Total No. of Questions : 8]

SEAT No. :

P2569

[Total No. of Pages : 2

[5021]-101

M.A./M.Sc. (Semester - I)

MATHEMATICS

MT - 501 : Real Analysis

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $X = \mathbb{R}^2$ and $d : X \times X \to \mathbb{R}$ defined by $d(x, y) = |x_1 - y_1| + |x_2 + y_2|$ where $x = (x_1, x_2), y = (y_1, y_2)$ then show that d is a metric on X. **[6]**

b) State and prove Cauchy-Schwarz's inequality. **[5]**

c) Define an inner product and state whether $\mathbb{R}^n$ and $\mathbb{C}^n$ are inner product spaces. **[5]**

Q2) a) Show that compact subsets of a metric space an closed. **[6]**

b) State and prove Heine-Borel theorem. **[5]**

c) Define a seperable space and verify whether $\mathbb{R}$ is seperable with **[5]**

i) usual metric?

ii) Discrete metric?

Q3) a) Let $A = [0,4]\times[1,10], B = [0,1]\times[0, 2]$ be subsets of $\mathbb{R}^2$. Draw pictures of S (A, B) and find D (A, B). **[6]**

b) Suppose $A, B \subseteq \mathbb{R}^n$ then with usual notations. Prove that D (A, B) = D (B, A). If D (A, B) = 0 then whether A = B? Justify. **[5]**

c) Let A, B, C be subsets of $\mathbb{R}^n$ then with usual notations prove that, $S(A,C) \subseteq S(A,B) \cup S(B,C)$ and $D(A,C) \subseteq D(A,B) + D(B,C)$. **[5]**

P.T.O.

Q4) a) Define an exterior measure of A subset of $\mathbb{R}^n$ and show that it is countably subadditive. **[6]**

b) With usual notations, prove that M_j is a ring. **[5]**

c) Define a measurable function on $\mathbb{R}^n$ and show that if f is measurable then |f| is also measurable. Whether converse is true? Justify. **[5]**

Q5) a) Prove that, every continuous function is measurable. **[6]**

b) Show that X_Q is lebesque integrable but not Riemann integrable function. **[5]**

c) Suppose f and g are Lebesque integrable then show that '~' as defined below is an equivalence relation. **[5]**

$f \sim g \Leftrightarrow f = g \ a.e.$

Q6) a) State and prove monotone convergence theorem for non-negative measurable functions. **[6]**

b) State fatou's lemma and show that strict inequality holds for fatou's lemma. **[5]**

c) Prove that $f = 0 \ a.e.$ on E $If \int_E f \ dm = 0 \forall E \in M$. **[5]**

Q7) a) Apply Gram-Scmidt process to $f_n(x) = x^n$ for n = 0, 1, 2, ... to obtain formulas for first three Legendre Polynomials. **[8]**

b) Show that the sequence $\frac{e^{inx}}{\sqrt{2\pi}}, n = 0, \pm 1, \pm 2, ...$ is completely orthonormal in $L^2[-\pi, \pi]$. **[8]**

Q8) a) State and prove Bessel's inequality. **[8]**

b) State and prove Riesz-Fischer theorem. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P2570

[Total No. of Pages : 4

[5021]-102

M.A./M.Sc. (Semester - I)

MATHEMATICS

MT - 502 : Advanced Calculus

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let the partial derivatives D_1 f, ... D_n f of scalar field f exist in some n-ball $B(\overline{a})$ and are continuous at $\overline{a}$. Prove that f is differentiable at $\overline{a}$. **[8]**

b) Let z be a function of x and y and $x = u^2 + v^2 - 2uv, y = u + v$.

Compute $(u+v)\frac{\partial z}{\partial u} + (u-v)\frac{\partial z}{\partial v}$. **[4]**

c) Find the directional derivative of the scalar field

$f(x, y, z) = x^2 + 2y^2 + 3z^2$ at $(1, 1, 0)$ in the direction $\overline{i} - \overline{j} + 2\overline{k}$. **[4]**

Q2) a) Let $\overline{f}, \overline{g} : S \to \mathbb{R}^m$ where $S \subset \mathbb{R}^n$ be a vector functions and $\overline{a} \in \mathbb{R}^n$. Let

$\lim_{\overline{x}\to\overline{a}} \overline{f}(\overline{x}) = \overline{b}$ and $\lim_{\overline{x}\to\overline{a}} \overline{g}(\overline{x}) = \overline{c}$ then prove that

$\lim_{\overline{x}\to\overline{a}} \left[\overline{f}(\overline{x}) \cdot \overline{g}(\overline{x})\right] = \overline{b} \cdot \overline{c}$ **[6]**

b) State only the chain rule for derivatives of vector fields in matrix form and explain the terms involved. **[5]**

c) Let the temperature of a thin plate is described by a scalar field f, the temperature at (x, y) is $f(x,y)$ Let $x = r\cos\theta, y = r\sin\theta$ so the temperature is determined as the function of r and θ

$\phi(r,\theta) = f(r\cos\theta, r\sin\theta)$.

P.T.O.

Express the partial derivatives $\frac{\partial \phi}{\partial r}$ and $\frac{\partial \phi}{\partial \theta}$ in terms of partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. **[5]**

Q3) a) Define a line integral of a vector field along the curve. Illustrate by an example that the value of the line integral is independent of the parametric representation of the curve. **[6]**

b) Calculate the line integral of vector field $\overline{f}$ along the path described by $\overline{f}(x, y) = (x^2 - 2xy)\overline{i} + (y^2 - 2xy)\overline{j}$ from (–1, 1) to (1, 1) along the parabola $y = x^2$. **[5]**

c) Show that the work done by the constant force depends only on the endpoints and not on the curve joining them. **[5]**

Q4) a) State and prove second fundamental theorem for line integrals. **[8]**

b) Let $\overline{f} = (f_1, ..., f_n)$ be a continuously differentiable vector field on an open set S in $\mathbb{R}^n$. If $\overline{f}$ is gradient on S, then prove that the partial derivatives of the component of $\overline{f}$ are related by the equations $D_i f_j(\overline{x}) = D_j f_i(\overline{x})$ for $ij = 1, ..., n$ and every $\overline{x}$ in S. **[5]**

c) Determine whether or not vector field $\overline{f}(x, y) = 3x^2 y\overline{i} + x^3 y\overline{j}$ is a gradient on any open subset of $\mathbb{R}^2$. **[3]**

Q5) a) Let P and Q be scalar fields that are continuously differentiable on an open set S in the xy-plane. Let C be a piecewise smooth Jordan curve, and let R denote the union of C and its interior. Suppose R is of special type $R = \{(x, y) \mid a \le x \le b \text{ and } f(x) \le y \le g(x)\}$. Where f and g are continuous on [a, b] with $f \le g$ and assume R is subset of S. Prove that $\iint_R \frac{\partial P}{\partial y} dxdy = \oint_C Pdx$ where the line integral is taken around C in the counter clockwise direction. **[8]**

b) Evaluate $\iint_{\theta} xy(x+y)dx\,dy$ where $\theta = [0,1]\times[0,1]$. **[4]**

c) Transform the integral to one or more iterated integrals in polar co-ordinate $\int_0^1 \left[\int_0^1 f(x,y)dy\right] dx$. **[4]**

Q6) a) In usual notation prove the transformation formula

$$\iint_S f(x,y)dxdy = \iint_T f\left(X(u,v), Y(u,v)\right) \, |J(u,v)| \, dudv.$$

Where $f(x,y) = 1$ on rectangle S. **[8]**

b) Use the Green's theorem to evaluate the line integral $\oint y^2 dx + xdy$ where C is the square (0, 0), (2, 0), (2, 2), (0, 2). **[4]**

c) Evaluate $\iiint_S xy^2 z^3 dx\,dy\,dz$ where S is the solid bounded by the surface $z = xy$ and the plane $y = x \;\; x = 1 \;\; z = 0$. **[4]**

Q7) a) Define the fundamental vector product show that the fundamental vector product for the surface $z = f(x,y)$ is never zero. **[6]**

b) Define the surface integral and explain the terms involved in it. **[5]**

c) A parametric surface is described by the vector equation.

$\overline{r}(u,v) = u\cos v\overline{i} + u\sin v\overline{j} + u^2\overline{k}$ where $0 \le u \le 4$ and $0 \le v \le 2\pi$. Compute the area of the surface. **[5]**

Q8) a) State and prove the Gauss divergence theorem. **[8]**

b) Let S be the surface of the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$, and let $\overline{n}$ be the unit outer normal to S. If $\overline{F}(x, y, z) = x^2\overline{i} + y^2\overline{j} + z^2\overline{k}$, use the divergence theorem to evaluate the surface integral $\iint_S \overline{F} \cdot \overline{n}\, ds$. **[4]**

c) Give an example of vector field with zero divergence and zero curl. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P2571

[Total No. of Pages : 3

[5021]-103
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 503 : Linear Algebra
(2008 Pattern) (Old)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *V denotes a finite dimensional vector space over the field K.*

Q1) a) Prove that a linearly independent subset of V can be extended to form a basis of V. Find a basis of $\mathbb{R}^4$ consisting of the vector $[1\ 0\ 1\ 0]^t$. **[6]**

b) Let T be a nilpotent operator on V over K. Show that $T^n = 0$, where n is the dimension of V. **[5]**

c) Find a basis and the dimension of the vector space over $\mathbb{R}$ consisting of all polynomials $p(x)$ of degree at most 7 such that $p(-x) = p(x)$. **[5]**

Q2) a) Let V and V' be vector spaces over K. Prove that V is isomorphic to V' if and only if $\dim V = \dim V'$ **[6]**

b) Let $T : \mathbb{R}[x] \to \mathbb{R}[x]$ defined by $T(p(x)) = xp(x)$. Show that T is a linear operator on $\mathbb{R}[x]$. Also show that if D is the differential operator on $\mathbb{R}[x]$. Then $DT - TD = I$. **[5]**

c) State the dimension formula for subspaces. Using it prove that if W_1 and W_2 are subspaces of finite dimensional vector space V with

$$\dim W_1 + \dim W_2 > \dim V, \text{then } W_1 \cap W_2 \neq \{0\}.$$ **[5]**

P.T.O.

