Total No. of Questions : 8]

SEAT No. :

P1407

[Total No. of Pages : 3

[5221]-101

M.A./M.Sc.

MATHEMATICS

MT - 501 : Real Analysis

(2013 Pattern) (Semester - I) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define an exterior measure of set $E \subseteq \mathbb{R}^d$. Show that exterior measure is countably subadditive. **[5]**

b) Show that closed sets of $\mathbb{R}^d$ are measurable. **[3]**

c) Is Lebesgue measure translation invarient? Justify. **[2]**

Q2) a) Define a measurable function defined on a measurable set. If f is a measurable function then show that following statements are equivalent: **[5]**

i) $\{x \in E / f(x) < \alpha\}$ is measurable for $\forall \alpha \in \mathbb{R}$.

ii) $\{x \in E / f(x) \leq \alpha\}$ is measurable for $\forall \alpha \in \mathbb{R}$.

iii) $\{x \in E / f(x) > \alpha\}$ is measurable for $\forall \alpha \in \mathbb{R}$.

iv) $\{x \in E / f(x) \geq \alpha\}$ is measurable for $\forall \alpha \in \mathbb{R}$.

b) State Littlewood's three principles. **[3]**

c) Give an example of a non-measurable function. **[2]**

P.T.O.

Q3) a) State and prove Egorov's theorem. **[5]**

b) Let $\{E_k\}$ be measurable subsets of $\mathbb{R}^d$ with $\sum_{k=1}^{\infty} m(E_k)$ is finite and if **[3]**

$$E = \lim_{k\to\infty} Sup(E_k)$$
$$= \{x \in \mathbb{R}^d / x \in E_k \text{ for infinitely many k's}\}$$

then show that E is measurable and m(E) = 0.

c) Show that a closed set is a G_δ set in $\mathbb{R}^d$. **[2]**

Q4) a) State and prove Riesz-Fischer theorem. **[5]**

b) If $f(x) = |x|^{-a}$ if $|x| \le 1$
$= 0$ otherwise
and $g(x) = |x|^{-b}$ if $|x| > 1$
$= 0$ otherwise

then show that f is integrable on $\mathbb{R}^d$ if $a<d$ and g is integrable on $\mathbb{R}^d$ if $b>d$. **[3]**

c) State Fatou's lemma. **[2]**

Q5) a) State Fubini's theorem and using it prove that, if f and g are integrable functions then $f*g$ is integrable and $\|f*g\|_{L^1(\mathbb{R}^d)} \le \|f\|_{L^1(\mathbb{R}^d)} \|g\|_{L^1(\mathbb{R}^d)}$. **[5]**

b) Show that if f is uniformely continuous on $\mathbb{R}$ and integrable then $\lim_{n\to\infty} Sup|f(x)| = 0$. **[3]**

c) If f and g are measurable on $\mathbb{R}^d$ then show that, $f(x-y)\, g(y)$ is integrable on $\mathbb{R}^{2d}$. **[2]**

Q6) a) Suppose F is real valued and of bounded variation on [a,b]. Then for all $a \le x \le b$ show that, **[5]**

$P_F(a,x) - N_F(a,x) = F(x) - F(a)$ and

$P_F(a,x) + N_F(a,x) = T_F(a,x)$

b) Prove that functions of bounded variations are bounded but converse does not hold. **[3]**

c) Define maximal function f^* of an Lebesgue integrable function f defined on $\mathbb{R}^d$. **[2]**

Q7) a) State and prove Lebesgue differentiation theorem. **[5]**

b) Define approximation to the identity. Suppose ϕ is bounded and integrable function on $\mathbb{R}^d$ having banded support with $\int_{\mathbb{R}^d} \phi(x)dx = 1$ and $K_\delta(x) = \delta^{-d}\ \phi(x/\delta)$ for $\delta > 0$ then verify that $\{K_\delta\}_{\delta > 0}$ is an approximation to the identity. **[5]**

Q8) a) State and prove rising sun lemma. **[5]**

b) Define Dini derivatives and find for **[5]**

$$f(x) = x\{1 + \sin(\log x)\} \text{ if } x > 0$$
$$= 0 \qquad \text{if } x = 0$$
$$= \sqrt{-x\sin^2(\log(x))} \text{ if } x < 0$$

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Total No. of Questions : 8]

SEAT No. :

P1408

[Total No. of Pages : 3

[5221] - 102

M.A./M.Sc.

