempt any two of questions 2, 3, 4.***
3) ***Figures to the right indicate full marks.***

Q1) Attempt the following questions : **[20]**

i) Write a function to find g.c.d. of two integers.

ii) Write a note on scope resolution operator.

iii) What is data encapsulation.

iv) Write a function to read a matrix using do_while loop.

v) Write a program to find a raised to 6.

vi) Give an example of a union in C++.

vii) Write a note on operator "new".

viii) Explain function overloading.

ix) Write a program to find product and division of two numbers.

x) What is function prototype?

Q2) Define a class rational having two data members Num and Dev. Overload necessary constructors and the operators * and /. Find multiplication and division of two rational numbers. **[15]**

Q3) Write a note on compile time polymorphism and run time polymorphism. **[15]**

Q4) Write a complete C++ program to find the roots of a quadratic equation. **[15]**

Total No. of Questions : 8]

SEAT No. :

P1008

[Total No. of Pages : 3

[5428]-31
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT -701 : Functional Analysis
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*
3) *Figures to the right indicate full marks.*

Q1) a) State and prove the Hahn-Banach theorem. **[8]**

b) Prove that if $p \geq q \geq 1$, then $l_q \subset l_p$. **[6]**

c) Give an example of a non-identity unitary operator on an infinite dimensional Hilbert space. **[2]**

Q2) a) Let H be a Hilbert space and f be a linear functional on H. Prove that there exists a unique vector $y \in H$ such that $f(x) = < x, y >$ for every $x \in H$. **[8]**

b) Let Y be a closed subspace of a normed linear space X. Show that a sequence $\{x_n + Y\}$ converges to $x + Y$ in X/Y if and only if there is a sequence $\{y_n\}$ in Y such that $\{x_n + y_n\}$ converges to x in X. **[4]**

c) If M is a closed subspace of a normed linear space N and if T is the natural mapping of N into N/M defined by $T(x) = x + M$, then show that T is a continuous linear transformation for which $\|T\| = 1$. **[4]**

Q3) a) If T is an operator on a Hilbert space H, then prove that T is normal if and only if its real and imaginary parts commute. **[6]**

b) Prove that if M is a proper closed linear subspace of a Hilbert space, then there exists a non-zero vector z_0 in H such that $z_0 \perp M$. **[6]**

c) If T is a continuous linear transformation of a normed linear space N into a normed linear space N' and if M is its null space then show that T induce a natural linear transformation. **[4]**

$T' : N / M \xrightarrow{\text{into}} N' \text{ and} \| T' \| = \| T \|$

P.T.O.

Q4) a) Let N be a non-zero normed linear space. Prove that N is a Banach space if and only if $\{x \in N : \|x\| = 1\}$ is complete. **[6]**

b) Show that the set of all unitary operators on a Hilbert space forms a group with respect to the operator multiplication. **[6]**

c) Given an example of normed linear space X whose subspace Y is not closed and **[4]**

$|||x + Y||| = \inf f\{\|x + y\|/y \in Y\}$

is not a norm.

Q5) a) Let X be an inner product space over $\mathbb{C}$ and let $A : X \to X$ be a linear map. **[6]**

Show that

$4 < A(x), y > = < A(x+y), x+y > - < A(x-y), x-y > +$

$i < A(x+iy), x+iy > -i < A(x-iy), x-iy >$

b) Let P be a projection on a Hilbert space H with range M and null space N. Prove that $M \perp N$ if and only if P is self-adjoint. **[6]**

c) Show that C [a, b] is a normed linear space with the norm

$$\| f \| = \left(\int_a^b |f(t)|^2 dt \right)^{\frac{1}{2}}$$

but not a Banach Space. **[4]**

Q6) a) Let E_1 and E_2 be subsets of a normed linear space X and $E_1 + E_2 = \{x + y/x \in E_1, y \in E_2\}$ prove that **[6]**

i) If E_1 and E_2 are compact then $E_1 + E_2$ is compact.

ii) If E_1 is compact and E_2 is closed then $E_1 + E_2$ is closed.

iii) $E_1 + E_2$ need not be closed even though E_1 and E_2 are closed.

b) Let $X = \mathbb{R}^3$ be a normed linear space with the Euclidean norm and let $f(x) = x_1 + x_2 + x_3$. where $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ Prove that f is a bounded linear functional on $\mathbb{R}^3$. Find the distance of the origin from the hyperplane $x_1 + x_2 + x_3 = 1$. **[6]**

c) Let $C'[0, 1]$ denote the vector space of all real-valued functions defined on [0, 1] having first order continuous derivatives. Show that the expression given below are equivalent norms on $C'[0,1]$. **[4]**

$\|f\|_1 = sup_{0 \le t \le 1}\{|f(t)| + |f'(t)|\}$

$\|f\|_2 = sup_{0 \le t \le 1}\{|f(t)\} + sup_{0 \le t \le 1}\{|f'(t)|\}$.

