Total No. of Questions : 8]

SEAT No. :

P2590

[Total No. of Pages : 2

[5021]-1001
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 501 : Real Analysis
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define a Cantor set. Show that length of Cantor set is zero and it is a perfect set. **[5]**

b) Prove that exterior measure of a closed cube in $\mathbb{R}^d$ is it's volume. **[3]**

c) Suppose E $\subset \mathbb{R}^d$ and m_* (E) = 0 then show that E is a Lebesgue measurable set. Also if F$\subset$E then show that F is Lebesgue measurable. **[2]**

Q2) a) Let $\{E_1, E_2, ...\}$ be a collection of measurable subsets of $\mathbb{R}^d$. With usual notations prove that if $E_k \to E$ and m $(E_k) < \infty$ for some *k* then m (E) = $\lim_{N\to\infty} m(E_N)$. **[5]**

b) Show that Lebesgue measure is translation and dilation invariant. **[3]**

c) Give an example of an algebra which is not a σ - algebra. **[2]**

Q3) a) Give construction of a non-measurable set. **[5]**

b) Define a real valued measurable function on $\mathbb{R}^d$. Show that a step function is measurable. **[3]**

c) If *f* is a measurable function then prove that $\{x \in \mathbb{R}^d \,/\, f(x) \le a\}$ is measurable $\Leftrightarrow \{x \in \mathbb{R}^d \,/\, f(x) < a\}$ is measurable for any a $\in \mathbb{R}$. **[2]**

P.T.O.

Q4) a) Let the function f: $[0,1] \to R$ defined by, **[5]**

$$f(x)=\begin{cases} \frac{1}{x} & if \quad 0<x<1 \\ 5 & if \quad x=0 \\ 7 & if \quad x=1 \end{cases}$$

then prove that f is a measurable function.

b) Give an example of a non-measurable function defined on (0,1). **[3]**

c) Show that product of two real valued measurable functions defined on $\mathbb{R}^d$ is a measurable function. **[2]**

Q5) a) State and prove Egorov's theorem. **[5]**

b) Show that $\lim_{n\to\infty}\int_0^1 fn(x)dx=0$ where $fn(x)=\frac{nx}{1+n^2x^2}$ using dominated convergence theorem. **[3]**

c) If A and B are subsets of $\mathbb{R}^d$ then prove $\chi_{A\cup B}=\chi_A+\chi_B-\chi_{A\cap B}$ **[2]**

Q6) a) Let $\{fn\}$ be a sequence defined as:

$f_{2n-1}=\chi_{[0,1]}$, $f_{2n}=\chi_{(1,2})$ for n= 1,2,..... **[5]**

Show that for above sequence, strict inequality occurs in Fatou's lemma.

b) If $\int_E f=0$ and $f(x)\geq 0$ on E then show that $f=0$ a.e. on E. **[3]**

c) State 2 properties of integral of a non-negative measurable functions. **[2]**

Q7) a) State and prove monotone convergence theorem for a sequence of non-negative measurable functions. **[5]**

b) Define Lebesgue integral of a measurable function f defined on $\mathbb{R}^d$. Suppose $f\leq g$ a. e. x and f, g both are integrable functions then show that $\int f\leq\int g$. **[5]**

Q8) a) Show that the vector space L' is complete in it's norm. **[5]**

b) Give statemens of Fubirṻ 's theorem and Lebesgue differentiation theorem. **[5]**

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Total No. of Questions : 8]

P2591

SEAT No. :

[Total No. of Pages : 3

[5021]-1002
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT : 502 :Advanced Calculus
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define the following terms:

i) Derivative of a scalar field with respect to a vector.

ii) Total derivative of a scalar field. Also explain the significance and difference of the above terms. **[5]**

b) Let $\overline{T}:\mathbb{R}^n \to \mathbb{R}^n$ be a given linear transformation compute the derivative $f'(\overline{x};\overline{y})$ for the scalar field defined on $\mathbb{R}^n$ by the equation. $f(\overline{x}) = \overline{x}.\overline{T}(\overline{x})$, **[3]**

c) Let f be a nonconstant scalar field differentiable every where in the plane, and let C be a constant. Assume the cartesian equation f (x,y) = C describes a curve C at each of its points. Prove that the gradient vector ∇f at each point of C is normal to C. **[2]**

Q2) a) Let $\overline{f}$ and $\overline{g}$ be vector fields such that the composition $\overline{h} = \overline{f}o\overline{g}$ is defined in the neighbourhood of a point $\overline{a}$. Assume that $\overline{g}$ is differentiable at a with total derivative. $\overline{g}'(\overline{a})$.*Let* $\overline{b} = \overline{g}(\overline{a})$ and assume $\overline{f}$ is differentiable at $\overline{b}$ with total derivative $\overline{f}'(\overline{b})$. Prove that $\overline{h}$ is differentiable at $\overline{a}$, and the total derivative is given by $\overline{h}(\overline{a}) = \overline{f}(\overline{b})o\overline{g}(\overline{a})$ **[5]**

b) Evaluate the directional derivative of. f (x,y,z) =3x –5y+2z at (2,2,1) in the direction of the out word normal to the sphere $x^2 + y^2 + z^2 = 9$ **[3]**

c) Prove that if a vector field $\overline{f}$ is differentiable at $\overline{a}$ then $\overline{f}$ is continuous at $\overline{a}$. **[2]**

P.T.O.

3) a) Define line integral and illustrate it by an example. State the basic properties of the line integral. **[4]**

b) A two- dimensional force field $\overline{f}$ is given by the equation $\overline{f}(x,y)=cxy\,\overline{i}+x^6y^2\overline{j}$ where c is a positive constant. This force acts on a particle which move from (0,0) to the line x = 1 along a curve $y = ax^b$ where a > 0 and b > 0. Find a value of 'a' (in terms of c) such that the work done by this force is independent of b. **[4]**

c) Calculate the line integral with respect to arc length. $\int_c z\,ds$ where C has the vector equation. $\overline{\alpha}(t)=t\cos t\;\overline{i}+t\sin t\;\overline{j}+t\overline{k}\;\;0\le t\le t_0$. **[2]**

Q4) a) State and prove the first fundamental theorem for line integral. **[5]**

b) Let S be the set of all (x,y) ≠ (0,0) in $\mathbb{R}^2$ and $\overline{f}$ is a vector field defined on S by the equation $\overline{f}(x,y)=\dfrac{-y}{x^2+y^2}\overline{i}+\dfrac{x}{x^2+y^2}\overline{j}$

Show that $D_1f_2 = D_2f_1$ everywhere on S but $\overline{f}$ is not a gradient on S. **[3]**

c) Calculate the line integral of the vector field

$\overline{f}(x,y,z)=x\overline{i}+y\overline{j}+(xz-y)\overline{k}$ along a line segment joining (0,0,0) to (1,2,4). **[2]**

Q5) a) State only the general formula for change of variables in double integrals. Explain the notation used. **[4]**

b) Transform the integral to polar co-ordinate and compute the value **[3]**

$$\int_0^1\left[\int_{x^2}^{x}\left(x^2+y^2\right)^{1/2}dy\right]dx$$

c) Use Green's theorem to compute the work done by the force field. $\overline{f}(x,y)=(y+3x)\overline{i}+(2y-x)\overline{j}$ in moving a particle once around the ellipse $4x^2+y^2=4$ **[3]**

Q6) a) Define a simple parametric surface. If $T = [0, 2\pi] \times [0, \pi/2]$ under the map $\bar{r}(u,v)$ = a cos u cos v $\bar{i}$ + asinu cosv $\bar{j}$ + a sin v $\bar{k}$ maps to a surface S, find singular points of this surface. Aslo explain whether S is simple. **[4]**

b) Define surface integral. State and prove the invariance of the surface integrals under smoothly equivalent parametric representations. **[4]**

c) .Let $\bar{f}(x,y,z) = (x^2 + yz)\bar{i} + (y^2 + xz)\bar{j} + (z^2 + xy)\bar{k}$ then find curl and divergence of $\bar{F}$ by computing its Jacobian matrix. **[2]**