MATHEMATICS

MT-502 : Advanced Calculus

(2013 Pattern) (Credit System) (Semester - I)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If a scalar field f is differentiable at $\bar{a}$ then show that all the directional derivatives of f exists at $\bar{a}$. **[5]**

b) If the vector fields $\bar{f}$, $\bar{g}:\mathbb{R}^n \to \mathbb{R}^m$ are continuous at $\bar{a}$, then prove that their dot product is also continuous at $\bar{a}$. **[3]**

c) Find the gradient vector at each point of the scalar field $f(x, y) = x^2 + y^2\sin(xy)$, if it exists. **[2]**

Q2) a) State and prove the chain rule for derivatives of scalar fields. **[5]**

b) Let $\bar{f}:\mathbb{R}^2 \to \mathbb{R}^2$ and $\bar{g}:\mathbb{R}^3 \to \mathbb{R}^2$ be two vector fields defined as follows :

$$\bar{f}(x,y) = e^{x+2y}\bar{i} + \sin(y+2x)\bar{j}$$

$$\bar{g}(u,v,w) = (u+2v^2+3w^3)\bar{i} + (2v-u^2)\bar{j}$$

i) Compute each of the Jacobian matrices $D\bar{f}(x,y)$ and $D\bar{g}(u,v,w)$.

ii) Compute the composition $\bar{h}(u,v,w) = \bar{f}[\bar{g}(u,v,w)]$.

iii) Compute the Jacobian matrix $D\bar{h}(1,-1,1)$. **[3]**

c) Find the directional derivative of scalar field $f(x, y, z) = 3x - 5y + 2z$ at (2, 2, 1) in the direction of the outward normal to the sphere $x^2 + y^2 + z^2 = 9$. **[2]**

P.T.O.

Q3) a) Define line integral of a vector field. Let $\bar{\alpha}$ and $\bar{\beta}$ be equivalent piecewise smooth paths. Let $\bar{f}$ be a vector field defined and bounded on the graph C of $\bar{\alpha}$ and $\bar{\beta}$. Prove that $\int_C \bar{f}.d\bar{\alpha} = \int_C \bar{f}.d\bar{\beta}$ if $\bar{\alpha}$ and $\bar{\beta}$ trace out C in the same direction. **[4]**

b) Find the amount of work done by the force $\bar{f}(x,y) = (x^2 - y^2)\bar{i} + 2xy\bar{j}$ in moving a particle (in a counter clockwise direction) once around the square bounded by the co-ordinate axes and lines $x = a$ and $y = a$, $a > 0$. **[4]**

c) Find the line integral of the vector field $\bar{f}(x,y) = (x^2 - 2xy)\bar{i} + (y^2 - 2xy)\bar{j}$ from (–1, 1) to (1, 1) along the parabola $y = x^2$. **[2]**

Q4) a) State and prove second fundamental theorem of calculus for line integrals. **[5]**

b) Compute the mass M of one coil of a spring having the shape of the helix whose vector equation is $\bar{\alpha}(t) = a\cos t\,\bar{i} + a\sin t\,\bar{j} + bt\,\bar{k}$ if the density at (x, y, z) is $x^2 + y^2 + z^2$. **[3]**

c) Determine whether or not the vector field $\bar{f}(x,y) = 3x^2y\bar{i} + x^3y\bar{j}$ is gradient on any open subset of $\mathbb{R}^2$. **[2]**