Q7) a) Let X and Y be normed linear spaces and $F : X \to Y$ be a linear map. Prove that F is continuous if and only if for every cauchy sequence (x_n) in X, the sequence $(F(x_n))$ is cauchy in Y. Show that this is not true for non-linear continuous map. **[6]**

b) Let E and F be closed subsets of Hilbert space H. If $E \perp F$ then show that $E + F$ is closed in H. **[6]**

c) Let X be an inner product space. For $x, y \in X$, show that $\|x + y\|^2 = \|x\|^2 + \|y\|^2$ if and only if $\operatorname{Re} < x, y > = 0$. **[4]**

Q8) a) Let $\{c_n\}$ be an orthogonal sequence in an inner product space X. Prove that $\sum_{n=1}^{\infty} |< x, c_n >< y, c_n >| \le \| x \| \; \| y \|$ for all $x, y \in X$. **[6]**

b) Define a function $\|.\|$ on $\mathbb{R}^3$ by

$$\| x \| = ((x_1^2 + x_2^2)^{3/2} + | x_3 |^3)^{1/3}$$

Show that a linear space $\mathbb{R}^3$ is normed linear space with respect to $\|.\|$. **[6]**

c) Let A subset of Hilbert Space H and let $\{x_n\}$ be a sequence in A such that

$$\lim_{n\to\infty} \| x_n \| = d = \inf_{x \in A} \| x \|.$$

Prove that $\{x_n\}$ converges in H. **[4]**

Total No. of Questions : 8]

SEAT No. :

P1009

[Total No. of Pages : 2

[5428]-32
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT -702 : Ring Theory
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Each question carries 16 marks.*

Q1) a) Define the term ring homomorphism. Find all the ring homomorphisms from $\mathbb{Z}_5$ to $\mathbb{Z}_6$. **[5]**

b) Define matrix ring. Let R be a non-zero commutative ring with unity. Show that $M_n(R)$ is always non-commutative if $n \geq 2$. **[5]**

c) Show that a Boolean ring with unity has characteristic two. Give an example of a Boolean ring. **[6]**

Q2) a) Let R be a commutative ring with $1 \neq 0$ and G be a finite group. Define group ring RG of G clearly defining the addition and multiplication on RG. Is RG always a commutative ring? Justify your answer. **[6]**

b) Is every UFD a PID? Justify. **[6]**

c) Show that in a commutative ring with unity every maximal ideal is a prime ideal. **[4]**

Q3) a) Show that the ring of integers $\mathbb{Z}[i]$ is an Euclidean domain. **[7]**

b) Show that in a commutative integral domain with 1, a prime element is irreducible but not conversely. **[5]**

c) Is every prime ideal in a commutative ring with unity maximal? Justify. **[4]**

P.T.O.

Q4) a) Show that if R is commutative ring such that the polynomial ring $R[X]$ is a PID then R is a field. **[8]**

b) Show that every finite field contains $\mathbb{Z}_p$ as subfield for some prime p. **[4]**

c) Is $\mathbb{Q}[X, Y]$ a Euclidean domain? Justify your answer. **[4]**

Q5) a) State and prove Eienstein's criterion. Illustrate by an example. **[7]**

b) Show that in a UFD a non-zero element is a prime if and only if it is irreducible. **[5]**

c) Is every integral domain a UFD? Justify. **[4]**

Q6) a) Show that $R = \mathbb{Q}[x]/(X^2+2X+1)$ is not a field by explicitly finding the element that does not have a multiplicative inverse. Is R an integral domain? Justify. **[6]**

b) Define the term affine algebraic set and give two examples of affine algebraic sets. Identify all the affine algebraic sets in $\mathbb{A}^1$ over $\mathbb{R}$. **[6]**

c) Give an example of a ring where every prime ideal is maximal. Justify. **[4]**

Q7) a) State and prove Hilbert's Basis theorem. **[8]**

b) Give a definition of Artinian ring and give two examples of Artinian rings. **[8]**

Q8) a) Show that $\mathbb{Z}_n$ has only one maximal ideal if and only if $n = p^k$ for some prime p. **[6]**

b) Let p be a prime. Show that the polynomial $f(x) = x^{p-1}+x^{p-2} + ... + x + 1$ is irreducible in $\mathbb{Z}[X]$. Is $f(x)$ irreducible in $\mathbb{Q}[X]$? **[6]**

c) Is $\mathbb{Z}[X, Y, Z]$ a UFD? Justify. **[4]**

Total No. of Questions : 8]

SEAT No. :