Q3) a) Show that two similar matrices have the same characteristic polynomials. Is the converse true? Justify your answer. **[6]**

b) Let T be an operator on V with $T^2 = 2016\, T$. Show that T is a projection on im T along ker T. **[5]**

c) Prove that if T is a linear operator on a finite dimensional vector space V such that rank $T = 1$ then det $(I + T) = 1 + \text{tr } T$. **[5]**

Q4) a) State the Cayley-Hamilton Theorem. Let A be a 8×8 matrix with minimal polynomial $x(x+1)^2(x+2)^3$. What are the possibilities for the characteristic polynomial of A? **[6]**

b) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(x, y, z) = (x + y, y + z)$. Find the matrix of T with respect to the bases {(1, 0, 0), (0, 1, 0), (0, 0, 1)} and {(1, 1), (1, –1)}. **[5]**

c) Let D be the differential operator on $\mathbb{R}_3[x]$. Find the matrix of D with respect to the basis $\{1, x, x^2, x^3\}$. Find eigenvalues and eigenvectors of D. **[5]**

Q5) a) Define the geometric multiplicity and algebraic multiplicity of an eigenvalue of a linear operator. Prove that geometric multiplicity of an eigenvalue can not exceed its algebraic multiplicity. **[6]**

b) Write all possible Jordan canonical forms of the matrix whose characteristic polynomial is $(x^2 - 4)^2$. **[5]**

c) Find a matrix whose characteristics polynomial is $1 - x + x^2 - x^3 + x^4$. What is the minimal polynomial of this matrix? **[5]**

Q6) a) State and prove Riesz representation theorem. **[6]**

b) Define an inner product space. Prove that $|< u, v >| \leq \| u \| \; \| v \|$ for all $u, v \in V$ where V is an inner product space. **[5]**

c) Give an example of a triangulable matrix which is not diagonalizable. Show that if T is diagonalizable, then V is the direct sum of ker T and im T. **[5]**

Q7) a) State and prove the Schur's theorem for triangulable operators. **[6]**

b) Prove the existence and uniqueness of the adjoint of a linear operator T on a finite dimensional vector space V. **[5]**

c) Let T, T_1, T_2 be operators on V such that $T_1 = (T + T^*)/2$ and $T_2 = (T - T^*)/2$. Prove that T is normal if and only if $T_1T_2 = T_2T_1$. **[5]**

Q8) a) Let T be a triangulable operator on a finite dimensional inner product space V on $\mathbb{C}$. Prove that T is normal if and only if V has an orthonormal basis consisting of eigenvectors of T. **[6]**

b) Let A be a skew-symmetric matrix of order n over $\mathbb{R}$. Show that $I_n + A$ is invertible. Also show that $(I_n - A)(I_n + A)^{-1}$ is orthogonal. **[5]**

c) Prove that a bilinear form is reflexive if and only if it is either symmetric or alternating. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2572

[Total No. of Pages : 3

[5021]-104

M.A./M.Sc. (Semester - I)

MATHEMATICS

MT - 504 : Number Theory

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove Wilson's lemma. **[6]**

b) Solve the congruence $x^3 + 4x + 8 \equiv 0 \pmod{15}$. **[5]**

c) Show that 1387 is composite. **[5]**

Q2) a) State and prove Gaussion reciprocity law. **[6]**

b) Prove that for every positive integer n, $\sum_{d|n} \phi(d) = n$. **[5]**

c) Prove that $101x + 37y = 3819$ has a positive solution in integers. **[5]**

Q3) a) Let p be an odd prime and g.c.d. $(a, p) = 1$. Consider the integer a, $2a$, $3a$, ..., $\left\{\frac{(p-1)}{2}\right\}$ a and their least positive residues modulo p. If n denotes the number of these residues that exceed p|2, then prove that $\left(\frac{a}{p}\right) = (-1)^n$. **[6]**

b) Prove that for every integer n, $\sum_{d|n} |\mu(d)| = 2^{w(n)}$. **[5]**

P.T.O.

c) Let x and y be real numbers. Then prove that **[5]**

i) $[x]+[y] \le [x+y] \le [x]+[y]+1$

ii) $\left[\frac{[x]}{m}\right] = \left[\frac{x}{m}\right]$, if m is a positive integer.

Q4) a) State and prove Chinese remainder theorem. **[6]**

b) Find the highest power of 15 that divides 1000! **[5]**

c) Let $f(n)$ be a multiplicative function and let $F(n) = \sum_{d|n} f(d)$. Then prove that $F(n)$ is multiplicative. **[5]**

Q5) a) If m_1 and m_2 denote two positive, relatively prime integers, then prove that **[6]**

$\phi(m_1 m_2) = \phi(m_1)\, \phi(m_2)$

b) Determine whether $x^2 - 31 \equiv 0 (\text{mod}\, 103)$ is solvable. **[5]**

c) Show that $61! + 1 \equiv 0 \ (\text{mod } 71)$. **[5]**

Q6) a) If α and β are algebraic numbers, then prove that $\alpha + \beta$ and $\alpha\beta$ are algebraic numbers. Also prove that if α and β are algebraic integers, so are $\alpha + \beta$ and $\alpha\beta$. **[8]**

b) Prove that 1 + i is a prime in $Q_{(i)}$. **[4]**

c) If α is algebraic of degree n, then prove that α^{-1} and $-\alpha$ are also algebraic of degree n. **[4]**

Q7) a) Show that the fields $Q\left(\sqrt{m}\right)$, for $m = -1, -2, -3, -7, 2, 3$ are Euclidean and so have the unique factorization property. **[8]**

b) Let $Q\left(\sqrt{m}\right)$ have the unique factorization property. Then prove that any rational prime p is either a prime Π of the field or a product of Π_1, Π_2 of two primes, not necessarily distict of $Q\left(\sqrt{m}\right)$. **[4]**

c) Find the minimal polynomial of $\left(1+\sqrt[3]{7}\right)/2$. **[4]**

Q8) a) If ξ is an algebraic number of degree n, then prove that every number in Q(ξ) can be written uniquely in the form $a_0 + a_1\xi + a_2\xi^2 + \ldots\ldots\ldots\ldots + a_{n-1}\xi^{n-1}$, where a_i are rational numbers. **[6]**

b) Let p be an odd prime. If there is an integer x such that $p \mid x^2 - 2$, then prove that $p \equiv 1 \text{ or } 7 \pmod 8$. **[5]**

c) Show that there is no x for which both $x \equiv 29 \pmod{52}$ and $x \equiv 19 \pmod{72}$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2573

[Total No. of Pages : 3

[5021]-105

M.A./M.Sc. (Semester - I)

MATHEMATICS

MT - 505 : Ordinary Differential Equations

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **[6]**

b) Find the general solution of differential equation $y'' + 3y' - 10y = 6e^{4x}$ by using method of undetermined coefficients. **[5]**

c) Verify that $y_1 = x$ is one solution of $x^2y'' + 2xy' - 2y = 0$ and find y_2 and general solution. **[5]**

Q2) a) If $y_1(x)$ and $y_2(x)$ are two solutions of equation $y'' + P(x)y' + Q(x)y = 0$ on interval [a, b], then prove that $y_1(x)$ and $y_2(x)$ are linearly dependent on this interval if and only if their Wronskian W $(y_1 y_2)$ is identically zero. **[8]**

b) Find the particular solution of $y'' + 2y' + 5y = e^{-x} \sec 2x$ by using method of variation of parameter. **[8]**

Q3) a) State and prove sturm comparison theorem. **[8]**

b) Find the power series solution of the differential equation $(1 + x^2)y'' + xy' + y = 0$. **[8]**

P.T.O.

Q4) a) Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$ where $q(x) > 0$ for all $x > 0$. If $\int_1^\infty q(x)dx = \infty$, then prove that $u(x)$ has infinitely many zeros on the positive x-axis. **[8]**

b) Verify that origin is regular singular point and calculate two independent Frobenius series solution for the equation $2x^2y'' + x(2x+1)y' - y = 0$. **[8]**

Q5) a) Find the general solution of the system **[8]**

$$\frac{dx}{dt} = 3x - 4y$$

$$\frac{dy}{dt} = x - y$$

b) Prove that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if a > 0 and $b^2 - 4ac < 0$. **[4]**

c) Find the critical points of **[4]**

$$\frac{dx}{dt} = y^2 - 5x + 6$$

$$\frac{dy}{dt} = x - y$$

Q6) a) Find the general solution $(1-e^x)y'' + \frac{1}{2}y' + e^x y = 0$ near the singular point $x = 0$. **[8]**

b) Prove that $e^x = \lim_{b\to\infty} \mathrm{F}\left(a, b, a, \frac{x}{b}\right)$. **[4]**

c) Show that e^x and e^{-x} are linearly independent solutions of $y'' - y = 0$ on any interval. **[4]**

Q7) a) Find the general solution near $x = 0$ of the hypergeometric equation.