Q7) a) Let $\bar{f}(x,y) = P(x,y)\bar{i} + Q(x+y)\bar{j}$ be a vector field that is continuously differentiable on an open connected set S in the plane. Prove that $\bar{f}$ is gradient on S if and only if $\frac{\partial p}{\partial y} = \frac{\partial \theta}{\partial x}$ every where on S. **[5]**

b) Let S_n(a) denote the n-dimensional solid sphere (or n- ball) of radius a given by $S_n(a) = \{(x_1....x_n) | x_1^2 + ... + x_n^2 \le a^2\}$ and

Let $v_n(a) = \int ......\int dx_1.....dx_n$ be the volume of S_n (a). prove that

$$V_n(a) = \frac{\pi^{n/2} a^n}{\sqrt{\left(\frac{1}{2}n+1\right)}}$$

[5]

Q8) a) State and prove stores theorem. **[5]**

b) Transform the surface integral $\iint_S (curl\bar{F}).\bar{n}$ ds to a line integral by the use of Stores theorem, where $\bar{F}(x,y,z) = y^2\bar{i} + xy\,\bar{j} + xz\,\bar{K}$, Where S is the hemisphere $x^2+y^2 + z^2 = 1$, $Z \ge 0$, and $\bar{n}$ is the unit normal with nonnegative z – component. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2592

[Total No. of Pages : 2

[5021]-1003
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 503 : Group Theory
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that for all integers $n \geq 1$, the set of n^{th} complex roots of unity is a group under multiplication. **[4]**

b) Show that in a group G, with finite number of elements, the number of elements x of G such that $x^3 = e$ is odd. Show that the number of elements x of G such that $x^2 \neq e$ is even. **[4]**

c) Show that $A = \left\{ \begin{bmatrix} x & x \\ x & x \end{bmatrix} / x \in \mathbb{R}, x \neq 0 \right\}$ is subgroup of $(GL_2(\mathbb{R}), X)$. **[2]**

Q2) a) Show that for each $a \in G$, where G is a group; centralizer of 'a' is subgroup of G. Find centralizer of an element in an Abelian group G. **[4]**

b) Let $a, b \in G$. If $|a| = 10$, $|b| = 21$, show that $< a > \cap < b > = \{e\}$. **[3]**

c) Determine the subgroup lattice for $\mathbb{Z}_8$. **[3]**

Q3) a) Show that order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. **[5]**

b) Find the number of elements of order 5 in alternating group with 6 elements. **[2]**

c) Find Aut $(\mathbb{Z}_{10})$. **[3]**

P.T.O.

Q4) a) Find all inner automorphisms in D_4. Do they form the group under compositions? Justify. **[4]**

b) Prove that converse of Lagrange's theorem is true for finite cyclic groups. **[4]**

c) State orbit-stabilizer theorem for finite groups. **[2]**

Q5) a) With all usual notations, show that $U(st) \approx U(s) \oplus U(t)$ whenever s and t are relatively prime. Also, prove that $U_s(st)$ is isomorphic to U(t). **[4]**

b) Find all subgroups of order 3 in $\mathbb{Z}_9 \oplus \mathbb{Z}_3$. **[4]**

c) Find the order of an element 5+<6> in the factor group $\mathbb{Z}_{18}/<6>$. **[2]**

Q6) a) Let G = U(16), H = {1,15}, K = {1,9}. Are H and K isomorphic? What about G/H and G/K? **[4]**

b) State and prove first isomorphism theorem for groups. **[4]**

c) Find Kernel of ϕ, where **[2]**

$\phi \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z}$ is defined by

$\phi(a,b) = a - b$. Also, find $\phi^{-1}(3)$.

Q7) a) In an Abelian group G with prime power order and an element 'a' with maximal order in G, show that G can be written in the form <a> × K. **[5]**

b) Find all Abelian groups (upto isomorphism) of order 360. **[3]**

c) State Cauchy's theorem for finite groups. **[2]**

Q8) a) Show that conjugacy is an equivalence relation on a group. **[4]**

b) Determine the groups of order 99. **[3]**

c) State and prove Index theorem for finite groups. **[3]**

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Total No. of Questions : 8]

P2593

SEAT No. :

[Total No. of Pages : 4

[5021]-1004
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 504 : Numerical Analysis
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) Prove that the order of convergence of the secant method is approximately 1.618 ($\alpha = 1.618$) and asymptotic error constant $\lambda \approx C^{1/\alpha} = \left(\dfrac{f''(p)}{2f'(p)}\right)^{\alpha-1}$ **[5]**

b) Show that when newton's method is applied to the equation $\dfrac{1}{x} - a = 0$, the resulting iteration function is $g(x) = x(2-ax)$. **[3]**

c) Compute the limit, $\lim\limits_{n\to\infty} \dfrac{2^n}{2^n+3}$ and determine the corresponding rate of convergence. **[2]**

Q2) a) Apply steffensen method to the iteration function $g(x) = \cos x$ using starting value of $P_o = 0$. Perform four iterations. Compute the absolute error of each approximation and verify that the convergence is quadratic to ten digit, the fixed point is $p = 0.7390851332$. **[5]**

b) The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero in the interval $(-1,0)$. Approximate this zero within an absolute tolerance of 5×10^{-5} using newton's method starting with $P_0 = 0$. **[3]**

c) Perform secant method to determine P_2, the second approximation to the location of root of the equation $\cos x - x = 0$ on the interval $(0,1)$. **[2]**

P.T.O.

Q3) a) For the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$ **[5]**

Determine matrices L,U and P such that LU = PA using Gaussian elimination with scaled partial pivoting.

b) Solve the following system using Gaussian elimination with partial pivoting. **[3]**

$2x + 3y + z = -4$

$4x + y + 4z = 9$

$3x + 4y + 6z = 0$

c) Compute the condition number k_∞ for the matrix $A = \begin{bmatrix} 3 & 1.5 & 1 \\ 1.5 & 1 & 0.75 \\ 1 & 0.75 & 0.6 \end{bmatrix}$

[2]

Q4) a) Solve the following system of linear equations by Gauss-seidal method, start with $x^{(0)} = [0\ 0\ 0]^T$ (Perform 3 iterations) **[5]**

$4x_1 - x_2 - x_3 = 3$

$-2x_1 + 6x_2 + x_3 = 9$

$-x_1 + x_2 + 7x_3 = -6$

b) For the coefficient matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 6 \end{bmatrix}$ and right hand side

vector $b = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$

Write out the components of the Jacobi method iteration equation. Then starting with $x^{(0)} = 0$, Perform two iterations of the Jacobi method. **[3]**

c) Define the terms: **[2]**

i) Rate of convergence

ii) Asymptotic error constant

Q5) a) Consider the matrix $A = \begin{bmatrix} 3 & 2 & -2 \\ -3 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

Perform 5 iterations of the power method starting with $x^{(0)} = [1\ 0\ 0]^T$ **[5]**

b) Solve the following system by Jacabi method starting with vector $x^{(0)} = [0\ 0\ 0]^T$ **[3]**

(Perform 3 iterations)

$4x_1 - x_2 + x_3 = 4$

$-x_1 + 4x_2 - x_3 = 2$

$x_1 - x_2 + 4x_3 = 4$

c) Find the vector valued function F associated with the following system and compute the Jacobian F. **[2]**

$x_1^3 - 2x_2 - 2 = 0$

$x_1^3 - 5x_3^2 + 7 = 0$

$x_2\, x_3^2 - 1 = 0$

Q6) a) Derive the closed newton-cotes formula with n=3; **[5]**

$$\int_a^b f(x)dx = \frac{b-a}{8}\left[f(a) + 3f(a+\Delta x) + 3f(a+2\Delta x) + f(b)\right]$$

b) Derive the following forward difference approximation for the second derivative. **[3]**

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0+h) + f(x_0+2h)}{h^2}$$

c) If f(x) = ln(x), find f'(2) for h = 0.1, 0.01 **[2]**

Q7) a) Determine the degree of precision of the simpson's $\frac{3^{th}}{8}$ rule **[5]**

b) Use householder's method to reduce the following symmetric matrix to tridiagonal form **[5]**

$$A = \begin{bmatrix} 5.5 & -2.5 & -2.5 & -1.5 \\ -2.5 & 5.5 & 1.5 & 2.5 \\ -2.5 & 1.5 & 5.5 & 2.5 \\ -1.5 & 2.5 & 2.5 & 5.5 \end{bmatrix}$$