P1010

[Total No. of Pages : 3

[5428]-33
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT -703 : Mechanics
(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Explain the terms **[6]**

i) Holonomic constraints.

ii) Angular momentum.

iii) Degree of freedom

iv) Brachistochrone Problem.

b) A particle is constrained to move on the surface of a cylinder of fixed radius. Find the Lagrange's equation of motion. **[6]**

c) Show that the Lagrange's equation of Motion can also be written as **[4]**

$$\frac{\partial L}{\partial t}-\frac{d}{dt}\left(L-\sum \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right)=0$$

Q2) a) Show that the geodesic in a Euclidean plane is a straight line **[5]**

b) Explain the D' Alembert's Principle and derive Lagrange's equation of motion using the same. **[7]**

c) Explain the degree of freedom and find the degree of freedom of a free particle moving in a plane. **[4]**

P.T.O.

Q3) a) Prove that, if $\frac{\partial f}{\partial x}=0$, then E-L equation is $y'\frac{\partial f}{\partial y'}-f=c_1$. **[6]**

b) Find E-L differential equation satisfied by twice differentiable function y(x) which extremizes the functional $I(y(x))=\int_{x_1}^{x_2} f(x,y,y')dx$ where y is prescribed at the end points. **[8]**

c) Explain the Basic lemma. **[2]**

Q4) a) Explain Hamiltonian Principle for non-conservative system. **[6]**

b) Find the extremals for an isoperimetric problem $I(y(x))=\int_0^1 (y'^2-y^2)dx$, subject to the conditions that $\int_0^{\pi} ydx=1, y(0)=0, y(\pi)=1$. **[8]**

c) Explain the Hamilton's Principle. **[2]**

Q5) a) Describe the motion of a particle of mass m moving near the surface of the Earth under the Earth's constant gravitational field by Hamilton's procedure. **[6]**

b) Prove that a co-ordinated which is cyclic in the Lagrangian is also cyclic in the Hamiltonian. **[4]**

c) Write Hamilton's equations of motion for a compound pendulum. **[6]**

Q6) a) Explain the Principle of least action. **[2]**

b) Prove that central force motion is always motion in a plane. **[6]**

c) Show that the determinant of an orthogonal matrix is ± 1. **[8]**

Q7) a) Prove that Poisson brackets are invariant under canonical transformation. **[8]**

b) Explain Legendre transformations and the Hamiltonian equations of motion. **[8]**

Q8) a) Prove that in case of an orthogonal transformation the inverse matrix is identified by the transpose of the matrix. **[8]**

b) If the matrix of transformation form space set of axes to body set of axes is equivalent to a rotation through an angle χ about some axis through the origin then show that

$$\cos\left(\frac{\chi}{2}\right)=\cos\left(\frac{\phi+\varphi}{2}\right).\cos\left(\frac{\theta}{2}\right).$$ **[8]**

Total No. of Questions : 8]

SEAT No. :

P1011

[Total No. of Pages : 3

[5428]-34
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT -704 : Measure and Integration
(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *All symbols have their usual meanings.*

Q1) a) Let $\{E_i\}$ be a sequence of measurable sets. Then

i) If $E_1 \subseteq E_2 \subseteq \ldots$, then show that $m(\lim E_i) = \lim m(E_i)$. **[3]**

ii) if $E_1 \supseteq E_2 \supseteq \ldots$, and $m(E_1) < \infty$, then show that $m(\lim E_i) = \lim m(E_i)$. **[3]**

b) Define following terms with suitable example.

i) σ-algebra **[2]**

ii) Inner Measure **[2]**

iii) Complete measure **[2]**

c) Let f be a continuous function and g a measurable function. Show that the composite function $f \circ g$ is measurable. **[4]**

Q2) a) If f and g are integrable functions and E is a measurable set the show that **[6]**

i) $\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g.$

ii) If $|h| \le |f|$ and h is a measurable then h is integrable.

iii) If $f \ge g$ a.e. then $\int f \ge \int g.$

P.T.O.

b) Define complete measure. Show that if μ is a complete measure and f is a measurable function, then $f = g$ a.e. implies g is measurable. **[6]**

c) Let $(X, \mathscr{B}, \mu)$ be a measure space and $Y \in \mathscr{B}$. Let $\mathscr{B}_Y$ consist of those sets of $\mathscr{B}$ that are contained in Y and $\mu_Y(E) = \mu(E)$ if $E \in \mathscr{B}_Y$. Then show that $(Y, \mathscr{B}_Y, \mu_Y)$ is a measure space. **[4]**