Q5) a) Define double integral of a step function. Explain how to use this to define the double integral of a function which is defined and bounded on rectangle. **[5]**

b) Use Green's theorem to evaluate the line integral $\oint_C y^2\,dx + x\,dy$ where C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). **[3]**

c) By using the transformation $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, $r > 0$, $0 \le \theta < 2\pi$. Transform an integral $\iiint_S f(x,y,z)\,dxdydz$ extended over a region S in xyz-space. **[2]**

Q6) a) Define fundamental vector product of a parametric surface. Find the fundamental vector product for the surface $\bar{r}(x,y)=x\bar{i}+y\bar{j}+f(x,y)\bar{k}$. What are the singular point of this surface? **[5]**

b) Compute the surface area of that portion of the sphere $x^2 + y^2 + z^2 = a^2$ lying within the cylinder $x^2 + y^2 = ay$ where $a > 0$. **[3]**

c) Define surface integral and explain the terms involved in it. **[2]**

Q7) a) State and prove Green's theorem for plane regions bounded by piecewise smooth jordan curves. **[5]**

b) Transform the integral to polar co-ordinate and compute its value $\int_0^{2a}\left[\int_0^{\sqrt{2ax-x^2}}\left(x^2+y^2\right)dy\right]dx$. **[5]**

Q8) a) Let S be a smooth parametric surface, $S=\bar{r}(T)$, where T is a region in the uv-plane bounded by a piecewise smooth Jordan curve Γ. Assume that $\bar{r}$ is a one-to-one mapping whose components have continuous second order partial derivatives on some open set containing $T\cup\Gamma$. Let C denote the image of Γ under $\bar{r}$ and let P, Q and R be continuously differentiable scalar fields on S.

Prove that

$$\iint_S\left(\frac{\partial R}{\partial y}-\frac{\partial \theta}{\partial z}\right)dy\wedge dz+\left(\frac{\partial P}{\partial z}-\frac{\partial R}{\partial x}\right)dz\wedge dx+\left(\frac{\partial \theta}{\partial x}-\frac{\partial P}{\partial y}\right)dx\wedge dy=\int_C P dx+Q dy+R dz$$

where the curve Γ is traversed in the positive (counter clockwise) direction and the curve C is traversed in direction inherited from Γ through the mapping function $\bar{r}$. **[5]**

b) If component of $\bar{F}$ has all mixed partial derivatives continuous then prove that $\text{Curl}\left(\text{Curl}\,\bar{F}\right)=\text{grad}\left(\text{div}\,\bar{F}\right)-\nabla^2\bar{F}$. **[5]**

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Total No. of Questions :8]

SEAT No. :

P1409

[Total No. of Pages : 2

[5221]-103
M.A/M.Sc.
MATHEMATICS
MT - 503 : Group Theory
(2013 Pattern) (Credit System) (Semester - I)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let $G=\left\{\begin{bmatrix} a & a \\ a & a \end{bmatrix} \middle| a\in \mathbb{R}, a\neq 0\right\}$. Show that G is a group under matrix multiplication. **[4]**

b) In a group G. Show that right and left cancellation laws hold. **[3]**

c) The integers 5 and 15 are among a collection of 12 integers that forms a group under multiplication modulo 56.
List all 12. **[3]**

Q2) a) State and prove Cayley's theorem. **[5]**

b) If ϕ is an isomorphism from G on to $\overline{G}$ then show that ϕ^{-1} is an isomorphism from $\overline{G}$ on to G. **[3]**

c) Find Inner automorphism of D_4. **[2]**

Q3) a) Define centralizer of a in group G. Further, show that centralizer of an element in a group G. Forms a subgroup of G. **[4]**

b) Is group of order 3 a cyclic? Justify. **[3]**

c) Suppose a group contains elements a and b such that $|a| = 4$, $|b| =2$, and $a^3b = ba$. Find $|ab|$. **[3]**

P.T.O.