$x(1-x)y'' + [c-(a+b+1)x]y' - ab\,y = 0$, where a, b and c are constants. **[8]**

b) If m_1 and m_2 are roots of the auxiliary equation of the system

$$\frac{dx}{dt} = a_1 x + b_1 y$$

$$\frac{dy}{dt} = a_2 x + b_2 y$$

which are real, distict and of the same sign, then prove that the critical point (0,0) is node. **[8]**

Q8) a) Show that the function $f(x, y) = xy^2$ satisfies Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le b$; but it does not satisfy a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y \infty$. **[8]**

b) Find the exact solution of intial value problem $y^1 = y^2, y(0) = 1$, satarting with $y_0(x) = 1$. Apply Picard's method to calculate $y_1(x),\ \ y_2(x),\ \ y_3(x)$ and compare it with the exact solution. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P2574

[Total No. of Pages : 3

[5021]-201

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 601 : General Topology

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\bigcap_\alpha \tau_\alpha$ is a topology on X. Is $\bigcup_\alpha \tau_\alpha$ a topology on X? Justify. **[8]**

b) Let B and B' be bases for the topologies τ and τ^1, respectively on X. Then prove that the following are equivalent : **[5]**

i) τ^1 is finer than τ.

ii) For each $x \in X$ and each basis element $B \in B$ containing x, there is a basis element $B' \in B'$ such that $x \in B' \subset B$.

c) Define : **[3]**

i) Discrete topology

ii) Indiscrete topology

iii) Finite complement topology

Q2) a) If B is a basis for the topology of X and C is a basis for the topology of Y, then prove that the collection $D = \{B \times C \mid B \in B \text{ and } C \in C\}$ is a basis for the topology of X×Y. **[8]**

b) If τ and τ^1 are topologies on X and τ' is strictly finer than τ, what can you say about the corresponding subspace topologies on the subset Y of X? **[5]**

P.T.O.

c) If X and Y are topological spaces and there is a product topology an $X \times Y$, then prove that the projection map $\pi_1 : X \times Y \to X$ is an open map. **[3]**

Q3) a) Let $A \subset X$ and $B \subset Y$, show that in the space $X \times Y$, $\overline{A \times B} = \overline{A} \times \overline{B}$. **[8]**

b) Let X be a space satisfying the T_1 axiom; let A be a subset of X. Prove that the point $x \in X$ is a limit point of A if and only if every neighborhood of x contains infinitely many points of A. **[5]**

c) Show that a subspace of a Hausdorff space is Hausdorff. **[3]**

Q4) a) Let X and Y be topological spaces; Let $f : X \to Y$. Prove that the following are equivalent : **[8]**

i) f is continuous

ii) For every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

iii) For every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

b) State and prove the pasting lemma. **[5]**

c) Let Y be an ordered set in the order topology. Let $f, g : X \to Y$ be continuous. Show that the set $\{x \mid f(x) \le g(x)\}$ is closed in X. **[3]**

Q5) a) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α Let $\prod X_\alpha$ have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous. **[8]**

b) Let $P : X \to Y$ be a continuous map. Show that if there is a continuous map $f : Y \to X$ such that p of equals the identity map of Y, then p is a quotient map. **[5]**

c) Define

i) quotient map,

ii) open map

Give an example of quotient map that is not open map. **[3]**

Q6) a) Prove that a finite cartesian product of connected spaces is connected. **[8]**

b) Whether there exists a space that is connected but not path connected? Justify. **[5]**

c) Show that the continuous image of path connected space is also path connected. **[3]**

Q7) a) Show that every compact subspace of a Hausdorff space is closed. **[8]**

b) State and prove the tube lemma. **[8]**

Q8) a) Prove that a product of regular spaces is regular. **[8]**

b) Show that every metrizable space is normal. **[6]**

c) State Tychonoff theorem. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2575

[Total No. of Pages : 3

[5021]-202

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 602 : Differential Geometry

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove Lagrange's Multiplier theorem for n-surface. **[6]**

b) Show that surface of revolution of the function $f(x_1, x_2) = -x_2$ is a 2 - surface. **[5]**

c) Show that gradient of a function f at $P \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at P. **[5]**

Q2) a) Let S be a n-surface which is expressed as level set of regular points of smooth function on an open set U. With smooth vector field $\overline{X}$. Restriction of $\overline{X}$ to S is tangent vector field on S. If $\overline{\alpha} : \mathrm{I} \to \mathrm{U}$ is any integral curve of $\overline{X}$ such that $\overline{\alpha}$ (to) $\in \mathrm{S}$ for some to $\in \mathrm{I}$, then $\overline{\alpha}(\mathrm{t}) \in \mathrm{S}$ for all $\mathrm{t} \in \mathrm{I}$. **[6]**

b) Show that the unit n - sphere $x_1^2 + x_2^2 + ... + x_n^2 = r^2$ is connected for $n > 1, \ r > 0$. **[5]**

c) Describe the spherical image of the paraboloid $-x_1 + x_2^2 + x_3^2 + ... + x_{n+1}^2 = 0$ where n - surface is oriented by $-\nabla f / \| \nabla f \|$. **[5]**

P.T.O.

Q3) a) Show that Gauss map of compact, connected and oriented n - surface in $\mathbb{R}^{n+1}$ is onto unit sphere S^n. **[8]**

b) Show that n - plane is an n - surface. **[4]**

c) Sketch the following vector fields on $\mathbb{R}^2 : \bar{X}(P) = (P, X(P))$. Where **[4]**

i) $\bar{X}(P) = P/2$

ii) $\bar{X}(x_1, x_2) = \left(2x_1, \frac{1}{2}x_2\right)$

Q4) a) Show that for n - surface S in $\mathbb{R}^{n+1}, p \in S, \bar{v} \in Sp$, there exists an open interval I containing O and a maximal geodesic in S passing through p with initial velocity $\bar{v}$. **[6]**

b) Show that if $\bar{\alpha} : I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then $\ddot{\bar{\alpha}}(t) \perp \dot{\bar{\alpha}}(t)$ for all $t \in I$. **[5]**

c) For a 2-surface S in $\mathbb{R}^3$ and $\bar{\alpha} : I \to S$ a geodesic in S, with $\dot{\bar{\alpha}} \neq 0$, show that a vector field $\bar{X}$ tangent to S along α is parallel along α iff both $\|\bar{X}\|$ and angle between $\bar{X}$ and $\dot{\bar{\alpha}}$ are constant along $\bar{\alpha}$. **[5]**

Q5) a) Show that parallel transport $P_\alpha : S_p \to S_q$ where S is n-surface in $\mathbb{R}^{n+1}; p,q \in S$. α is peicewise smooth parametrized curve from p to q is a one one, onto, linear map which preserves the dot products. **[6]**

b) Define Weingarten map. Compute the same for the n - surface oriented by outward normal where n - surface is unit n - sphere. **[5]**

c) Show that Weingarten map is self adjoint. **[5]**

Q6) a) Define curvature of plane curve and compute the same for curve given by the equation $x_2 - ax_1^2 = c, \quad a \neq 0.$ **[6]**

b) Show that for oriented plane curve S, there exists a global parametrization of S iff S is connected. **[10]**

Q7) a) Show by an example that the integral of an exact 1-form over a compact connected oriented plane curve is always zero. **[6]**

b) Compute the principal curvatures of the hyperboloid $-x_1^2 + x_2^2 + x_3^2 = 1$ in $\mathbb{R}^3$, oriented by $\bar{N}(P) = \left(P, \frac{-x_1}{\|P\|}, \frac{-x_2}{\|P\|}, \frac{-x_3}{\|P\|}\right)$. **[5]**

c) Show that on each compact oriented n - surface S in $\mathbb{R}^{n+1}$, there exists a point p such that the second fundamental form at p is definite. **[5]**

Q8) a) Compute the Gaussian curvature of the ellipsoid $x_1^2 + \frac{x_2^2}{9} + \frac{x_3^2}{16} = 1$ oriented by outward normal at $P \equiv (1,9,16)$. **[5]**

b) State and prove Inverse function theorem for n - surfaces. **[5]**

c) Compute the integral curve through P = (1, 1) and P = (a, b) for the vector field given by the equation $\bar{X}(x_1, x_2) = (-x_2, x_1)$. **[6]**

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Total No. of Questions : 8]

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[5021]-203

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-603: Groups and Rings

(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that set of 2×2 real matrices forms a group under addition. Is this set a group under matrix multiplication? Justify. **[5]**

b) Prove that in a group there exists unique identity element. Further prove that for every element there exists unique inverse in that group. **[5]**

c) Define a cyclic group. Define as abelian group. Is every abelian group cyclic? Justify. **[6]**

Q2) a) Prove that $(\mathbb{Z},+)$ and $(\mathbb{Q}^*,.)$ are not isomorphic. Also, prove that $(\mathbb{Z}_4,+)$ and $(\mathbb{Z}_2 \oplus \mathbb{Z}_2,+)$ are not isomorphic. **[5]**

b) Give examples of two non isomorphic groups of order 6 with justification. **[5]**

c) Suppose that H is a proper subgroup of $\mathbb{Z}$ under addition and H contains 18, 30 and 40. Determine H. **[6]**

Q3) a) Define Center of a group G. Prove that the center of a group is a normal subgroup of G. **[5]**

b) List all the six cyclic subgroups of U(15). **[5]**

c) Find $Aut(\mathbb{Z}_6)$, the group of automorphisms of $\mathbb{Z}_6$. **[6]**

P.T.O.