Q3) a) Define outer measure. Show that the outer measure of an interval equals its length. **[6]**

b) Show that for any set A and real number $\epsilon > 0$, there is an open set O containing A such that $\mu^*(O) \leq \mu^*(A) + \epsilon$. **[5]**

c) Let f be defined on [0,1] by $f(0) = 0$, $f(x) = x \sin 1/x$ for $x > 0$. Find the measure of the set $[x : f(x) \geq 0]$. **[5]**

Q4) a) State and prove Fatou's Lemma. **[6]**

b) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathscr{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there exist a unique measurable extended real valued function f on X such that $f \leq \alpha$ on B_α and $f \geq \alpha$ on $X \sim B_\alpha$. **[6]**

c) If f and g are non negative measurable functions and a and b are non negative constants, then show that $\int af + bg = a\int f + b\int g$. **[4]**

Q5) a) Let v be a signed measure on the measurable space $(X, \mathscr{B})$ then prove that there is a positive set A and a negative set B such that $X = A \cup B$ and $A \cap B = \phi$. **[6]**

b) Define Product Measure. Let E be a set $\mathbb{R}_{\sigma\delta}$ with $\mu \times v(E) < \infty$ then show that the function g defined by $g(x) = vE_x$ is a measurable function of x and $\int g d\mu = \mu \times v(E)$. **[6]**

c) Show that a step function defined on $[a, b]$ is of bounded variation on $[a, b]$. **[4]**

Q6) a) Let μ, v and λ be σ-finite. Show that the Radon-Nikodym derivative $\left[\frac{dv}{d\mu}\right]$ has the following properties : **[8]**

i) If $v \ll \mu$ and f is a non negative measurable function, then $\int f\,dv = \int f\left[\frac{dv}{d\mu}\right]d\mu$.

ii) $\left[\frac{d(v_1+v_2)}{d\mu}\right] = \left[\frac{dv_1}{d\mu}\right] + \left[\frac{dv_2}{d\mu}\right]$.

iii) If $v \ll \mu \ll \lambda$ then $\left[\frac{dv}{d\lambda}\right] = \left[\frac{dv}{d\mu}\right]\left[\frac{d\mu}{d\lambda}\right]$.

b) Let μ be a measure on an algebra $\mathscr{G}$ and μ^* the outer measure induced by μ. Then prove that the restriction $\bar{\mu}$ of μ^* to the μ^*- measurable sets is an extension of μ to σ-algebra containing $\mathscr{G}$. **[8]**

Q7) a) If μ is a measure on a ring $\mathscr{R}$ and the set function μ^* is defined on $\mathscr{H}(\mathscr{R})$ by $\mu^*(E) = \inf\,[\sum_{n=1}^{\infty}\mu(E_n) : E_n \in \mathscr{R},\ n = 1, 2, ..., E \subset \bigcup_{n=1}^{\infty} E_n]$. Then show that **[5]**

i) for $E \in \mathscr{R}$, $\mu^*(E) = \mu(E)$,

ii) μ^* is an outer measure on $\mathscr{H}(\mathscr{R})$.

b) If μ is a σ-finite measure on a ring $\mathscr{R}$ then show that it has unique extension to the σ-ring $\mathscr{S}(\mathscr{R})$. **[5]**

c) Let μ^* be a topologically regular outer measure on X then prove that each Borel set is μ^* - measurable. **[6]**

Q8) a) Define positive set. Show that **[8]**

i) every measurable subset of a positive set is itself positive.

ii) the union of a countale collection of positive sets is positive.

b) Let B be a μ^*-measurable set with $\mu^* B < \infty$ then prove that $\mu_* B = \mu^* B$. **[4]**

c) If E and F are disjoint sets then show that **[4]**

$$\mu_* E + \mu_* F \le \mu_* (E \cup F) \le \mu_* E + \mu^* F \le \mu^* (E \cup F) \le \mu^* E + \mu^* F.$$

Total No. of Questions : 8]

SEAT No. :

P1012

[Total No. of Pages : 2

[5428]-35
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT -705 : Graph Theory-I
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that the hypercube Q_k is bipartite and is k regular. Also find the number of edges in Q_k. **[6]**

b) Prove that a simple graph of girth 5 in which every vertex has degree at least k has at least $k^2 + 1$ vertices, with equality achieveable when $k \in \{2, 3\}$. **[5]**

c) Show that the Petersen graph has no cycle of length 7. **[5]**

Q2) a) Prove that K_n decomposes into two isomorphic (self-complementary) subgraphs if and only if n or $n - 1$ is divisible by 4. **[5]**

Draw all the non isomorphic simple graphs on a fixed set of four vertices.

b) Prove that every u, v-walk contains *a* u, v-path. **[5]**

c) Prove that a graph G is Eulerian if it has at most one non trivial component and its vertices all have even degree. **[6]**

Q3) a) Prove that a graph is connected if and only if for every partition of its vertices into two nonempty sets, there is an edge with endpoints in both sets. **[6]**

b) Show that if G is a simple n-vertex graph with $\delta(G) \geq \frac{(n-1)}{2}$, then G is connected. **[5]**

c) Show that the number of vertices in a self-complementary graph is either 4k or 4k + 1, where k is a positive integer. **[5]**

P.T.O.