Q8) a) Apply euler's method to approximate the solution of the initial value problem $\frac{dx}{dt} = tx^3 - x$, $(0 \le t \le 1)$, $x(0) = 1$ using 4 steps. **[5]**

b) Use the fourth order runge kutta method to solve the initial value problem $\frac{dx}{dt} = \frac{t}{x}$ $(0 \le t \le 5$, $x(0) = 1$ The exact solution for this problem is $x(t) = \sqrt{t^2 + 1}$ with what rate does the fourth order runge kutta method converge to this exact solution. **[5]**

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Total No. of Questions : 8]

P2594

SEAT No. :

[Total No. of Pages : 3

[5021]-1005
M.A./M.Sc. (Semester - I)
MATHEMATICS
MT - 505 : Ordinary Differential Equations
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ is one solution of the differential equation $y'' + p(x)y' + Q(x)y=0$, then find the other solution. **[5]**

b) If K and b are positive constants, the find the general solution of $y''+k^2y = \sin bx$ by using method of undetermined coefficients. **[3]**

c) Show that e^x and e^{-2x} are linearly independent solutions of $y''+y'-2y=0$ on any interval. **[2]**

Q2) a) If $y_1(x)$ and $y_2(x)$ are two solutions of equation $y''+P(x)y'+Q(x)y=0$ on interval [a,b], then prove that $y_1(x)$ and $y_2(x)$ are linearly dependent on this interval if and only if their wronskian $W(y_1,y_2)$ is identically zero. **[5]**

b) Find a particular solution of $y''+2y'+y = e^{-x} \log x$ by using method of variation of parameters. **[3]**

c) Verify that $y_1 = x$ is one solution of differential equation $x^2y'' - x(x + 2)y' + (x + 2)y = 0$ and then find another solution y_2 and general solution. **[2]**

Q3) a) State and prove sturm comparison theorem. **[5]**

b) If $q(x)<0$ and $u(x)$ is non trivial solution of $u''+q(x)u=0$, then prove that $u(x)$ has at most one zero. **[3]**

c) Find the normal form of differential equation $x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y=0$. **[2]**

P.T.O.

Q4) a) Find the general solution of differential equation $(1+x^2)y'' + 2xy'-2y=0$ in terms of power series in x. can you express this solution by means of elementary function.

b) Show that $x=\infty$ is regular singular point of differential equation $x^2y''+4xy'+2y=0$. **[3]**

c) Locate and classify the singular points on the x–axis of $x^3(x-1)y''-2(x-1)y'+3xy=0$. **[2]**

Q5) a) Find two independent frobenius series solution of the differential equation $2xy''+(x+1)y'+3y=0$. **[5]**

b) Find the general solution of the system **[3]**

$$\frac{dx}{dt} = -3x + 4y$$
$$\frac{dy}{dt} = -2x + 3y$$

c) Show that $y = c_1\sin x + c_2\cos x$ is general solution of $y'' + y = 0$ on any interval. **[2]**

Q6) a) Find the general solution near x=0 of the hypergeometric equation. **[5]**

$x(1-x)y''+[c-(a+b+1)x]y'-aby=0$ where a,b and c are constants.

b) For the following system **[3]**

$$\frac{dx}{dt} = x$$
$$\frac{dy}{dt} = -x + 2y$$

i) Find the differential equation of path

ii) Solve the equations to find the path

iii) Find the critical points.

c) State picard's existence and uniqueness theorem. **[2]**

Q7) a) If m_1 and m_2 are roots of the auxiliary equation of the system.

$$\frac{dx}{dt} = a_1x + b_1y$$
$$\frac{dy}{dt} = a_2x + b_2y$$

Which are real, distict and of same sign, then prove that the critical point (0,0) is a node. **[5]**

b) Solve the following initial value problem **[5]**

$$\frac{dy}{dx} = z, \quad y(0) = 1$$
$$\frac{dz}{dx} = -y, \quad z(0) = 0$$

Q8) a) Let f(x,y) be a continuous function that satisfies a lipschitz condition $|f(x,y) - f(x,y_2)| \leq k\,|y_1 - y_2|$ on a strip defined by $a \leq x \leq b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$ has one and only one solution $y = y(x)$ on the interval $a \leq x \leq b$. **[5]**

b) Find the general solution of differential equation $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ near the singular point x=0. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2595

[Total No. of Pages : 3

[5021]-2001

M.A./M.Sc. (Semester - II)

MATHEMATICS

MT - 601 : Complex Analysis

(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions of the following.***
2) ***Figures to the right indicate full marks.***

Q1) a) If f is holomorphic at z_0 then prove that **[4]**

$$\frac{\partial f(z_0)}{\partial \overline{z}} = 0 \text{ and } f'(z_0) = \frac{\partial f(z_0)}{\partial z} = 2\frac{\partial u(z_0)}{\partial z}.$$

b) If f and g are continuous function on a smooth curve γ then prove that,

$$\int_\gamma (\alpha f(z) + \beta g(z)dz = \alpha\int_\gamma f(z)dz + \beta\int_\gamma g(z)dz.$$ **[4]**

c) Suppose f is continuous in a region Ω. Prove that any two primitives of f (if they exist) differ by a constant. **[2]**

Q2) a) Show that a power series is infinitely complex differentiable in it's disc of convergence. **[5]**

b) Can every continuous function on the closed unit disc is approximated by polynomials in the variable z? Justify. **[3]**

c) Determine the radius of convergence of the series **[2]**

$$\sum_{n=1}^{\infty}\left(\frac{n^2}{4^n + 3n}\right)z^n .$$

Q3) a) Suppose that f is holomorphic in an open set Ω except possibly at a point z_0 in Ω. If f is bounded on $\Omega - \{z_0\}$ then prove that z_0 is a removable singularity. **[5]**

P.T.O.

b) If f is holomorphic in an open set that contains the closure of a disc D contered at z_0 and radius R then prove that,

$$|f^{(n)}_{(z_0)}| \leq \frac{n!\|f\|_c}{R^n}. \text{ Where } \|f\|_c = \sup_{z \in C} |f(z)|.$$ **[3]**

c) Describe geometrically the sets of points z in the complex plane defined by the relation. **[2]**

i) $\frac{1}{z} = \overline{z}$

ii) $\text{Re}(z) = 3$

Q4) a) State and prove Morera's theorem. **[5]**

b) If f is holomorphic function in Ω^+ that extend continuously to I and such that f is real valued on I then prove that there exists a function F holomorphic in all of Ω such that F = f on Ω^+. **[3]**

c) Show that the complex zeros of $\sin(\pi z)$ are exactly at the integers and each of order 1. **[2]**

Q5) a) Using Residue formula prove that, **[5]**

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^4} = \frac{\pi}{\sqrt{2}}$$

b) If f has a pole of order n at z_0 then prove that, **[3]**

$$f(z) = \frac{a_{-n}}{(z-z_0)^n} + \frac{a_{-n+1}}{(z-z_0)^{n-1}} + --- + \frac{a_{-1}}{(z-z_0)} + G(z)$$

Where G is a holomorphic function in a neighbourhood of z_0.

c) If f and g are holomorphic in a region Ω and $f(z) = g(z)$ for all z in some sequence of distinct point with limit point in Ω then prove that $f(z) = g(z)$ throughout Ω. **[2]**

Q6) a) Suppose f is a meromorphic function in an open set containing a cricle c and it's interior. If f has no poles and never vanishes on c then prove that,

$$\frac{1}{2\pi_i}\int_c \frac{f'(z)}{f(z)}dz = \{\text{Number of zeros off inside } c\}-$$

$$\{\text{Number of poles of } f \text{ inside } c\}.$$ **[5]**

b) Prove that if the real part of an entire function f is bounded then f is constant. **[3]**

c) Find the nature of isolated singularity at origin for the function $f(z)=\frac{\sin z}{z}$ **[2]**