Q4) a) Find the inverse and the order of each of the following permutations in S_{14}
(i) (6 14 4 7) (3 2 1 5) (ii) (5 10 4) (8 2 9) (13 7 12) **[5]**

b) Find the group of inner automorphism of dihedral group D_4 i.e. find $Inn(D_4)$. **[5]**

c) State and prove the Lagrange's theorem for finite groups. Is the converse of the theorem true? Justify. **[6]**

Q5) a) State and prove Cayley's theorem. **[5]**

b) Prove that the group of complex numbers under addition is isomorphic to $\mathbb{R} \oplus \mathbb{R}$. **[5]**

c) If $\tau = (7\ 9\ 4)\ (5\ 1)$, $\rho = (7\ 4\ 8\ 6\ 5\ 1)\ (3\ 2\ 9) \in S_9$. Then find $\tau^{-1}\rho\tau$ and $\rho^{-1}\tau\rho$. **[6]**

Q6) a) Let Z be the center of a group G and *Inn*(G) be the group of all inner automorphisms of G. Prove that $G/Z \approx Inn(G)$. **[5]**

b) Determine all the homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{24}$. **[5]**

c) Find all the non isomorphic abelian groups of order 1400. **[6]**

Q7) a) Determine all the groups of order atmost 5. **[5]**

b) Define the index of a subgroup of a group G. Prove that a subgroup of G with index 2 is normal in G. **[5]**

c) If H is a subgroup of a finite group G and |H| is a power of a prime *p*, then prove that H is contained in some Sylow *p*–subgroup of G. **[6]**

Q8) a) Prove that the number of Sylow *p*-subgroup of a group G is equal to 1 modulo *p* and divider |G|. **[5]**

b) Prove that a non cyclic group of order 21 has 14 elements of order 3. **[5]**

c) Prove that the groups of order 26 and 51 are not simple. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2577

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[5021]-204

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 604: Complex Analysis

(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let f be analytic in B(a; R), then show that $f(z)=\sum_{n=0}^{\infty}a_n(z-a)^n$ for $|z-a|<R$, where $a_n=\frac{1}{n!}f^{(n)}(a)$ and also show that this series has radius of convergence $\geq R$. **[7]**

b) Let z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$. Then prove that the cross ratio (z_1, z_2, z_3, z_4) is a real number if and only if all four points lie on a circle. **[6]**

c) If $f(z)$ is analytic and $|f(z)|$ is constant in a domain D, then prove that $f(z)$ is constant in D. **[3]**

Q2) a) Let G be an open subset of $\mathbb{C}$ and $f:G\to\mathbb{C}$ be an analytic function. If γ is a closed rectifiable curve in G such that $n(\gamma;\omega)=0$ for all $\omega\in\mathbb{C}-G$, then prove that for $a\in G-\{\gamma\}$ **[8]**

$$n(\gamma;a)f(a)=\frac{1}{2\pi i}\int_\gamma\frac{f(z)}{(z-a)}dz.$$

b) Let D = $\{z : |z| < 1\}$. Find all Möbius transformations T such that T(D) = D. **[5]**

c) Define the concept of winding number of a closed rectifiable curve γ in $\mathbb{C}$ for $a\notin\{\gamma\}$ and prove that it is an integer. **[3]**

P.T.O.

Q3) a) Let G_1 and G_2 be open sets in $\mathbb{C}$. If f and g are analytic on G_1 and G_2 respectively and $f(G_1) \subseteq G_2$, then prove that gof is analytic on G_1 and $(gof)'(z) = g'(f(z))f'(z)$ for all $z \in G_1$. **[7]**

b) Prove that every non-constant polynomial has at least one zero in $\mathbb{C}$. **[5]**

c) Let G be a region and let f and g be analytic functions on G such that $f(z)g(z) = 0$ for all $z \in G$. Show that either $f \equiv 0$ or $g \equiv 0$. **[4]**

Q4) a) If f has an isolated singularity at a, then prove that the point $z = a$ is a removable singularity if and only if $\lim_{z \to a}(z-a)f(z) = 0$. **[7]**

b) Show that the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$ is 1 and discuss convergence for $z = i$. **[5]**

c) Does there exist a function $f(z)$ analytic in $|z| < 1$ and satisfying $f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{2n}$?, $(n = 1, 2, ...)$. Justify your answer. **[4]**

Q5) a) Let G be a region set and $f : G \to \mathbb{C}$ be a continuous function such that $\int_T f = 0$ for every triangular path T in G then prove that f is analytic in G. **[7]**

b) For a given power series $\sum_0^{\infty} a_n z^n$, define the number R, $0 \leq R \leq \infty$, by $\frac{1}{R} = lim\ sup |a_n|^{\frac{1}{n}}$. Prove that, if $0 < r < R$ then the series converges uniformly on $\{z : |z| \leq r\}$. **[6]**

c) With the help of Argument Principle, find $\int_{\left|z-\frac{\pi}{2}\right|=1} \tan z \, dz$. **[3]**

Q6) a) If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \geq |f(z)|$ for all $z \in G$, then prove that f is constant. **[6]**

b) Let f be analytic in the disk B(a; R) and suppose that γ is a closed rectifiable curve in B(a; R). Then prove that $\int_\gamma f = 0$. **[4]**

c) State and prove Casorati-Weierstrass theorem. **[6]**

Q7) a) Let G be a region and suppose that f is a non constant analytic function on G. Then show that for any open set U in G, f(U) is open. **[7]**

b) Let f be analytic on B(0, 1) and suppose $|f(z)| \leq 1$ for $|z| < 1$. Show that $|f'(0)| \leq 1$. **[5]**

c) Using Rouche's Theorem, determine the number of zeros of the polynomial $P(z) = z^{10} - 6z^9 - 3z + 1$ inside the unit circle $|z| = 1$. **[4]**

Q8) a) Let $z = a$ be an isolated singularity of f and let $f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$ be its Laurrent series expansion in $ann(a; 0, R)$. Then prove that $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \leq -1$. **[3]**

b) Let $G = C - \{0\}$ and show that every closed curve in G is homotopic to a closed curve whose trace is contained in $\{z : |z| = 1\}$. **[5]**

c) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. Then prove that the following conditions are equivalent : **[8]**

i) $f \equiv 0$;

ii) there is a point a in G such that $f^{(n)}(a) = 0$ for each $n \geq 0$;

iii) $\{z \in G \mid f(z) = 0\}$ has a limit point in G.

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Total No. of Questions : 8]

SEAT No. :

P2578

[Total No. of Pages : 3

[5021]-205

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-605: Partial Differential Equations

(2008 Pattern) (Old Course)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Find PDE by eliminating f and g from $u = f(x+ay) + g(x-ay)$. **[5]**

b) Solve the PDE $p^2 + q^2 = m^2$. **[5]**

c) Let $u(x, y)$ and $\nu(x, y)$ be two functions of x and y such that $\frac{\partial \nu}{\partial y} \neq 0$. If, further $\frac{\partial(u,\nu)}{\partial(x,y)} = 0$, then prove that there exists a relation $F(u,\nu) = 0$ between u and ν not involving x and y explicity. **[6]**

Q2) a) Show that the equations $f = xp - yq - x = 0$ and $g = x^2 p + q - xz = 0$ are compatible and find the one parameter family of common solutions. **[8]**

b) Classify the following PDEs as linear/semilinear/quasilinear/nonlinear. **[4]**

i) $yp - xq = xyz + x$.

ii) $e^x p - xyq = xz^2$.

iii) $(x^2 + z^2)p - xyq = z^3 x + y^2$.

iv) $pq = z$.

c) Verify that the PDE $(1+yz)dx + z(z-x)dy - (1+xy)dz = 0$ is integrable. **[4]**

P.T.O.