Q4) a) Prove that a graph G is bipartite if and only if for every subgraph H of G, there is an independent set containing at least half of the vertices of H. **[7]**

b) Show that a simple graph is a forest if and only if every induced subgraph has a vertex of degree at most 1. **[3]**

c) Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph. **[6]**

Q5) a) Prove that the center of a tree is a vertex or an edge. **[7]**

b) Prove that deleting a vertex of maximum degree cannot increase the average degree, but deleting a vertex of minimum degree can reduce the average degree. **[3]**

c) Prove that if G is a simple graph with *diam* G ≥ 3, *then diam* $\bar{G} \leq 3$. **[6]**

Q6) a) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **[8]**

b) Prove that a graph with at least 3 vertices is connected if and only if at least two of the subgraphs obtained by deleting one vertex are connected. **[3]**

c) Prove that a weighted graph with distinct edge weights has a unique minimum-weight spanning tree. **[5]**

Q7) a) State and prove the Havel-Hakimi theorem. **[10]**

b) Find the minimum size of a maximal matching in C_n. **[6]**

Q8) a) Prove that in a connected weighted graph G, Kruskal's Algorithm constructs a minimum-weight spanning tree. **[6]**

b) Define clique number and independence number of a graph G with an example. **[2]**

c) Prove that every tree T has at most one perfect matching. **[3]**

d) Prove that if G is a connected graph, then an edge cut F is a bond if and only if $G - F$ has exactly two components. **[5]**

Total No. of Questions : 8]

SEAT No. :

P1013

[Total No. of Pages : 2

[5428]-41
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT - 801 : Field Theory
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions of the following.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $p(x)$ be an irreducible polynomial in F[x] then prove that there exists an extension E of F in which $p(x)$ has a root. **[8]**

b) Determine all cubic irreducible polynomials over $\mathbb{Z}_2$. **[4]**

c) Is $\mathbb{R}$ a normal extension of $\mathbb{Q}$? Justify. **[4]**

Q2) a) Show that degree of the extension of the splitting field of $x^3-2 \in \mathbb{Q}[x]$ is 6. **[8]**

b) Let F be a finite field then prove that the number of elements of F is P^n for some positive integer n and prime p. **[4]**

c) Show that $f(x) = x^p-x-1$ is irreducible over $\mathbb{Z}_p$. **[4]**

Q3) a) If E be a finite extension of a field F such that $E = F(\alpha)$ for some $\alpha \in E$ then prove that there are only a finite number of intermediate fields between F and E. **[6]**

b) Show that $\mathbb{Q}\left(\sqrt{2},\sqrt{3}\right) = \mathbb{Q}\left(\sqrt{2}+\sqrt{3}\right)$. **[5]**

c) Let E be the splitting field of a degree n over a field F. Show that $[E:F] \leq n!$. **[5]**

P.T.O.

Q4) a) If $E = \mathbb{Q}(\sqrt[3]{2}, w)$ be an extension of a field $\mathbb{Q}$ where $w^3=1$, $w \neq 1$ and $H = \{1, \tau_2\}$ be subgroup of $G(E/\mathbb{Q})$ then find E_H, **[8]**

Where $\sigma_2 : \begin{cases} \sqrt[3]{2} \longrightarrow \sqrt[3]{2}\, w^2 \\ w \longrightarrow w^2 \end{cases}$

b) State fundamental theorem of Galois theory. **[4]**

c) If E is a finite extension of a field F then prove that $|G(E/F)| \leq [E : F]$. **[4]**

Q5) a) If E be a finite normal separable extension of a field F then prove that F is the fixed field of G (E/F). **[6]**

b) Show that Galois group of $x^4 + 1 \in \mathbb{Q}[x]$ is the kelin four group. **[5]**

c) Find the group of $\mathbb{Q}$ - automorphism of $\mathbb{Q}(\sqrt[3]{2})$. **[5]**

Q6) a) Show that a finite field F of p^n elements has exactly one subfield with p^m elements for each divisor m of n. **[8]**

b) Show that a regular n-gon is constructible if and only if $\phi(n)$ is a power of 2. **[8]**

Q7) a) Prove that a finite extension of a finite field is separable. **[8]**

b) Let F be a field then prove that there exists an algebraically closed field K containing F as a subfield. **[8]**

Q8) a) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over $\mathbb{Q}$. **[8]**

b) If F contain a primitive n^{th} root ω of unity and E is a finite cyclic extension of degree n over F then prove that E is a splitting field of an irreducible polynomial $x^n - b \in F[x]$. **[8]**