Q7) a) Prove that any holomorphic function in a simply connected domain has a primitive. **[5]**

b) Prove that every non-constant polynomial $p(z) = a_n z^n + a_{n-1} z^{n-1} + \text{------} + a_0$ with complex coefficients has a root in $\mathbb{C}$. **[5]**

Q8) a) If Ω is an open set in $\mathbb{C}$ and $T \subset \Omega$ a triangle whose interior is also contained in Ω and f is holomorphic in Ω then prove that, $\int_T f(z)dz = 0$. **[6]**

b) If f is an entire function that satisfies $\sup_{|z|=R} |f(z)| \leq AR^k + B$, for all $R > 0$ and for some $K \geq 0$ and some constants $A,B > 0$ then prove that f is a polynomial of degree $\leq K$. **[4]**

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Total No. of Questions : 8]

P2596

SEAT No. :

[Total No. of Pages : 2

[5021]-2002
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT - 602 : General Topology
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions*
2) *Figures to the right side indicate full marks.*

Q1) a) Let X be a set; let τc be the collection of all subsets U of X such that X\U is either countable or is all of X. Prove that τc is a topology on X. Is the collection $\tau_\infty = \{U : X \setminus U$ is infinite or empty or all of X$\}$ a topology on X? **[5]**

b) Show that the topologies of $\mathbb{R}_1$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another. **[3]**

c) Let X be a set; let B be a basis for a topology τ on X. Then prove that τ equals the collection of all unions of elements of B. **[2]**

Q2) a) Show that a space X is hausdorff if and only if the diagonal $\Delta = \{x \times x \mid x \in X\}$ is closed in X×X. **[5]**

b) Show that every simply ordered set is a hausdorff space in the order topology. **[3]**

c) Let A be a subset of the topological space X. let A' be the set of all limit points of A. Then prove that $\overline{A} = A \cup A'$. **[2]**

Q3) a) If A is a subspace of X and B is a subspace of Y, then prove that the product topology on A×B is the same as the topology A×B inherits as a subspace of X×Y. **[5]**

b) Let $\pi_1 : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be the projection map. Show that π_1 is continuous, open map but not a closed map. **[3]**

c) Find the boundary and interior of each of the followig subsets of $\mathbb{R} \times \mathbb{R}$

i) $A = \{x \times y \mid y = 0\}$ ii) $B = \{x \times y \mid x > 0 \text{ and } y \neq 0\}$. **[2]**

Q4) a) Let X and X' denote a single set in two topologies τ and τ' respectively. Let $i: X' \to X$ be the identity map. **[5]**

i) Show that i is continuous $\Leftrightarrow \tau'$ is finer than τ.

ii) Show that i is a homeomorphism $\Leftrightarrow \tau' = \tau$.

b) State and prove the pasting lemma. **[3]**

c) Suppose that $f : x \to y$ is continuous. If x is a limit point of the subset A of X, is it necessarily true that f(x) is a limit point of f(A)? Justify. **[2]**

Q5) a) Prove that the union of collection of connected sets that have a point in common is connected. **[5]**

b) Prove that the components of X are connected disjoint subspaces of X whose union is X, such that each non-empty connected subspaces of X intersects only one of them. **[3]**

c) Determine the components of discrete topological space. **[2]**

Q6) a) Prove that every compact subspace of a Hausdorff space is closed. **[5]**

b) Prove that compactness implies limit point compactness. Is the converse true? Justify. **[3]**

c) Show that in the finite complement topology on R, every subspace is compact. **[2]**

Q7) a) State and prove the tube lemma. **[5]**

b) Prove that a product of regular spaces is regular. **[5]**

Q8) a) Prove that every compact hausdorff space is normal. **[5]**

b) State

i) The urysohn lemma **[2]**

ii) The urysohn metrization theorem **[1]**

iii) The tietze extension theorem **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2597

[Total No. of Pages : 2

[5021]-2003
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT - 603 : Ring Theory
(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If R is a ring with 1 and I is an ideal in R such that $I \neq R$, then prove that there is a maximal ideal M of the same kind I such that $I \subseteq M$. **[4]**

b) Show that (Q,+) has no maximal subgroup. **[4]**

c) Describe the units in the ring given by $\mathbb{Z}\times Q\times\mathbb{Z}$ **[2]**

Q2) a) Prove that the characteristic of a local ring is either zero or power of a prime. **[4]**

b) If $I \subseteq J$ are both two sided ideals in a ring R, then prove that (R/I)/(J/I) is naturally isomorphic to R/J. **[4]**

c) Prove or disprove : The ring $\mathbb{Z}_6$ [x] is an integral domain. **[2]**

Q3) a) If R is a commulative ring with unity and if $p(x) = a_0 + a_1 x + ---- + a_r x^r \in R(x)$ is unit in R(x) then prove that a_0 is unit in R and $a_1, a_2, ----, a_r$ are all nilpotent elements in R. **[5]**

b) If n is a power of a prime, then show that the ring $\mathbb{Z}/n\mathbb{Z}$ is a local ring. **[3]**

c) Give an example of non-trivial commutative ring in which square of every element is zero. **[2]**

P.T.O

Q4) a) Let 'a' and 'b' are nilpotent elements of commutative ring R then prove that a+b is nilpotent in R. **[4]**

b) If I and J are ideals in a ring R with 1 which are co-maximal i.e. I + J = R, then show that I^m and J^n are co-maximal, for all $m, n \in \mathbb{Z}$. **[4]**

c) Show that $\frac{Q(x)}{<x+2>}$ is a field. **[2]**

Q5) a) Prove that every euclidean domain is principal ideal domain. **[5]**

b) Show that $1 + x + x^2 + \text{----} + x^{p-1}$ is irreducible in $\mathbb{Z}(x)$ for any prime p. **[5]**

Q6) a) Prove that every prime element in a commutative integral domain R with 1 is irreducible in R. Is the converse true? Justify. **[5]**

b) Show that $1+2i\sqrt{5}$ is irreducible in $Z\left[i\sqrt{5}\right]$ Is it prime? **[5]**

Q7) a) Let I be an ideal in a ring R. Prove that I is a 2- sided ideal in a ring R if and only if I is the kernel of some homomorphism f : R $\rightarrow$ S for a suitable ring S. **[5]**

b) Let R = C ([0.1], R) be the ring of all continuous functions defined on [0,1] Show that R is an integral domain. **[5]**

Q8) a) Define **[5]**

i) Free module

ii) Torsion free module

Also give an example of torsion free module which is not a free module **[5]**

b) Show that any unitary module over a ring wiht unity is a quotient of a free module.

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Total No. of Questions : 8]

SEAT No. :

P2598

[Total No. of Pages : 3

[5021]-2004
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT - 604 : Linear Algebra
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) If U,V and W are subspaces of a vector space T such that $U \oplus V = U \oplus W$ then show that **[5]**

i) $V \cong W$

ii) V is not necessarily equal to W.

b) Let $F^{2\times2}$ be the vector space of all 2×2 matrices over F and $W = \left\{ \begin{pmatrix} x & y \\ z & 0 \end{pmatrix} \middle| x, y, z \in F \right\}$ show that W is subspace of $F^{2\times2}$. **[3]**

c) Find a basis of the subspace of R^3 generated by the vectors $V_1 = (2,0,-2)$, $V_2 = (2,4,2)$, $V_3 = (0,-6,4)$ **[2]**

Q2) a) Let U,V be vector spaces over F and $F:U \rightarrow V$ be a linear mapping then show that : **[5]**

i) Ker(f) is a subspace of U.

ii) The range of f is a subspace of V.

iii) F is one-one if and only if ker(f) = (0).

b) Let $f:R^2 \rightarrow R^2$ be a linear mapping, where f(a,b) = (a–3b, 9a+b). Find a basis for a range of f and hence determine the rank of f. **[3]**

c) Let $f:R^3 \rightarrow R^3$ be defined by f(x,y,z) = (x-3,y,2z). Determine whether f is linear transformation. **[2]**

P.T.O.