Q3) a) Prove the following theorem : **[8]**

If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$, then h_1 and h_2 satisfy

$$\frac{\partial(f,h)}{\partial(x,u_x)}+\frac{\partial(f,h)}{\partial(y,u_y)}+\frac{\partial(f,h)}{\partial(z,u_z)}=0, \text{ where } h = h_i\ (i = 1, 2).$$

b) Find the complete integral of $p^2x + q^2y = z$ by Jacobi method. **[8]**

Q4) a) Classify the following PDEs as hyperbolic/elliptic/parabolic and reduce them to canonical forms :

i) $x^2u_{xx} - y^2u_{yy} = 0 (x > 0, y > 0).$ **[6]**

ii) $u_{xx} + yu_{yy} = 0 (y > 0).$ **[6]**

b) Prove that the singular integral is also a solution. **[4]**

Q5) a) Find the d'Alembert's Solution of the one-dimensional wave equation $y_{tt} = c^2 y_{xx}, (-\infty < x < \infty, t > 0).$ **[8]**

b) State and prove Harnack's theorem. **[8]**

Q6) a) State and prove maximum principle and deduce minimum principle using it. **[8]**

b) State Dirichlet problem. Moreover, prove that Dirichlet problem has unique solution, if it exist. **[8]**

Q7) a) Solve the following heat equation using variable separable method : **[8]**

$$u_t = ku_{xx}, 0 < x < l, t > 0,$$
$$u(0,t) = u(l,t) = 0, t > 0,$$
$$u(x,0) = f(x), 0 \le x \le l.$$

b) By considering the Dirichlet problem for circle, derive the expression of Poisson integral formula. **[8]**

Q8) a) Solve the following using Duhamel's principle : **[8]**

$$u_t - ku_{xx} = \mathrm{F}(x,t), 0 < x < l, t > 0,$$

$$u(x,0) = f(x), 0 < x < l,$$

$$u(0,t) = u(l,t) = 0, t > 0.$$

b) Find the integral surface of the equation $(p^2 + q^2)x = pz$ containing the curve C: $x_0 = 0$, $y_0 = s^2$, $z_0 = 2s$. **[8]**

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Total No. of Questions : 4]

SEAT No. :

P2579

[Total No. of Pages : 2

[5021]-206

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT-606: Object Oriented Programming using C++

(2008 Pattern) (Old Course)

Time : 2 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Question one is compulsory.***
2) ***Attempt any two questions from Q.2, Q.3 and Q.4.***
3) ***Figures to the right indicate full marks.***

Q1) Attempt any ten of the following : **[20]**

a) What is data encapsulation?

b) Give an example of union in C++?

c) Define hybrid inheritance.

d) What is reference variables?

e) State one difference between break and continue statement.

f) Define friend function.

g) What is technique to determine correct function in function overloading?

h) Write a program to multiply and divide two real numbers $a = 2.5$ and $b = 1.5$ using inline function.

i) Explain the term "message passing".

j) Define static member.

k) Write the function to read a matrix of size 5×6 from the keyboard using "for" loop.

l) Interpret the following statement
int * inarray[10];.

Q2) a) Write a program in C++ to find Euclidean distance between two points in XY plane with output. **[5]**

b) Write a note on applications of object oriented programming. **[5]**

c) Differentiate between a class and structure in C++. **[5]**

Q3) a) Write a C++ program to find maximum of two numbers. **[5]**

b) Write a note on general form of class declaration. **[5]**

c) What are rules to define constructor function? **[5]**

Q4) a) Define : **[9]**

i) Call by value

ii) Call by reference

iii) Return by reference with examples.

b) Write a note on local classes. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2580

[Total No. of Pages : 2

[5021]-301

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-701: Functional Analysis

(2008 Pattern) (Old Course)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that if N is a normed linear space and $x_0 \neq 0,\ x_0 \in \mathrm{N},$ then there exists a functional f_0 in N* such that $f_0(x_0) = \| x_0 \|$ and $\| f_0 \| = 1.$ **[6]**

b) Show that in a Normed linear space N, the closed unit sphere S* in N* is a compact Hausdorff space in weak* topology. **[6]**

c) Show that every positive operator on a finite dimensional Hilbert space has a unique positive root. **[4]**

Q2) a) State and prove Open Mapping theorem for Normed linear spaces. **[8]**

b) Show that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **[6]**

c) Show that norm of an isometry on a normed linear spaces is 1. **[2]**

Q3) a) State and prove uniform boundedness theorem for Banach spaces. **[6]**

b) State and prove Schwarz in equality for Hilbert spaces. **[5]**

c) State and prove Bessel's in equality for the Hilbert spaces. **[5]**

Q4) a) Show that every nonzero Hilbert space contains a complete orthonormal set. **[6]**

b) Show that for any Hilbert space H and arbitrary functional f in H*, there exists a unique vector Y in H such that $f(x) = (x, y)$ for every x in H. **[10]**

Q5) a) Show that the self adjoint operators in $\mathscr{B}(\mathscr{H})$, forms a closed real linear subspace of $\mathscr{B}(\mathscr{H})$. **[6]**

b) Show that set of Normal operators on Hilbert space is closed under addition and multiplication. **[4]**

c) Show that if P is a projection on H with range M and null space N then $M \perp N \Leftrightarrow P$ is self adjoint. **[6]**

Q6) a) Prove that if P and Q are the projections on closed linear subspaces M and N of H, then $M \perp N \Leftrightarrow PQ = 0 \Leftrightarrow QP = 0$. **[6]**

b) Prove that two matrices in An are similar $\Leftrightarrow$ they are the matrices of a single operator on a Hilbert space relative to different bases. **[6]**

c) Prove that an operator T on a Hilbert space is singular iff O belongs to the spectrum $\sigma(T)$. **[4]**

7) a) State and prove spectral theorem for Hilbert spaces. **[10]**

b) Show that an operator T on H is normal iff its adjoint T* is a polynomial in T. **[6]**

8) a) State and prove Gram Schmidt orthogonalisation for the Hilbert spaces. **[10]**

b) Show that if T is an operator on H for which $(Tx, x) = 0$ for all x, then $T = 0$. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2581

[Total No. of Pages : 3

[5021]-302

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT-702: Ring Theory

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) State true or false and justify your answers : **[10]**

i) If X = [0, 1] is a subspace of $\mathbb{R}$ with standard topology induced from $\mathbb{R}$ then the ring R = C(X, $\mathbb{R}$) of all continuous real valued functions is an integral domain.

ii) Field of fractions of $\mathbb{Q}$ is $\mathbb{R}$.

iii) $\mathbb{Z}_{10}[X]$ is an integral domain.

iv) In $\mathbb{Z}_{85}$, every zero-divisor is a nilpotent.

v) Quotient of an integral domain by an ideal is always an integral domain.

b) If R is a Boolean ring with unity then show that $x = -x$ for all $x \in$ R and that R is commutative. Is there a Boolean ring without unity? **[6]**

Q2) a) Show that every Euclidean domain is PID. Give two different examples of Euclidean domains other than a field. **[6]**

b) Let R = $\mathbb{C}\{z\}$ be the ring of complex entire functions. For $a \in \mathbb{C}$, let M_a be the ideal of all entire functions which have a zero at a. show that M_a is maximal ideal of R. Is every maximal ideal of R of this form for some $a \in \mathbb{C}$? **[6]**

P.T.O.

c) Let P be a prime ideal of an integral domain R and let $f(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ be a polynomial in R[x] where $n \geq 1$. Suppose $a_{n-1}, \dots a_1, a_0$ are elements of P and suppose a_0 is not an element of P^2 then show that $f(x)$ is irreducible. **[4]**

Q3) a) Let R be a commutative ring with $1 \neq 0$ and G be a finite group. Define group ring RG of G clearly defining the addition and multiplication on RG. Is RG always a commutative ring? Justify your answer. **[7]**

b) Show that in a commutative integral domain with 1, a prime element is irreducible but not conversely. **[5]**

c) Prove that the rings $2\mathbb{Z}$ and $3\mathbb{Z}$ are not isomorphic. **[4]**

Q4) a) Show that in a ring with identity every proper ideal is contained in a maximal ideal. **[8]**

b) Prove that any subfield of $\mathbb{C}$ must contain $\mathbb{Q}$. **[4]**

c) Is $\mathbb{Z}[X]$ a Euclidean domain? Justify your answer. **[4]**

Q5) a) Give a definition of PID. Show that every non-zero prime ideal in a PID is a maximal ideal. **[7]**

b) Show that in a UFD a non-zero element is a prime if and only if it is irreducible. **[5]**

c) Is every UFD a PID? Justify your answer. **[4]**

Q6) a) Let K be a field. Show that K[x]/($f(x)$) is field if and only if $f(x)$ is irreducible polynomial in K[x] **[6]**

b) Define the term affine algebraic set and give two examples of affine algebraic sets. Identify all the affine algebraic sets in $\mathbb{A}^1$ over $\mathbb{R}$. **[6]**

c) Give a definition of Artinian ring and give two examples of Artinian rings. **[4]**

Q7) a) Let V be a finite dimensional vector space over a field F and given a linear transformation $T : V \rightarrow V$ give a F[x]–module structure to V. Also show that if V is a F[x]–module then there is a linear transformation associated to it. **[8]**

b) Define the term free module. Show that every abelian group G can be made into $\mathbb{Z}$–module. Is every finite abelian group free $\mathbb{Z}$–module? **[8]**

Q8) a) Let $\phi : R \rightarrow S$ be a ring homomorphism of commutative rings with 1. Show that $\ker(\phi)$ is an ideal of R. Is *image*(ϕ) an ideal of S? Justify your answer. **[6]**

b) Find all units of ring of Gaussian integers $\mathbb{Z}[i]$. Decide whether 2 is irreducible in the ring $\mathbb{Z}[i]$. **[6]**

c) Prove that a quotient of a PID by a prime ideal is again a PID. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P2582

[Total No. of Pages : 3

[5021] - 303

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT - 703 : Mechanics

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Explain the concept of degrees of freedom and find the degree of **[5]**

i) a simple pendulum

ii) a compound pendulum

iii) a free particle moving in a plane.

b) Two particles of mass m_1 and m_2 are joined by a rod of fixed length. The particle of mass m_2 is constrained to move along a horizontal axis X. Apply D' Alembert's principle and show that the equation of motion of the system is given by

$m_1 y\ddot{x} - m_2 x\ddot{y} - m_2 gx = 0,$ where g being the acceleration due to gravity.**[7]**

c) Find the equation of motion of a simple pendulum by using D' Alembert's principle. **[4]**

Q2) a) Show that the Lagrange's equation of motion can also be written as

$$\frac{\partial L}{\partial t} - \frac{d}{dt}\left(L - \sum \dot{q}_j \frac{\partial L}{\partial \dot{q}_j}\right) = 0$$ **[6]**