Total No. of Questions : 8]

SEAT No. :

P1014

[Total No. of Pages : 3

[5428]-42
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT - 802 : Combinatorics
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) What is the probability of randomly choosing a permutation of the 10 digits 0, 1, 2, ---------- 9 in which **[6]**

i) An odd digit is in the first position and 1, 2, 3, 4 or 5 is in the last position.

ii) 5 is not in first position and 9 is not in the last position.

b) Prove by combinatorial argument that **[6]**

$$\binom{r}{r}+\binom{r+1}{r}+\binom{r+2}{r}+----+\binom{n}{r}=\binom{n+1}{r+1}$$

Hence evaluate the sum $1^2 + 2^2 +$ --------- $+ n^2$.

c) Find a rook polynomial for a full 5×5 board. **[4]**

Q2) a) How many numbers between 0 and 10,000 have a sum of digits equal to 13. **[6]**

b) How many arrangements of 'MISSISSIPPI' are there with no pair of consecutive S's? **[6]**

c) Solve the recurrence relation, assuming that n is a power of 2. **[4]**

$a_n = 2a_{n/2} + 5$ with $a_2 = 1$.

P.T.O.

Q3) a) How many ways are there to select 300 chocolate candies from seven types if each type comes in boxes of 20 and if at least one but not more than five boxes of each type are chosen. **[6]**

b) Find ordinary generating function. Whose coefficient a_r equals $3r^2$. Hence evaluate the sum $0 + 3 + 12 + \text{---------} + 3n^2$. **[6]**

c) How many arrangements of letters in REPETITION are there with the first E occurring before the first T? **[4]**

Q4) a) Use generating functions to find the number of ways to select 10 balls from a large pile of red, white and blue balls if the selection has at most two red balls. **[6]**

b) How many arrangements are there of 'MATHEMATICS' with both T's before both A's or both A's before both M's or both M's before the E? **[6]**

[By before, we mean any where before, not just immediately before]

c) Among 40 toy robots, 28 have a broken wheel or rusted but not both, 6 are not defective and the number with a broken wheel equals the number with rust. Find how many robots are rusted? **[4]**

Q5) a) How many ways are there to distribute eight different toys among four children if the first child gets atleast two toys? **[6]**

b) Using inclusion exclusion principle, find the number of ways to distribute 25 identical balls into 6 distinct boxes with at most 6 balls in any of the first three boxes. **[6]**

c) Solve the recurrence relation **[4]**

$a_n = a_{n-1} + 3n^2$ with $a_0 = 10$

Q6) a) Using generating functions, solve the recurrence **[6]**

relation $a_n = a_{n-1} + 2$ with $a_0 = 1$

b) How many ways are there to divide five pears, five apples, five doughnuts, five lollipops, five chocolate cats and five candy rocks into two (unordered) piles of 15 objects each? **[6]**

c) Show that in any set of n integers, $n \geq 3$ there always exists a pair of integers whose difference is divisible by $n-1$. **[4]**

Q7) a) Find recurrence relation for the number of n-digit ternary sequences with an even number of 0's and an even number of 1's. **[8]**

b) i) Solve the recurrence relation **[4]**

$a_n^2 = 2a_{n-1}^2 + 1$ with $a_0 = 1$.

ii) Show that if n + 1 distinct numbers are chosen from 1, 2, ------- 2n, then two of the numbers must always be consecutive integers. **[4]**

Q8) a) A computer dating service wants to match six men, denoted M_1, M_2, M_3, M_4, M_5, M_6 each with one of six women, denoted W_1, W_2, W_3, W_4, W_5, W_6. If man M_1 is incompatible with women W_2 and W_4 ; man M_2 is incompatible with W_1 and W_5 ; man M_3 is compatible with all womens ; man M_4 is incompatible with women W_2 and W_5 ; man M_5 is incompatible with woman W_4 and man M_6 is incompatible with woman W_6. How many matches of the six men are there? **[8]**

b) In a class of 30 children, 20 take Latin, 14 take Greek and 10 take Hebrew. If no child takes all three languages and 8 children take no language, how many children take Greek and Hebrew? **[8]**

Total No. of Questions : 8]

SEAT No. :

P1015

[Total No. of Pages : 2

[5428]-43
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT-803 : Differentiable Manifolds
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let M be an orientable k-manifold with non-empty boundary. Then prove that ∂M is orientable. **[8]**

b) State Green's theorem for compact, oriented 2-manifold. **[4]**

c) Define a closed form and give an example. **[4]**

Q2) a) Define orientation of a manifold M and induced orientation on ∂M. **[4]**

b) State Stokes' theorem. **[4]**

c) Let $\alpha : (0, 1)^2 \rightarrow \mathbb{R}^3$ be given by $\alpha\,(u, v) = (u, v, u^2 + v^2 + 1)$. Let Y be the image set of α . **[8]**