Q3) a) If V and U are vector spaces over F and f: V $\rightarrow$ U is a linear mapping from V onto U with Kernel K then show that $U \cong V/k$. Further, there is a one-to-one correspondence between the set of subspaces of V containing K and the set of subspaces of U. **[5]**

b) Let f : $R^3 \rightarrow R^3$ be a linear mapping defined by f(a,b,c) = (2a+3c,b-c,4c). Find the matrices A and B respectively of the linear mapping f with respect to the standard basis (e_1,e_2,e_3) and the basis (e^1_1, e^1_2, e^1_3) where $e^1_1 = (1,1,0), e^1_2 = (0,1,1), e^1_3 = (1,1,1)$. **[5]**

Q4) a) If $\phi \in$ Hom (v,v) and suppose $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ are distinct eigenvalues of ϕ and $x_1, x_2, \text{---}, x_n$ are eigenvectors associated with $\lambda_1, \ldots\ldots \lambda_n$ respectively. Then show that the set $\{x_1, x_2, \ldots, x_n\}$ is a linearly independent set. **[5]**

b) Let U,V be finite dimensional vector spaces over F and Let ϕ : V $\rightarrow$ U be a linear mapping. Then prove that, rank(ϕ) + nullity(ϕ) = dim(V) **[3]**

c) Define **[2]**

i) Basis of a vector space

ii) Jordan block

Q5) a) If $A \in F^{n\times m}$ matrix has n distinct eigen values $\lambda_1, \lambda_2, \ldots\ldots \lambda_n$ then show that there exists an invertible matrix P such that $P^{-1} AP = diag(\lambda_1, \lambda_2, \ldots\ldots \lambda_n)$ **[5]**

b) The three eigen vectors $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ of a 3×3 matrix A are associated respectively with eigen values 1,–1 and 0. Find matrix A **[3]**

c) Determine the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 3 & 1 & 0 \end{bmatrix}$, If exist. **[2]**

Q6) a) If matrix $A=\begin{bmatrix}1 & 1 & 2\\ -1 & 2 & 1\\ 0 & 1 & 3\end{bmatrix}$ then find a matrix P such that

$$P^{-1}AP=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$ **[5]**

b) Find the Jordan canonical form of the matrix, $A=\begin{bmatrix}0 & 4 & 2\\ -3 & 8 & 3\\ 4 & -8 & -2\end{bmatrix}$ **[3]**

c) Determine whether the given set of vectors are orthogonal. S = {(1,0,1), (0,1,1), (0,0,1)} **[2]**

Q7) a) Let B be a symmetric bilinear form on a finite dimensional vector space V over F. If ($e_1, e_2, \ldots, e_n$) is any orthogonal basis of V, then prove that the number of ei's such that B(ei,ei)=0 is equal to the dimension of $V^{\perp}$. **[5]**

b) Reduce the following matrix into triangular form $A=\begin{bmatrix}1 & -3 & 3\\ 0 & -1 & 2\\ 0 & -3 & 4\end{bmatrix}$ **[5]**

Q8) a) State and prove Sylvester's theorem. **[5]**

b) If T is a self-adjoint operator on a finite dimensional euclidean vector space E. Then prove that there is an orthonormal basis of E consisting of eigenvector of T. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2599

[Total No. of Pages : 3

[5021]-2005
M.A./M.Sc. (Semester - II)
MATHEMATICS
MT - 605 : Partial Differential Equations
(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the arbitrary function 'F' from the equation : **[4]**

$F(x - z, y - z) = 0$

b) Find the general integral of the equation : **[4]**

$z(xp - yq) = y^2 - x^2$

c) State the condition for the equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible on domain D. **[2]**

Q2) a) Verify that the following pfafifian differential equation is integrable and find it's primitive. **[5]**

$z(z + y^2)\, dx + z(z + x^2)\, dy - xy(x + y)\, dz = 0$

b) Explain the method of solving the first order partial differential equation $g(x, p) = h(y, q)$ **[3]**

c) Find the complete integral of the equation : **[2]**

$zy^2 dx + zx^2\, dy - x^2 y^2\, dz = 0$

P.T.O.

Q3) a) If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$ then prove that h_1 and h_2 satisfy : $\frac{\partial(f.h)}{\partial(x.u_x)} + \frac{\partial(f.h)}{\partial(y.u_y)} + \frac{\partial(f.h)}{\partial(z.u_z)} = 0$ **[4]**

b) Find the complete integral of the equation : **[4]**

$(1 + yz)\, dx + z(z - x)\, dy - (1 + xy)\, dz = 0$

c) Find the general integral of : $yz\, p + xz\, q = xy$ **[2]**

Q4) a) Find the complete integral of the first order partial differential equation : $(p^2 + q^2)\, y - qz = 0$ by charpitis method. **[4]**

b) Solve by Jacobi's method : $U_x^2 + U_y^2 + U_z = 1$ **[4]**

c) Derive the analytic expression for the Monge Cone at (x_0, y_0, z_0). **[2]**

Q5) a) F ind the integral surface for the differential equation : $z(xz_x - yz_y) = y^2 - x^2$, passing through the initial data curve (2s, s, s) **[4]**

b) Reduce the equation : $u_{xx} + 2u_{xy} + 17u_{yy} = 0$ to canonical form. **[4]**

c) Find the initial strip for the equation $pq = xy$ which passes through the curve : $z = x,\ y = 0$. **[2]**

Q6) a) Solve : $u_{xx} + u_{yy} = 0,\ 0 < x < a,\ 0 < y < b$ **[5]**

with the boundary conditions :

$u(x,0) = f(x), \quad 0 \le x \le a$

$u(x,b) = 0, \quad 0 \le x \le a$

$u(0,y) = 0, \quad 0 \le y \le b$

$u(a,y) = 0, \quad 0 \le y \le b$

b) State Dirichlet problem for rectangle and find it's solution. **[5]**

Q7) a) State and prove Kelvin's Inversion theorem. **[5]**

b) Prove that for the equation : $u_{xx} + \frac{1}{4}u = 0$ **[5]**

The Riemann function is : $V(x, y, \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$

Where J_0: denote the Bessel's function of the first kind of order zero.

Q8) a) Solve :

$$u_{tt} - c^2 u_{xx} = F(x,t), \ 0 < x < e, \ t > 0$$

$$u(x,0) = f(x), \quad 0 < x < e$$

$$u_t(x,0) = g(x), \quad 0 < x < e$$

$$u(0,t) = u(e,t), = 0, \quad t > 0$$

by using Duhamel's principle. **[5]**

b) Classify the following equation into Hyperbolic, Parabolic or Elliptic type. **[3]**

$e^z u_{xy} - u_{xx} = \log(x^2 + y^2 + z^2 + 1)$

c) State Harnack's theorem. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P2600

[Total No. of Pages : 4

[5021]-3001

M.A./M.Sc.

MATHEMATICS (Credit System)

MT - 701 : Combinatorics

(2013 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:-

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) How many arrangements of the letters in 'MISSISSIPPI' in which **[5]**

i) The M is immediately followed by an I?

ii) The M is beside an I, that is, an I is just before or just after the M.

b) Find a recurrence relation for the number of n-digit binary sequences with no pair of consecutive 1's? **[3]**

c) Using a generating function, find the number of distributions of 18 chocolate bunny rabbits into four Easter baskets with at least 3 rabbits in each basket. **[2]**

Q2) a) How many arrangements of letters in 'INSTITUTIONAL' have all of the following properties : **[5]**

i) No consecutive T's

ii) The two N's are consecutive and

iii) Vowels in alphabetical order.