Q4) a) State and prove Orbit-Stabilizer theorem for the finite group of permutations. **[5]**

b) Suppose that K is a proper subgroup of H and H is a proper subgroup of G. If | K | = 42 and |G|=420 then what are the possible orders of H? **[5]**

Q5) a) Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β **[3]**

b) Prove that A_n is a subgroup of S_n. **[2]**

c) Let G = <a> be a cyclic group of order n. Show that G = <a^k> if and only if (k,n) =1. **[5]**

Q6) a) If ρ and t are relatively prime then show that U(ρt) is isomorphic to external direct product of U(ρ) and U(t).

that is, $U(\rho t) \approx U(\rho) \oplus U(t)$. **[5]**

b) How many elements of order 5 are there in $Z_{25} \oplus Z_5$? List them. **[3]**

c) Prove or disprove, $Z \oplus Z$ is a cyclic group. **[2]**

Q7) a) Prove that, factor group of cyclic group is cyclic and hence find the elements of H and G/H, where G = Z/<20> and H=<4>/<20>. **[4]**

b) Explain Greedy algorithm for an abelian group of order p^n. **[3]**

c) Let ϕ be a group homomorphism from G to $\overline{G}$. Prove that, ker.ϕ is a normal subgroup of G. **[3]**

Q8) a) Define Sylow p-subgroup. Also, determine the groups of order 99. **[5]**

b) State Fundamental theorem of finite abelian groups of order $p^n.m$, where p is a prime that does not divide m. Also, express G as a internal direct product of cyclic groups of prime power order, where G ={1, 8, 17, 19, 26, 28, 37, 44, 46, 53, 62, 64, 71, 73, 82, 89, 91, 98, 107, 109, 116, 118, 127, 134} is the group under multiplication modulo 135. **[5]**

Total No. of Questions : 8]

SEAT No. :

P1410

[Total No. of Pages : 4

[5221]-104

M.A./M.Sc.

MATHEMATICS

MT-504 : Numerical Analysis

(2013 Pattern) (Semester-I) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable scientific calculator is allowed.*

Q1) a) Let g be continuous on the closed interval $[a, b]$ with $g:[a,b]\rightarrow[a,b]$. Then g has fixed point $p\in[a,b]$. Further more, if g is differentiable on the open interval (a, b) and there exists a positive integer k < 1 such that $|g'(x)|\leq k<1$ for all x in (a, b), then show that fixed point in $[a, b]$ is unique. **[5]**

b) Show that when Newton's method is applied to the equation $x^2 - a = 0$, the resulting iteration function is $g(x)=\frac{1}{2}\left(x+\frac{a}{x}\right)$. **[3]**

c) Compute the following limit and determine the corresponding rate of convergence $\lim_{n\to 0}\frac{\sin n}{n}$. **[2]**

Q2) a) Show that the function $g(x)=e^{-x^2}$ has unique fixed point on [0, 1] by using fixed point iteration method and starting value $P_0 = 0$. **[5]**
(Do at least 5 iterations).

b) The function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero in the interval (1, 2). Approximate this zero within an absolute tolerance of 5×10^{-5} using Newton's method starting with $P_0 = 1$. **[3]**

c) Define the terms: **[2]**

i) Order of convergence ii) The degree of precision

P.T.O.

Q3) a) Solve the following system using the Gaussian elimination with scaled partial pivoting. Show all intermediate matrices and the row vector at each step. Also show the constants of scaled vector. **[5]**

$$2x - y + z = 2,$$
$$4x + 2y + z = 7,$$
$$6x - 4y + 2z = 4.$$

b) Determine the Doolittle decomposition of the given matrix and then solve the system AX = b for the right hand side vector. **[3]**

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix}, b = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$$

c) Find the number of arithmetic operations required to find solution of an arbitrary system of 'n' equations by using Gaussian elimination with back substitution. **[2]**

Q4) a) Show that the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 0 & 2 \end{bmatrix}$ has no LU decomposition.