Evaluate $\int_Y (x_2 dx_2 \wedge dx_3 + x_1 x_3 dx_1 \wedge dx_3.)$

Q3) a) Define the differential operator d and for any k-form ω, show that $d\,(d\omega) = 0$. **[7]**

b) Show that $g\,(X, Y, Z) = \det \begin{pmatrix} x_i & y_i & z_i \\ x_j & y_j & z_j \\ x_k & y_k & z_k \end{pmatrix}$ is an alternating 3-tensor on $\mathbb{R}^n$. Further, express g as a combination of elementary tensors. **[6]**

c) Define volume of parametrized surface in $\mathbb{R}^n$. **[3]**

P.T.O.

Q4) a) Let U be an open set in $\mathbb{R}^n$ and $f : U \to \mathbb{R}^n$ be of class C^r. Let $M = \{x : f(x) = 0\}$ and $N = \{x : f(x) \geq 0\}$. If M is nonempty and $Df(x)$ has rank one at each point of M, then prove that N is an n-manifold in $\mathbb{R}^n$ and $\partial N = M$. **[8]**

b) Define an alternating tensor and give an example. **[4]**

c) Give an example of a manifold which can be covered by a single coordinate patch. **[4]**

Q5) a) Let F be a k-tensor. With usual notation, if $AF = \sum_{\sigma \in S_k} (\text{sign } \sigma) F^{\sigma}$, then prove that AF is an alternating tensor. Find AF if F is already alternating. **[7]**

b) If $\omega = x^2yzdx + xyzdy + ze^xydz$ and $\eta = yz \sin xdx + xyzdy + 2xyzdz$, then find $(\omega \wedge \eta)$. **[5]**

c) Define an exact form and give an example. **[4]**

Q6) a) Let M be a k-manifold in $\mathbb{R}^n$. If ∂M is nonempty, then prove that ∂M is a $k-1$ manifold without boundary. **[7]**

b) If $\omega = x^2yz^2dx + 2xz \cos ydy + xye^zdz$ find $d\omega$. **[5]**

c) Show that a unit n-ball $\mathbb{B}^n$ is an n-manifold in $\mathbb{R}^n$. What is its boundary? **[4]**

Q7) a) What is the dimension of $A^k(V)$, the space of alternating k-tensors on an n dimensional vector space V? Justify. **[8]**

b) Let $A = \mathbb{R}^2 - \{0\}$. If $\omega = \dfrac{xdx + ydy}{x^2 + y^2}$, then show that ω is closed and exact on A. **[8]**

Q8) a) If ω and η are k and l forms respectively, then prove that $d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$ **[8]**

b) Let $\omega = y^2zdx + x^2zdy + x^2ydz$, and $\alpha(u, v) = (u - v, uv, u^2)$. Find $\alpha^*(d\omega)$. **[8]**

Total No. of Questions : 8]

SEAT No. :

P1016

[Total No. of Pages : 2

[5428]-44
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT-804 : Algebraic Topology
(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define homotopy relation between two continuous functions. Prove that this relation is an equivalence relation. **[6]**

b) Let $f : S^1 \to X$ be a continuous map. Show that f is null homotopic if and only if there is a continuous map $g : B^2 \to X$ with $f = g|S^1$. **[5]**

c) Define a contractible space. Show that B^n is contractible. Give an example of a non-contractible space. **[5]**

Q2) a) Prove that the relation of being the same homotopy type is an equivalence relation. **[6]**

b) Define the retraction. Prove that S^1 is a retract of $\mathbb{R}^2 - \{0\}$. **[5]**

c) Show that a retract of a Housdorff space is a closed subset. **[5]**

Q3) a) Prove that every non-empty open connected subset of $\mathbb{R}^n$ is path connected. **[6]**

b) Define a path connected space. Prove that every path connected space is connected. **[5]**

c) Let f be a path. Prove that $f * \overline{f}$ and $\overline{f} * f$ are homotopic to null paths. **[5]**

P.T.O.