P.T.O.

b) There are 15 different apples and 10 different pears. How many ways are there for Jack to pick an apple or a pear and then for Jill to pick an apple and a pear? **[3]**

c) Find two different chessboards (not row or column rearrangements of one another) that have the same rook polynomial. Write the rook polynomial. **[2]**

Q3) a) How many arrangements of 1, 1, 1, 1, 2, 3, 3 are there with the 2 not beside either 3? **[5]**

b) Solve the recurrence relation **[3]**

$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3},\ a_0 = a_1 = 1,\ a_2 = 2$

c) How many r-digit ternary sequences are there with an even number of O's? **[2]**

Q4) a) How many arrangements are there of 'MATHEMATICS' with both T's before both A's or both A's before both M's or both M's before the E? [By before, we mean anywhere before, not just immediately before] **[5]**

b) How many ways are there to distribute eight different toys among four children if the first child gets at least two toys? **[3]**

c) Using the identify $\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \dots + \binom{n}{r} = \binom{n+1}{r+1}$, evaluate the sum

$1\times2\times3 + 2\times3\times4 + \dots + (n-2)(n-1)n$ **[2]**

Q5) a) Find ordinary generating function whose coefficient a_r equals $r(r-1)(r-2)(r-3)$. Hence evaluate the sum :

$$4\times3\times2\times1 + 5\times4\times3\times2 + ...+n(n-1)(n-2)(n-3)$$ **[5]**

b) Solve the recurrence relation : **[3]**

$$a_n = -na_{n-1} + n!\ ,\ \text{When } a_o = 1$$

c) Find the coefficient of x^{25} in $(1+x^3+x^8)^{10}$ **[2]**

Q6) a) Using generating functions, solve the recurrence relation :

$$a_n = a_{n-1} + n(n-1),\ a_o = 1$$ **[5]**

b) Suppose a school with 120 students offers Yoga and Karate. If the number of students taking Yoga alone is twice the number taking Karate (possibly, Karate and Yoga), if 25 more students study neither skill than study both skills, and if 75 students take at least one skill, then how many students study Yoga? **[3]**

c) Find a generating function for the number of ways to make r cents' change in pennies, nickels, dimes and quarters. **[2]**

Q7) a) How many ways are there to paint the 10 identical rooms in a hotel with five colors if at most three rooms can be painted green, at most three painted blue, at most three red, and no constraint on the other two colors, black and white? **[5]**

b) How many permutations of the 26 letters are there that contain none of the sequences MATH, RUNS, FROM or JOE? **[5]**

Q8) a) A computer dating service wants to match seven women W_1, W_2, W_3, W_4, W_5, W_6, W_7 each with one of the seven Men M_1, M_2, M_3, M_4, M_5, M_6, M_7. **[5]**

If woman W_1 is incompatible with men M_1 or M_3;

woman W_2 is incompatible with men M_1 or M_5;

woman W_4 is incompatible with men M_3 or M_6;

woman W_5 is incompatible with men M_2 or M_7;

and woman W_7 is incompatible with man M_4. Also, women W_3 and W_6 are compatible with all men, How many matches of the seven women are there?

b) Assuming that n is a power of 2, solve the recurrence relation. **[5]**

$a_n = 4\ a_{n/2} + 3n$

(leaving a constant A to be determined)

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Total No. of Questions : 8]

SEAT No. :

P2601

[Total No. of Pages : 3

[5021]-3002

M.A./M.Sc.

MATHEMATICS

MT - 702 : Field Theory

(2013 Pattern) (Credit System) (Semester - III)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:-

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let F be a field and $p(x) \in F(x)$ be an irreducible polynomial. If K is an extension field of F containing a root α of $p(x)$ and $F(\alpha)$ denote the subfield of K generated over F by α then prove that

$$F(\alpha) \cong \frac{F(x)}{(p(x))}$$ **[5]**

b) Show that $x^2 + x + 1$ is irreducible polynomial over $\mathbb{F}_2$ (finite field with two elements) and hence obtained extension of $\mathbb{F}_2$ with four elements. **[3]**

c) Show that $\sqrt{2} \notin \phi(\alpha)$, where α is the real root of $f(x) = x^3 - 3x - 1$ between 0 and 2. **[2]**

Q2) a) Prove that if α is algebraic over F then there exist a unique monic irreducible polynomial $M_{\alpha,F}(x)$ in F(x) which has α as a root. **[5]**

b) If $[F(\alpha) : F]$ is odd then prove that $F(\alpha) = F(\alpha^2)$. **[3]**

c) Find the discriminant of the polynomial $P(x) = x^3 - x^2 - 4 \in \phi[x]$. **[2]**

P.T.O.

Q3) a) Prove that the extension $F\left(\sqrt[n]{a}\right)$ for $a \in F$ is cyclic over F of degree dividing n, where F be a field of characteristic not dividing n and F contains the n^{th} root of unity. **[5]**

b) Define the n^{th} cyclotomic polynomial $\Phi_n(x)$. Find cyclotomic polynomial $\Phi_n(x)$ for n=1, 2, 3, 4, 5. **[3]**

c) Define the following terms **[2]**

i) Cyclic extension

ii) Lagrange resolvent

Q4) a) Determine the splitting field and it's degree over Q for the polynomial $f(x) = x^3 - 2$. **[5]**

b) Prove that a polynomial over a field of characteristic O is separable if and only if it is the product of distinct irreducible polynomials. **[3]**

c) Show that squaring the circle is impossible by using straightedge and compass. **[2]**

Q5) a) Prove that there exists a finite field of order p^n and they are unique up to isomorphism. **[5]**

b) Find all automorphisms of $Q\left(\sqrt[3]{2}\right)$ over Q and hence show that $Q\left(\sqrt[3]{2}\right)$ is not Galois over Q. **[3]**

c) Determine the minimal polynomial over Q for the element $\sqrt{2}+\sqrt{5}$. **[2]**

Q6) a) Show that Galois group of $x^{p^n} - 2$ over $\mathbb{F}_p$ is a cyclic group of order n. **[5]**

b) If K/F be any finite extension then prove that $|Aut(K|F)| \leq [K:F]$. **[3]**

c) Show that $Q\left(\sqrt{2}, \sqrt[6]{2}\right) = Q\left(\sqrt[6]{2}\right)$. **[2]**

Q7) a) Show that $Q\left(\sqrt{2}, \sqrt{3}\right)$ is Galois over Q and it is isomorphic to klein 4 group. **[5]**

b) If K is algebraic over F and L is algebraic over K then prove that L is algebraic over F. **[5]**

Q8) a) If K_1, K_2 are two finite extension of a field F contained in a field K such that $[K_1, F] = n$, $[K_2, F] = m$ and m & n are relatively prime then prove that $[K_1K_2:F] = [K_1:F]\ [K_2:F]$. **[5]**

b) i) Show that all subfields of $Q\left(\sqrt{2}, \sqrt{3}\right)$ are Galois extension of Q.

ii) Determine the Galois group of the polynomial $f(x)=x^3+x^2-2x-1$. **[5]**

Total No. of Questions : 8]

SEAT No. :

P2602

[Total No. of Pages : 3

[5021]-3003

M.Sc.

MATHEMATICS

MT - 703 : Functional Analysis

(2013 Pattern) (Credit System) (Semester - III)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:-

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove the principle of Uniform Boundedness. **[5]**

b) Let M = {{xn}$\in l^2$: xn = 0 for all but finitely many n }. Is M a closed subspace of l^2? Justify. **[3]**

c) Define a reflexive space and give an example. **[2]**

Q2) a) State and prove Riesz Representation Theorem for a Hilbert space. **[5]**

b) Give an example of an orthonormal basis of L^2 ([0, 2π]). Justify. **[5]**

Q3) a) If A is a normal operator and λ, μ are distinct eigen values of A, then prove that ker $\left(A-\lambda\right)\perp$ ker $\left(A-\mu\right)$ **[4]**

b) Give an example of an isometry on a Hilbert space which is not surjective. Justify. **[4]**

c) State Hahn - Banach theorem. for Hilbert spaces. **[2]**

P.T.O.