P.T.O

b) Explain Atwood machine and discuss it's motion. **[6]**

c) A particle of mass m moves in one dimension such that it has the Lagrangian $L=\frac{m^2\dot{x}^4}{12}+m\dot{x}^2\ V(x)-V^2(x)$, where V is some differentiable function of x. Find equation of motion for V(x). **[4]**

Q3) a) Show that the curve is a catenary for which the area of surface of revolution is minimum when revolved about Y-axis. **[8]**

b) Find the extremal of the functional **[8]**

$$I=\int_0^{\pi/2}\left(\dot{y}^2-y^2+2xy\right)dx,$$ subject to the conditions that

$$y(0)=0, y\left(\frac{\pi}{2}\right)=0.$$

Q4) a) Obtain the Hamilton's principle for conservative system, from D′ Alembert's principle. **[6]**

b) Deduce Newton's second law of motion from Hamilton's principle. **[7]**

c) Explain the conservative and scleronomic system. **[3]**

Q5) a) A planet moves under the inverse square law of attractive force. Find Lagrangian L, Hamiltonion H, and the Routhian R of the planet. **[8]**

b) If $f=f\left(q_j,\dot{q}_j,t\right)$ then show that $\Delta f=\delta f+\Delta t\frac{df}{dt}$. **[8]**

Q6) a) State and prove the Kepler's third law of planetary motion. **[8]**

b) Obtain the differential equation of a central orbit in the form **[8]**

$\frac{l^2u^2}{m}\left(u+\frac{d^2u}{d\theta^2}\right)=-f\left(\frac{1}{u}\right)$, where f is a law of force and $u=\frac{l}{r}$ and l is the constant of angular momentum, m is the mass of the particle.

Q7) a) Prove that the inverse matrix of an orthogonal transformation identifies the transpose of the matrix. **[8]**

b) If the matrix of transformation from space set of axes to body set of axes is equivalent to a rotation through an angle χ about some axis through the origin then show that $\cos\left(\frac{\chi}{2}\right) = \cos\left(\frac{\phi+\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)$. **[8]**

Q8) a) Show that the transformation **[8]**

$$P = \frac{1}{Q}, q = PQ^2$$ is canonical and find the generating function.

b) Prove that Poisson brackets are invariant under canonical transformations. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P2583

[Total No. of Pages : 4

[5021] - 304
M.A./M.Sc. (Semester - III)
MATHEMATICS
MT - 704 : Measure and Integration
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *All symbols have their usual meanings.*

Q1) a) Let $\{(X_\alpha, \mathscr{B}_\alpha, \mu_\alpha)\}$ be a collection of measurable spaces, and suppose that the sets $\{X_\alpha\}$ are disjoint and define

$X = \bigcup X_\alpha, \mathscr{B} = \{B : (\alpha)[B \cap X_\alpha \in \mathscr{B}_\alpha]\}$ and $\mu(B) = \sum \mu_\alpha (B \cap X_\alpha)$.

i) Show that $\mathscr{B}$ is a σ- algebra. **[6]**

ii) Show that μ is a measure., **[5]**

b) Let $\{A_n\}$ be a countable collection of measurable sets. Then show that

$$\mu\left(\bigcup_{k=1}^{\infty} A_k\right) \leq \sum_{k=1}^{\infty} \mu A_k.$$ **[5]**

Q2) a) Define following terms with suitable example. **[6]**

i) σ-algebra

ii) Outer Measure

iii) Signed measure

b) Show that each nonempty open set G in $\mathbb{R}$ is a union of disjoint open intervals at most countable in number. **[4]**

c) Let c be any real number and let f and g be real valued measurable functions defined on the same measurable set E. Show that $f + c$, cf, $f + g$, $f - g$ and fg are measurable. **[6]**

Q3) a) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathscr{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there exist a unique measurable extended real valued function *f* on X such that $f \leq \alpha$ on B_α and $f \geq \alpha$ on $X \sim B_\alpha$. Also, if *g* is any other function with this property, then show that $g = f \; a \; e$ **[8]**

b) State and prove Fatou's Lemma. **[8]**

Q4) a) If $\{f_n\}$ be a sequence of nonnegative measurable functions then show that $\int \sum_{n=1}^{\infty} f_n = \sum_{n=1}^{\infty} \int f_n$. **[6]**

b) Let E be a measurable set such that $0 < \nu E < \infty$. Then show that there is a positive set A contained in E with $\nu A > 0$. **[6]**

c) Define positive set. Show that **[4]**

i) every measurable subset of a positive set is itself positive.

ii) the union of a countable collection of positive sets is positive.

Q5) a) Let $(X, \mathscr{B}, \mu)$ be a σ-finite measure space, and let υ be a measure defined on $\mathscr{B}$ which is absolutely continuous with respect to μ. Then prove that there is a nonnegative measurable function *f* such that for each set E in $\mathscr{B}$ we have $\upsilon E = \int_E f d\mu$. **[8]**

b) Give an example to show that the Hahn Decomposition need not be unique. **[4]**

c) Let $f, g \in L^P(\mu)$ and *a*, *b* be constants then show that $af + bF \in L^P(\mu)$. **[4]**

Q6) a) Let F be a bounded linear functional on $L^P(\mu)$ with $1 \leq p < \infty$ and μ a σ-finite measure. Then show that there is a unique element *g* in L^q where $1/p + 1/q = 1$, such that $F(f) = \int f \, g d\mu \; with \; \|F\| = \|g\|_q$. **[6]**

b) Let μ, ν and λ be σ-finite. Show that the Radon-Nikodym derivative $\left[\frac{d\nu}{d\mu}\right]$ has the following properties: **[8]**

i) If $\nu \ll \mu$ and f is a nonnegative measurable function, then

$$\int f d\nu = \int f \left[\frac{d\nu}{d\mu}\right] d\mu.$$

ii) $$\left[\frac{d(\nu_1 + \nu_2)}{d\mu}\right] = \left[\frac{d\nu_1}{d\mu}\right] + \left[\frac{d\nu_2}{d\mu}\right].$$

iii) If $\nu \ll \mu \ll \lambda$ then $\left[\frac{d\nu}{d\lambda}\right] = \left[\frac{d\nu}{d\mu}\right]\left[\frac{d\mu}{d\lambda}\right].$

c) Show that the outer measure of an interval equals its length. **[2]**

Q7) a) i) Define an outer Measure μ^*.

ii) Show that the class $\mathbb{B}$ of μ^*-measurable sets is a σ-algebra.

iii) If $\bar{\mu}$ is μ^* restricted to $\mathbb{B}$, then prove that $\bar{\mu}$ is a complete measure on $\mathbb{B}$.

[6]

b) Let (X, $\mathscr{G}$, μ) and (Y, $\mathscr{B}$, v) be two complete measure spaces and f an integrable function an X × Y. Then prove the following: **[6]**

i) For almost all x the function f_x defined by $f_x(y) = f(x, y)$ is an integrable function Y.

ii) For almost all y the function f^y defined by $f^y(x) = f(x, y)$ is an integrable function X.

iii) $\int_y f(x, y) dv(y)$ is an integrable function on X.

iv) $\int_x f(x, y) d\mu(y)$ is an integrable function on Y.

c) If E and F are disjoint sets then show that **[4]**

$$\mu_* E + \mu_* F \le \mu_*\left(E \cup F\right) \le \mu_* E + \mu^* F \le \mu^*\left(E \cup F\right) \le \mu^* E + \mu_* F.$$

Q8) a) If μ^* is a Caratheodory outer measure with respect to Γ then prove that every function in Γ is μ^*-measurable. **[8]**

b) Let μ^* be a topologically regular outer measure on X then prove that each Borel set is μ^* - measurable. **[4]**

c) Show that the Hausdorff dimension of the Canter ternary set is $\log 2 / \log 3$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P2584

[Total No. of Pages : 2

[5021] - 305

M.A./M.Sc. (Semester - III)

MATHEMATICS

MT - 705 : Graph Theory

(2008 Pattern)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that a graph is bipartite if it has no odd cycle. **[6]**

b) Let A be the adjacency matrix of a simple graph G. Prove that the $(i, j)^{th}$ entry in A^k is the number of different edge sequences of k edges between vertices v_i and . **[5]**

c) Prove that an edge is a cut edge if and only if it belongs to no cycle. **[5]**

v_j $\delta(G) \geq \frac{(n-1)}{2}$,

Q2) a) Find the number of labeled trees with n vertices. Draw all labeled trees with 4 distinct vertices. **[6]**

b) Show that every set of six people contains at least three mutual acquaintances or three mutual strangers. **[6]**

c) Prove that the Petersen graph has ten 6-cycles. **[4]**

Q3) a) Prove that a graph in which every vertex has degree 3 has no decomposition into paths with at least five vertices each. **[8]**

b) Show that if G is a simple n - vertex graph with then G is connected. **[4]**

c) Show that the number of vertices in a self-complementary graph is either $4k$ or $4k + 1$, where k is a positive integer. **[4]**