Q4) a) Define :

i) Fundamental group of a space

ii) Simply connected space

Prove that a contractible space is simply connected. **[6]**

b) Suppose A is a strong deformation retract of X. Show that the inclusion map $i : A \to X$ induces an isomorphism $i^*: \pi_1(A, a) \to \pi_1(X, a)$. **[5]**

c) Prove that the fundamental group of the torus is isomorphic to $\mathbb{Z}\oplus\mathbb{Z}$. **[5]**

Q5) a) Using the techniques of algebraic topology, prove that every non-constant complex polynomial has a root. **[6]**

b) Prove that the fundamental group of $S^1 \times \mathbb{R}$ is isomorphic to $\mathbb{Z}$. **[5]**

c) Determine the fundamental groups of $\mathbb{R} - \{0\}$ and $\mathbb{R}^n$. **[5]**

Q6) a) Let $p : \tilde{X} \to X$ be a fibration with the unique path lifting. Suppose that f and g are paths in $\tilde{X}$ with $f(0) - g(0)$ and pf is homotopic to pg relative to $\{0, 1\}$. Prove that f is homotopic to g relative to $\{0, 1\}$. **[6]**

b) Prove that the product of fibrations with unique path lifting is a fibration with unique path lifting. **[5]**

c) Let $p : E \to B$ be a fibration. Prove that $p(E)$ is a union of path components of B. **[5]**

Q7) a) Suppose X is a locally connected space. Prove that a continuous map $p : \tilde{X} \to X$ is a covering map if and only if for each component H of X. the map $p|p^{-1}(H) : p^{-1}(H) \to H$ is a covering map. **[6]**

b) Define a covering map. Show that the map $p : \mathbb{R} \to S^1$ defined by $p(t) = e^{2\pi i t}$ is a covering map. **[5]**

c) Give an example of a local homeomorphism that is not a covering map. **[5]**

Q8) a) Prove that the closed ball B^n has the fixed point property. **[6]**

b) Show that two different complexes may have the same polyhedron. **[5]**

c) Prove that if A is a compact subset of $\mathbb{R}^n$ then A has topological dimension at most n. **[5]**

Total No. of Questions : 8]

SEAT No. :

P1017

[Total No. of Pages : 2

[5428]-45
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT-805 : Lattice Theory
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*
3) *Figures to the right indicate full marks.*

Q1) a) Prove that a poset $< L; \leq >$ is a lattice if and only if inf (H) and sup (H) exist for any finite nonempty subset H of L. **[6]**

b) Let L be a lattice, let Θ be a congruence relation on L, and let L_1 be sublattice of L. If for every $a \in L$, there exists exactly one $b \in L_1$ satisfying $a \equiv \mathrm{b}\ (\Theta)$, then prove that $L/\Theta \cong L_1$. **[5]**

c) Prove that a nonempty subset I of a lattice L is an ideal if and only if, for $a, b \in L$, $a \vee b \in I$ is equivalent to $a, b \in I$. **[5]**

Q2) a) If inf (H) exists for all nonempty subset H of a poset P then prove that sup (H) also exists in P. **[6]**

b) Prove that every element of a finite distributive lattice has a unique irredundant representation as a join of join irreducible elements. **[5]**

c) If a poset satisfies DCC then prove that it has minimal element. **[5]**

Q3) a) Prove that any lattice L can be embedded in $I_d(L)$. **[8]**

b) Let I be an ideal and let D be dual ideal if $I \cap D \neq \phi$, then prove that $I \cap D$ is a convex sublattice and every convex sublattice can be expressed in this form in one and only one way. **[8]**

P.T.O.

Q4) a) Prove that the collection of normal subgroups of a group G is modular lattice under set inclusion. **[5]**

b) If in a lattice L a median for all $a, b, c \in L$ then prove that L is distributive. **[6]**

c) If L and K two lattices, then show that $L \times K$ is a lattice under componentwise meet and join. **[5]**

Q5) a) Prove that in any distributive lattice $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ together implies that $x = y$. **[4]**

b) Prove that a lattice L is distributive if and only if every element has at the most one relative complement in any interval containing it. **[6]**

c) Let L be distributive lattice, $a, b \in L$ and $a \neq b$, then prove that there exist a prime ideal P of L containing exactly one of a and b. **[6]**

Q6) a) Let L be finite distributive lattice. Prove that the map $\phi : L \rightarrow H(J(L))$ is an isomorphism. **[8]**

b) Prove that a lattice is Boolean if and only if it is isomorphic to field of sets. **[8]**

Q7) a) Prove that every maximal chain of a finite distributive lattice L is of length $|J(L)|$. **[5]**

b) State and prove Stone's Theorem. **[7]**

c) If L is pseudo-complemented lattice and $a \vee b \in S(L)$ then prove that $a \vee b = (a \vee b)^{**}$. **[4]**

Q8) a) Prove that every neutral element is standard. **[5]**

b) Let $p_i = q_i$, $0 \leq i < n$, be lattice identities. Then prove that there is a single identity $p = q$ such that all $p_i = q_i$, $0 \leq i < n$, hold in a lattice L if and only if $p = q$ holds in L. **[7]**

c) Show that $N_5 \simeq L \times K$ implies that L or K has only one element. **[4]**