Q4) a) For an operator A on a Hilbert space H, if A*=A and <Ah,h>=o for all $h \in H$, then prove that A=0. **[4]**

b) Show that for a separable Hilbert space H with a orthonormal basis $\{e_n\}$, if A is an operator defined by $Ae_n = \frac{1}{n} e_n$, then A is compact operator. **[3]**

c) Let P and Q be projections, show that P + Q is a projection if and only if ran P ⊥ ran Q. **[3]**

Q5) a) If H is a Hilbert space, M is closed subspace of H, then show that $\left(M^{\perp}\right)^{\perp} = M$ **[4]**

b) State open mapping theorem for Banach spaces and use it to prove that the inverse of an invertible, bounded linear map from a Banach space X to a Banach sapce Y is bounded. **[4]**

c) Give an example of a (non - identity) Unitary operator on Hilbert space. **[2]**

Q6) a) State and prove closed graph theorem for Banach spaces. **[5]**

b) Give an example of a Banach space which is not Hilbert space. **[2]**

c) If T is a positive compact operator, then prove that there is a unique positive compact operator A such that $A^2=T$. **[3]**

Q7) a) Let X be a normed space and f be a linear functional on X. If ker f is colsed then prove that f is continuous. **[4]**

b) Give an example of a convex set. **[2]**

c) If T is compact self - adjoint operator on a Hibert space then prove that either $\pm\|T\|$ is an eigen value of T. **[4]**

Q8) a) Prove that the operator T is normal if and only if real and imaginary parts of T commute. **[4]**

b) For any operator A, prove that Ker A = $(\text{ran } A^*)^{\perp}$. **[4]**

c) A linear operator T : $l^2 \rightarrow l^2$ is defined by $T(x_1, x_2, x_3, ...) = (0, x_1, x_2, x_3 ...)$ Find T*. **[2]**

Total No. of Questions : 8]

SEAT No. :

P2603

[Total No. of Pages : 3

[5021] - 4001

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT - 801 : Number Theory

(2013 Pattern) (Credit System)

Time : 3 Hours] ***[Max. Marks : 50***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) State and prove Wilson's theorem. **[5]**

b) Find a positive integer *n* such that $\mu(n)+\mu(n+1)+\mu(n+2)=3$ **[3]**

c) Let p be a prime and let (a, p)= (b, p) = 1. If $x^2 \equiv$ b (mod p) and $x^2 \equiv$ a (modp) are not solvable, then prove that $x^2 \equiv$ ab (mod p) is solvable. **[2]**

Q2) a) If p is an odd prime and (a, 2p) =1, then prove that $\left(\frac{a}{p}\right)=(-1)^t$, where $t=\sum_{j=1}^{(p-1)/2}\left[\frac{ja}{p}\right]$. Also prove that $\left(\frac{2}{p}\right)=(-1)^{\frac{p^2-1}{8}}$. **[5]**

b) Solve the congruence, $x^3+4x+8 \equiv 0$ (mod 15). **[3]**

c) Find all solutions of the congruence if it exists $20x \equiv 4$ (mod 30). **[2]**

P.T.O.

Q3) a) State and prove Chinese remainder theorem. **[5]**

b) Let x and y be real numbers, then prove that **[3]**

i) $[x]+[y]\le[x+y]\le[x]+[y]+1$

ii) $[x]+[-x]=\begin{cases}0, & \text{if x is an integer}\\ 1, & \text{otherwise}\end{cases}$

c) Show that 1387 is composite. **[2]**

Q4) a) i) If m_1 and m_2 denote two positive, relatively prime integers, then prove that $\phi(m_1m_2)=\phi(m_1)\phi(m_2)$.

ii) If m has the cannonical factorization $m=\prod p^{\alpha}$, then prove that

$$\phi(m)=\prod_{p|m}\left(p^{\alpha}-p^{\alpha-1}\right)=m\prod_{p|m}\left(1-\frac{1}{p}\right).$$

[5]

b) Let p be an odd prime, then prove that $\left(\frac{a}{p}\right)\equiv a^{\left(\frac{p-1}{2}\right)}$ (mod p) **[3]**

c) Evaluate $\left(\frac{51}{71}\right)$. **[2]**

Q5) a) If ξ is an algebraic number of degree n, then prove that every number in $\mathbb{Q}(\xi)$ can be written uniquely in the form $a_0+a_1\xi+a_2\xi^2+---+a_{n-1}\xi^{n-1}$ where a_i are rational numbers. **[5]**

b) Prove that $101x+37y = 3819$ has a positive solution in integer. **[5]**

Q6) a) Let m be a negative square free rational integer. Then prove that

i) The field $\mathbb{Q}\left(\sqrt{m}\right)$ has units ± 1, and these are the only units except in the cases $m = -1$ and $m = -3$

ii) The units in $\mathbb{Q}\ (i)$ are ± 1 and $\pm (i)$

iii) The units for $\mathbb{Q}\left(\sqrt{-3}\right)$ are $\pm 1,\ 1 \pm \sqrt{-3/2}$ and $\left(-1 \pm \sqrt{-3}\right)/2$.

[5]

b) If P and Q odd and positive integer and $(P, Q)=1$, then prove that $\left(\frac{P}{Q}\right)\left(\frac{Q}{P}\right) = (-1)^{\left[\frac{(P-1)}{2} \times \frac{(Q-1)}{2}\right]}$ **[5]**

Q7) a) Prove that an algebraic number ξ satisfies a unique irreducible monic equation $g(x) = 0$ over $\mathbb{Q}$. Also prove that every polynomial equation over $\mathbb{Q}$ satisfied by ξ is divisible by $g(x)$. **[5]**

b) Find the minimal polynomial of $\frac{\left(1+\sqrt[3]{7}\right)}{2}$. **[3]**

c) Show that $(1 - i)$ is an associate of $(1 + i)$ in $\mathbb{Q}\ (i)$. **[2]**

Q8) a) If $f(n) = \sum_{d|n} \mu(d) F(n|d)$ for every positive integer n, then prove that $\mathrm{F}(n) = \sum_{d|n} f(d)$. Where μ(d) is mobius μ-function. **[5]**

b) Prove that $\sum_{d|n} \phi(d) = n$ for every positive integer n. **[3]**

c) Define identical congruence and give an example of identical congruence.

[2]

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Total No. of Questions : 8]

SEAT No. :

P2604

[Total No. of Pages : 3

[5021]-4002
M.A./M.Sc.
MATHEMATICS
MT - 802 : Differential Geometry
(2013 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Show that for a smooth vector field $\bar{X}$ on an open set $U \subset \mathbb{R}^{n+1}$ and $P \in U$, there exists an open interval I containing O and a maximal integral curve $\alpha : I \to C$ passing through P. **[5]**

b) Find and sketch the gradient field of function $f(x, y) = x^2 - y^2 / g$. **[3]**

c) Show that gradient of f at $P \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at P. **[2]**

Q2) a) State and prove Lagranges Multiplier theorem for an n-surface. **[5]**

b) Show that graph of a smooth function on an open set in $\mathbb{R}^n$ is an n-surface in $\mathbb{R}^{n+1}$. **[3]**

c) Sketch the cylinder $f^{-1}(0)$ where $f(x_1, x_2, x_3) = x_1 - x_2^2$. **[2]**

Q3) a) Show that a connected n-surface in $\mathbb{R}^{n+1}$ has exactly two smooth unit normal vector fields. Find the normal vector fields of an n - sphere with unit radius. **[5]**

P.T.O.

b) Show that the mobius band is an unorientable 2 - surface. **[3]**

c) Define Gauss map. Compute the same for 2-sheeted hyperboloid $x_1^2 - x_2^2 - .... - x_{n+1}^2 = 4 \quad x_1 > 0$ **[2]**

Q4) a) Show that parallel transport $P_\alpha : S_p \to S_q$ where S is an n-surface in $\mathbb{R}^{n+1}$, p,q$\in$S and α is a peicewise smooth parametrized curve from p to q is an one one, onto linear map preserving the dot product. **[5]**

b) Show that Levi-civita parallel smooth vector field $\bar{X}$ on an n - surface S in $\mathbb{R}^{n+1}$ has constant length and if $\bar{Y}$ is some other Levi-civita parallel smooth vector field then $\bar{X}.\bar{Y}$ is constant along α. **[3]**

c) Find velocity, the acceleration and the speed of the parametrized curve given by $\bar{\alpha}(t) = (\cos 3t, \sin 3t)$ **[2]**