P.T.O

Q4) a) Prove that the center of a tree is a vertex or an edge. **[7]**

b) Prove that every tree T of even order has exactly one subgraph in which every vertex has odd degree. **[6]**

c) Show that every graph has an even number of vertices of odd degree. **[3]**

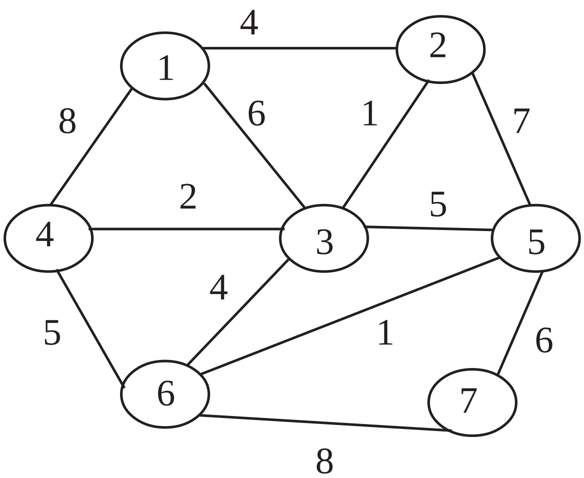
Q5) a) Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph. **[6]**

b) Prove that is G is a simple graph with *diam* $G \geq 3$, then *diam* $\overline{G} \leq 3$. **[6]**

c) Let T be a tree with average degree a. Determine $n(T)$ in terms of a. **[4]**

Q6) a) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **[10]**

b) Use Dijkstra's algorithm to find shortest distance from a vertex 1 to any other vertex in the following graph. **[6]**



Q7) a) State and prove the Havel-Hakimi Theorem. **[10]**

b) Show that the connectivity of the hypercube Q_k is k. **[6]**

Q8) a) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. **[5]**

b) Define clique number and independence number of a graph G with an example. Prove that every tree T has at most one perfect matching. **[5]**

c) Prove that if G is a connected graph, then an edge cut F is a bond if and only if G - F has exactly two components. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P2585

[Total No. of Pages : 3

[5021] - 401

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT-801 : Field Theory

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to right indicate full marks.*

Q1) a) If $f(x) = a_0 + a_1x + \text{------} + a_{n-1}x^{n-1} * x^n \in Z[x]$ is a monic polynomial and $f(x) = 0$ has a root a $\in Q$ then prove that $a \in Z$ and a divides a_0. **[6]**

b) Show that $x^3 + 3x + 2 \in Z_7[x]$ is irreducible over the field Z_7. **[6]**

c) Determine all quadratic irreducible polynomials over Z_2. **[4]**

Q2) a) If p(x) is an irreducible polynomial in F[x] then prove that there exist an extension E of F in which p(x) has a root. **[8]**

b) Show that $p(x) = x^2 - x - 1 \in Z_3[x]$ is irreducible over Z_3. Show that there exist an extension K of Z_3 with nine elements having all roots of $p(x)$. **[6]**

c) Determine the minimal polynomial of $\sqrt{2}+5$ over the field Q. **[2]**

Q3) a) If E is a finite extension of the field F. Then prove that E is an algebraic extension of F. **[8]**

Is the converse true? If not give counter example.

b) If K is an algebraically closed field then prove that every irreducible polynomial in K[x] is of degree one. **[6]**

c) What are the algebraic closures of the followings fields in the field of complex numbers. **[2]**

i) The field of real numbers, $\mathbb{R}$.

ii) The field of rational numbers, Q.

P.T.O

Q4) a) If the field K is a splitting field of $f(x) \in F[x]$, over the field F, then prove that K is an algebraic extension of F. **[8]**

b) Find the splitting field of the polynomial $x^3 + x^2 + 1 \in Z_2[x]$. **[8]**

Is it an algebraic extension of Z_2? Factorize $x^3 + x^2 + 1$ into linear factors in its splitting field.

Q5) a) Prove that every extension of the field F of degree two is normal extension. **[5]**

b) If F is a finite field then prove that number of elements in F is of the form P^n where P is prime number and n is some positive integer. **[5]**

c) If F is a finite field then prove that there exists an irreducible polynomial of any given degree n over F. **[6]**

Q6) a) Define **[6]**

i) Perfect field

ii) Separable extension.

Prove that an algebraic extension of a perfect field is separable.

b) Let K be an extension of the field F and a, b $\in$ K be separable over F. **[6]**

Prove that if F is finite field or characteristic of the field F is zero then F(a, b) is separable over F.

c) Prove that any finite extension of a finite field is simple. **[4]**

Q7) a) If E is a finite extension of a field F, then prove that $|G(E/F)| \leq [E:F]$. **[8]**

b) With usual notations prove that $G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$ is a trivial group. **[8]**

Q8) a) If F is a field of characteristic $\neq z$ and $f(x) = x^2 - a \in F[x]$ is an irreducible polynomial over F, then prove that its Galois group is of order 2. **[6]**

b) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radical over Q. **[6]**

c) Show that it is impossible to construct a cube with a volume equal to twice the volume of a given cube by using ruler and compass only. **[4]**

Total No. of Questions : 8]

SEAT No. :

P2586

[Total No. of Pages : 3

[5021] - 402

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT-802 : Combinatorics

(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) How many numbers between 0 and 10,000 have a sum of digits equal to 13? **[6]**

b) How many ways are there to pick 2 different cards from a standard 52 - card deck such that : **[6]**

i) The first card is an Ace and the second card is not a queen?

ii) The first card is a spade and the second card is not a queen?

c) Find the rook polynomial for a full $n \times n$ board **[4]**

Q2) a) How many arrangements are possible with five letters choosen from MISSISSIPPI? **[6]**

b) Find a recurrence relation for the number of n-digit quaternary (0, 1, 2, 3) sequences With at least one 1 and the first 1 occuring before the first 0. **[6]**

c) Find a generating function for the number of integers between 0 and 9,99,999 whose sum of digits is r. **[4]**

Q3) a) How many arrangements are there of 'MURMUR' with no pair of consecutive letters the same? **[6]**

b) How many r-digit quaternary sequences are there in which the total number of O'S and 1'S is even? **[6]**

c) Give a combinatiorial proof of **[4]**

$$\binom{n}{o}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots\ldots\ldots\ldots + \binom{n}{n}^2 = \binom{2n}{n}$$

P.T.O

Q4) a) How many arrangements of the 26 letters of the alpnabet in which **[6]**

i) a occurs before b?

ii) a occurs before b and c occurs before d?

b) How many ways are there to distribute eight different toys among four children if the first child gets at least two toys? **[6]**

c) Solve the recurrence relation $a_n = 2a_{n-1} - a_{n-2}$, $a_0 = a_1 = 2$ **[4]**

Q5) a) Using generating functions, solve the recurrence relation **[6]**

$a_n = 2a_{n-1} + 2^n$; $a_0 = 1$.

b) How many numbers between 1 and 280 are relatively prime to 280? **[6]**

c) Show that if n+1 distinct numbers are choosen from 1, 2, _____, 2n, then two of the numbers must always be consecutive integers. **[4]**

Q6) a) Find ordinary generating function whose coefficient $a_r = (r + 1)\, r\, (r-1)$ **[6]**

Hence evaluate the sum:

$3 \times 2 \times 1 + 4 \times 3 \times 2 +$ ______ $+ (n + 1)\, n\, (n - 1)$

b) Solve the recurrence relation **[6]**

$a_n^2 = 2a_{n-1}^2 + 1$; when $a_0 = 1$

c) Find the coefficient of x^9 in $(x^2 + x^3 + x^4 + x^5)^5$ **[4]**

Q7) a) How many ways are there to assign 6 cars, denoted $c_1, c_2, c_3, c_4, c_5, c_6$ to six men $m_1, m_2, m_3, m_4, m_5, m_6$, if man m_1 will not drive cars c_2 and c_4 ; man m_2 will not drive cars c_1 or c_5 ; if man m_3 drives all cars; man m_4 will not drive c_2 or c_5; man m_5 will not drive c_4 and man m_6 will not drive c_6? **[8]**

b) Solve the recurrence relation, assuming that n is a power of 2 (leaving a constant A to be determined) **[5]**

$a_n = 16\, a_{n/2} + 5n$.

c) Find a generating function for the number of ways to write the integer r as a sum of positive integers in which no integer appears more than three times. **[3]**

Q8) a) How many ways are there for a child to take 12 pieces of candy with four types of candy if the child does not take exactly two pieces of any type of candy? **[6]**

b) How many ways are there to split 6 copies of one book, 7 copies of a second book and 11 copies of a third book between two teachers if each teacher gets 12 books and each teacher gets at least 2 copies of each book? **[6]**

c) Show that given any set of seven distinct integers, there must exist two integers in this set whose sum or difference is a multiple of 10. **[4]**

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Total No. of Questions : 8]

P2587

SEAT No. :

[Total No. of Pages : 2

[5021]-403
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT - 803 : Differential Manifolds
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let U be an open set in $\mathbb{R}^n$ and f : U$\rightarrow$ $\mathbb{R}^n$ be of class C^r. Let $M = \{x : f(x) = 0\}$ and $N = \{x : f(x) \geq 0\}$. If M is non -empty and D $f(x)$ has rank one at each point of M, then prove that N is an n-manifold in $\mathbb{R}^n$ and $\partial N = M$. **[8]**

b) Define a closed form and give an example. **[4]**

c) Give an example of a manifold which can be covered by a single coordinate patch. **[4]**

Q2) a) Let F be a K-tensor. With usual notation, if AF = $\lim_{\sigma \in S_k} (\sin \sigma) F^\sigma$, then prove that AF is an alternating tensor. Find AF if F is already alternating. **[7]**

b) Show that g (X,Y,Z) = det $\begin{pmatrix} x_i & y_i & z_i \\ x_j & y_j & z_j \\ x_k & y_k & z_k \end{pmatrix}$ is an alternating 3-tensor on $\mathbb{R}^n$. Further express g as a combination of elementary tensors. **[6]**

c) Define volume of parametrized surface in $\mathbb{R}^n$. **[3]**

Q3) a) Define orientation of a manifold M and induced orientation on ∂M. **[4]**

b) State Green's theorem for compact, oriented 2-manifold. **[4]**

c) Let $w = y^2zdx + x^2zdy + x^2ydz$ and α $(u,v) = (u - v,\ uv,\ u^2)$. Find α^* (dw). **[8]**

P.T.O.