Q5) a) Define Weingarten map of a smooth n - surface and compute the same for the circular cylinder $x_2^2 + x_3^2 = a^2$ in $\mathbb{R}^3 (a \neq 0)$. **[5]**

b) Show that the Weingarten map is self adjoint. **[3]**

c) Compute $\nabla_v f$, where $f : \mathbb{R}^2 \to \mathbb{R}$ is given by $f(x_1, x_2) = 2x_1^2 + 3x_2^2$ at $\bar{v} = (1, 0, 2, 1)$. **[2]**

Q6) a) Show that for connected oriented plane curve C and a unit speed global parametrization $\beta : I \to C$, β is either one one or periodic. Discuss the case when C is compact. **[5]**

b) Find the length of connected oriented plane curve $f^{-1}(c)$ oriented by $-\nabla f / \| \nabla f \|$ where f is defined by

$$f(x, y) = \frac{1}{2}x^2 + \frac{1}{2}(y-1)^2 \text{ on } \mathbb{R}^2, c = 3.$$ **[3]**

c) Define Normal curvature and Gaussian curvature of a n - surface. **[2]**

Q7) a) Show that on each compact oriented n - surface S in $\mathbb{R}^{n+1}$, there exists a point P such that the second fundamental form at P is definite. **[5]**

b) Compute Gaussian curvature of the ellipsoid $\frac{x_1^2}{a^2}+\frac{x_2^2}{b^2}+\frac{x_3^2}{c^2}=1 \ (a,b,c \neq 0)$ oriented by inward normal. **[5]**

Q8) a) Show that second fundamental form Yp of an oriented n - surface S in $\mathbb{R}^{n+1}$ which is convex at $p \in S$, is semi-definite. **[5]**

b) Prove that if S is compact, connected oriented n - surface in $\mathbb{R}^{n+1}$ whose Gauss Kronecker curvature is no where zero then Gauss map $N:S \rightarrow S^n$ (unit n - sphere) is one-one and onto. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P2605

[Total No. of Pages : 4

[5021]-4003

M.A./M.Sc. (Semester - IV)

MATHEMATICS

MT - 803 : Fourier Analysis and Boundary Value Problems

(2013 Pattern) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) If a function *g(u)* is piecewise continuous on the interval $0 < u < \pi$ and that the right hand derivative $g'_R(0)$ exists, then prove that

$$\lim_{N \to \infty} \int_0^{\pi} g(u) D_N(u)\,du = \frac{\pi}{2}\, g\ (0^+).$$ **[5]**

Where $D_N(u)$ is the Dirichlet kernel.

b) Find the Fourier cosine series for the function $f(x) = \sin x$ on the interval $0 < x < \pi$. **[3]**

c) Find the Fourier sine series for the function $f(x) = \pi - x$ $(0 < x < \pi)$. **[2]**

Q2) a) Let f denote a function that is piecewise continuous on the interval $-\pi < x < \pi$ and periodic, with period 2π, on the entire x axis. Then prove that its Fourier series converges to the mean value $\frac{f(x^+) + f(x^-)}{2}$ of the one -sided limits of f at each point $x\left(-\infty < x < \infty\right)$ where both of the one sided derivatives $f'_R(x)$ and $f'_L(x)$ exists. **[5]**

P.T.O.

b) Find the Fourier series for the function $f(x)=\begin{cases}0 \text{ when } -\pi < x \le 0\\ x \text{ when } 0 < x < \pi\end{cases}$ **[3]**

c) Find Fourier sine series for function $f(x) = x^2$ $(0 < x < c)$. **[2]**

Q3) a) Solve the following boundary value problem **[5]**

$$y_{tt}(x,t) = a^2 y_{xx}(x,t) \qquad (0 < x < c,\ t > 0)$$

$$y(0,t) = 0, \quad y(c,t) = 0, \quad y_t(x,0) = 0\,.$$

b) Verify that the function $u_0 = y$, $u_n = \sinh ny \cos nx$ (n = 1, 2, 3...) satisfies Laplace's equation $u_{xx}(x, y) + u_{yy}(x, y) = 0$ $(0 < x < \pi, 0 < y < 2)$ and three boundary conditions $u(x, 0) = 0$, $u_x(0, y) = u_x(\pi, y) = 0$. **[3]**

c) If $f(x) = \sqrt[3]{x}$ $(-\pi < x < \pi)$, then show that $f(x)$ is piecewise continuous on the interval $-\pi < x < \pi$ but $f'(0^+)$ and $f'(0^-)$ do not exists. **[2]**

Q4) a) Solve the following boundary value problem. **[5]**

$$\rho^2 u_{\rho\rho}(\rho,\phi) + \rho u_{\rho}(\rho,\phi) + u_{\phi\phi}(\rho,\phi) = 0 \quad (1 < \rho < b, 0 < \phi < \pi)$$

$$u(\rho,0) = 0, \quad u(\rho,\pi) = 0, \quad (1 < \rho < b)$$

$$u(1,\phi) = 0, \quad u(b,\phi) = u_0, \quad (0 < \phi < \pi)$$

b) Solve the following boundary value problem. **[5]**

$$u_t(x,t) = k u_{xx}(x,t) \qquad (0 < x < \pi,\ t > 0)$$

$$u(0,t) = 0, \quad u_x(\pi,t) = 0 \qquad (t > 0)$$

$$u(x,0) = 0 \qquad (0 < x < \pi)$$

Q5) a) Prove that a necessary and sufficient condition for an orthonormal set $\{\phi_n(x)\}$ ($n = 1, 2, 3$ .....) to be complete is that for each function f in the space considered, Parseval's equation $\sum_{n=1}^{\infty} C_n^2 = \| f \|^2$ where C_n are the Fourier constants $C_n = (f, \phi_n)$, be satisfied. **[5]**

b) If C_n ($n = 1, 2, 3$....) are the Fourier constants for a function f in $C_p(a, b)$ with respect to an orthonormal set in that space, then prove that $\lim_{n \to \infty} C_n = 0$. **[3]**

c) If $\phi_0(x) = \frac{1}{\sqrt{2\pi}}$, $\phi_{2n-1}(x) = \frac{1}{\sqrt{\pi}} \cos nx$, $\phi_{2n}(x) = \frac{1}{\sqrt{\pi}} \sin nx$ ($n=1, 2, 3, ...$), then show that the set $\{\phi_n(x)\}$ (n = 0, 1, 2, ...) is orthonormal on the interval $-\pi < x < \pi$. **[2]**

Q6) a) If λ be an eigenvalue of the regular Sturm-Liouville problem $(rX')' + (q + \lambda p)X = 0 \ (a < x < b)$

$a_1 X(a) + a_2 X'(a) = 0, \ b_1 X(b) + b_2 X'(b) = 0$. If the conditions

$q(x) \le 0 \ (a \le x \le b)$ and $a_1 a_2 \le 0, \ b_1 b_2 \ge 0$ are satisfied, then prove that $\lambda \ge 0$. **[5]**

b) Find eigenvalues and normalized eigenfunction of Sturm - Liouville problem $X'' + \lambda X = 0$, $X(0) = 0$, $X'(1) = 0$. **[3]**

c) Show that each of the functions $y_1 = \frac{1}{x}$ and $y_2 = \frac{1}{1+x}$ satisfies nonlinear differential equation $y^1 + y^2 = 0$. Also show that if c is constant where $c \ne 0$ and $c \ne 1$, neither cy_1 nor cy_2 satisfies the equation. **[2]**

Q7) a) Derive Bessel's integral form **[5]**

$$J_n(x) = \frac{1}{\pi} \int_o^{\pi} \cos(n\phi - x \sin\phi)\, d\phi \quad (n = 0, 1, 2, 3, ...)$$

b) Let n be any fixed non-negative integer ($n = 0, 1, 2, ...$). Prove that the positive zeros of the function $J_n(x)$ or positive roots of the equation $J_n(x) = 0$ form an increasing sequence of numbers x_j ($j = 1, 2, 3$ ......) such that $x_j \to \infty$ as $j \to \infty$. **[5]**

Q8) a) Find the power series solution of Legendre's differential equation $(1-x^2)y'' - 2xy' + \lambda y = 0$. **[5]**

b) Derive Rodrigues formula for the Legendre's polynomials

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n . (n = 0, 1, 2, ...)$$ **[5]**

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