Q4) a) Let M be a K-manifold in $\mathbb{R}^n$. If ∂M is non-empty, then prove that ∂M is a K–1 manifold without boundary. **[7]**

b) If $w = x^2 z^2 dx + 2 (\cos y)z\, dy + e^z dz$. Find dw. **[5]**

c) Define exact form and give an example. **[4]**

Q5) a) Define the differential operator d and for any k-form w, show that $d\,(dw)=0$. **[7]**

b) If $w = x^2 dx + ydy + ze^x dz$ and

$\eta = y\cos x\, dx + xdy + 2xy\, dz$ then find $(w \wedge \eta)$. **[5]**

c) Find the tangent plane to the unit sphere at (1,0,0). **[4]**

Q6) a) What is the dimension of A^k (V), the space of alternating K-tensors on n-dimensional vector space V? Justify. **[8]**

b) State Stokes' theorem. **[4]**

c) Show that the unit n-ball B^n is an n-manifold in $\mathbb{R}^n$. What is its boundary? **[4]**

Q7) a) If w and η are k and l forms respectively then prove that $d\,(w\wedge \eta) = dw\wedge \eta + (-1)^k w \wedge d\eta$. **[8]**

b) Let α: $(0,1)\times(0,1) \rightarrow \mathbb{R}^3$ given by $\alpha\,(u,v) = (u,v, u^2+v^2+1)$ Let Y be the image set of α. Evaluate $\int_Y x_2 dx_2 \wedge dx_3 + x_1 x_3\, dx_1 \wedge dx_3$ **[8]**

Q8) a) If T: V $\rightarrow$ W is a linear transformation and if f and g are alternating tensors on W then prove that $T^*(f\wedge g) = (T^*f) \wedge (T^*g)$ **[8]**

b) Let $A = \mathbb{R}^2 - \{0\}$. If $w = \dfrac{xdx + ydy}{x^2 + y^2}$

then show that w is closed and exact on A. **[8]**

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Total No. of Questions : 8]

P2588

SEAT No. :

[Total No. of Pages : 3

[5021]-404
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT : 804 : ALGEBRAIC TOPOLOGY
(2008 Pattern)

Time : 3 Hour] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that the homotopy relation is an equivalence relation. **[6]**

b) Let f and g be continuous maps from X into S^n such that $f(x) \neq g(x)$ for all $x \in X$. Show that f is homotopic to g. **[5]**

c) Show that a retract of a Hausdorff space is closed. **[5]**

Q2) a) Define: a retract, deformation retract and a strong deformation retract.Bring out the relationships among them with suitable examples. **[6]**

b) Prove that the map $p : \mathbb{R} \to S^1$ given by $p(x) = (\cos 2\pi x, \sin 2\pi x,)$ is a covering map. **[5]**

c) Let $A \subset B \subset X$. Suppose B is a retract of X and A is a retract of B. Show that A is a retract of X. **[5]**

Q3) a) Prove that a non-empty open connected subset of $\mathbb{R}^2$ is path connected. **[6]**

b) Prove that $\mathbb{R}^2 - A$ is path connected if A is a countable subset of $\mathbb{R}^2$. **[5]**

c) Let $A \subset X$ be a path connected subset and $\{A_n : n \in \mathbb{Z}^+\}$ is a collection of path connected subsets of X each of which intersects with A. Show that $A \cup \{\cup_n A_n\}$ is path connected. **[5]**

Q4) a) Let f:X→Y be a continuous map. Prove that there exists a homomorphism $f^*:\pi_1(X,x_0)\to\pi_1(Y,f(x_0))$, where x_0 is any point of X. **[6]**

b) Prove that the fundamental group of real projective plane is the cycle group of order two. **[5]**

c) State Brouwer's fixed point theorem. Using it, prove that if A is a retract of B^2. Then every continuous function from A into A has a fixed point. **[5]**

Q5) a) Using the theory of homotopy relation and lifting lemma, prove that every non-constant complex polynomial has a root. **[6]**

b) Determine the fundamental groups of the following spaces: **[5]**

i) $R^2-(\mathbb{R}_+\times 0)$

ii) $\{x\in R^2 \mid \|x\|<1\}$

iii) The solid sphere

iv) Torus T with one point removed.

c) Determine the fundamental groups of $\mathbb{R}^2-0, \mathbb{R}^3-0$ and $S^2\times\mathbb{R}$. **[5]**

Q6) a) Prove that a covering map is an open map. **[6]**

b) Let $p:\tilde{X}\to X$ and $q:\tilde{Y}\to Y$ be covering maps. Show that $p\times q:\widetilde{X}\times\tilde{Y}\to X\times Y$ is a covering map. **[5]**

c) Let $p:\tilde{X}\to X$ be a covering map and let $f_1,f_2:Y\to\tilde{X}$ be two liftings of $f: Y\to X$. Suppose Y is connected and there exists $y_0\in Y$ such that $f_1(y_0)=f_2(y_0)$. Then prove that $f_1=f_2$. **[5]**

Q7) a) Define a fibration. Give an example of a fibration which is not a covering projection. **[6]**

b) Let $p: E\to B$ be a fibration. Show that $p(E)$ is a union of path components of B. **[5]**

c) Let p : $E\to B$ be a fibration such that every fibre has no non-null path. Prove that p has a unique path lifting. **[5]**

Q8) a) Prove that the diameter of a p-simplex $S_p = (a_0, a_1,..., a_p)$ is the length of its longest edge. **[6]**

b) Using topological dimension, prove that $\mathbb{R}^n$ is homeomorphic to $\mathbb{R}^m$ if and only if $n = m$. **[5]**

c) Prove that every simplex is complex. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2589

[Total No. of Pages : 2

[5021]-405
M.A./M.Sc. (Semester - IV)
MATHEMATICS
MT - 805 : Lattice Theory
(2008 Pattern)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let the algebra $L=\langle L;\wedge,\vee\rangle$ be a lattice. Set $a \leq b$ if and only if $a\wedge b = a$. Prove that $L^{P} = \langle L;\leq\rangle$ is a poset and as a poset L^{P} is a lattice. **[7]**

b) Prove that I is a prime ideal of lattice L if and only if there is a homomorphism ϕ of L onto C_2 with $I = \phi^{-1}(0)$. **[5]**

c) Prove that every complete lattice is bounded. Is the converse true? Justify your answer. **[4]**

Q2) a) Prove that every homomorphic image of a lattice L is isomorphic to a suitable quotient lattice of L. **[7]**

b) Let L be a pseudocomplemented meet semilattice. Assuming $S(L)=\{a^*\,|\,a\in L\}$ is a bounded lattice, prove that S(L) is distributive. **[5]**

c) Prove that dual of distributive lattice is distributive. **[4]**

Q3) a) Prove that a lattice L is distributive if and only if for any two ideals I,J of L, $I \vee J = \{i \vee j \mid i \in I, j\in J\}$. **[6]**

b) Prove that in a Boolean lattice, an ideal is maximal if and only if it is prime. **[6]**

c) Show that the following in equalities hold in any lattice. **[4]**

i) $(x\wedge y)\vee(x\wedge z)\leq x\wedge(y\vee z)$

ii) $(x\wedge y)\vee(x\wedge z)\leq x\wedge(y\vee(x\wedge z))$

P.T.O.

Q4) a) Let L be a lattice and a, b $\in$ L. If aMb in L and bMa in the dual of L, then prove that $[a \wedge b, b] \cong [a, a \vee b]$. **[7]**

b) Prove that any lattice can be embedded into its ideal lattice. **[5]**

c) Prove that every distributive lattice is modular, but not conversely. Find the smallest modular but non distributive lattice. **[4]**

Q5) a) Prove that a lattice L is modular if and only if it does not contain a pentagon. **[8]**

b) Let L be a distributive lattice, Let I be an ideal, Let D be a dual ideal of L and let $I \cap D = \phi$. Then prove that there exists a prime ideal P of L such that $P \supseteq I$ and $P \cap D = \phi$. **[8]**

Q6) a) Let L be a bounded distributive lattice with $0 \neq 1$. Prove that L is a Boolean lattice if and only if P(L), the set of all prime ideals of L, is unordered. **[8]**

b) Prove that a bounded conditionally complete lattice is complete. **[5]**

c) Give an example of a lattice which is semi modular but not modular. **[3]**

Q7) a) Prove that a lattice is distributive if and only if it is isomorphic to ring of sets. **[8]**

b) Let L be a lattice of finite length. If L is semimodular, then prove that any two maximal chains of L are of the same length. **[8]**

Q8) a) State and prove fixed point theorem for complete lattices. **[7]**

b) Prove that a modular lattice satisfies both the upper and lower covering condition. **[5]**

c) Prove that every lattice homomorphism is an isotone map. Is the converse true? Justify your answer. **[4]**

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