Rearrange the rows of A so that the resulting matrix does have an LU decomposition. **[5]**

b) For the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & 4 & 1 \\ -1 & 1 & 4 \end{bmatrix}$ and the right hand side vector $b = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$,

write out components of Jacobi method iteration equation. Then starting with initial vactor $X^{(0)} = 0$, perform two iterations of the Jacobi method. **[3]**

c) Show that the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ has no LU decomposition. **[2]**

Q5) a) Consider the matrix $A=\begin{bmatrix}1 & 4 & 5\\ 4 & -3 & 0\\ 5 & 0 & 7\end{bmatrix}$. Perform 5 iterations of power method starting with $X^{(0)} = [1\ 0\ 1]^T$. **[5]**

b) Define Householder matrix and show that it is symmetric and orthogonal. **[3]**

c) For the non-linear system **[2]**

$$1+x_2-e^{-x_1}=0$$
$$x_1^3-x_2=0,$$

Compute Jacobian of F.

Q6) a) Derive the closed Newton cotes formula with $n = 3$: **[5]**

$$\int_a^b f(x)dx=\left(\frac{b-a}{8}\right)[f(a)+3f(a+\Delta x)+3f(a+2\Delta x)+f(b)].$$

b) Derive the following backward difference approximation for the second derivative: **[3]**

$$f''(x_0)\approx\frac{f(x_0-2h)-2f(x_0-h)+f(x_0)}{h^2}$$

c) If $f(x) = 1 + x + x^3$, find $f'(2)$ for h = 0.01, 0.001. **[2]**

Q7) a) Verify that the composite trapezoidal rule has rate of convergence $0(h^2)$ by approximating the value of $\int_0^1 \sqrt{1+x^3}\,dx$. **[5]**

b) Use Householder's method to reduce the following symmetric matrix to tridiagonal form: **[5]**

$$A=\begin{bmatrix}-1 & -2 & 1 & 2\\ -2 & 3 & 0 & -2\\ 1 & 0 & 2 & 1\\ 2 & -2 & 1 & 4\end{bmatrix}.$$

Q8) a) Apply Euler's method to approximate the solution of initial value problem: **[5]**

$$\frac{dx}{dt} = \frac{t}{x}, (0 \leq t \leq 5),$$
$$X(0) = 1 \text{ using } 4 \text{ steps.}$$

b) Find solution of initial value problem **[5]**

$\frac{dx}{dt} = 1 + \frac{x}{t}$, $(1 \leq x \leq 6)$, $x(1) = 1$ using second-order Runge-Kutta method with $n = 0.5$.

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Total No. of Questions :8]

SEAT No. :

P1411

[Total No. of Pages :3

[5221] - 105

M.A./M.Sc.

MATHEMATICS

MT-505: Ordinary Differential Equations

(2013 Pattern) (Semester - I) (Credit System)

Time : 3 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $y_1(x)$ and $y_2(x)$ are two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that they are linearly dependent on this interval if and only if their Wronskian $W(y_1, y_2)$ is identically zero. **[5]**

b) Verify that $y_1 = x$ is one solution of differential equation $x^2 y'' + 2xy' - 2y = 0$ and find y_2 and the general solution. **[3]**

c) Show that $y = c_1 e^x + c_2 e^{-x}$ is general solution of $y'' - y = 0$ on any interval. **[2]**

Q2) a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **[5]**

b) Find the general solution of $y'' + 10y' + 25y = 14e^{-5x}$ by using method of undetermined coefficients. **[3]**

c) Change the independent variable x by $x = e^z$ and solve the differential equation $x^2 y'' + 3xy' + 10y = 0$. **[2]**

P.T.O.

Q3) a) State and prove sturm comparison theorem. **[5]**

b) Show that the zeros of the functions asinx + bcosx and csinx + dcosx are distict and occurs alternately if $ad - bc \neq 0$. **[3]**

c) Replace the differential equation $\frac{d^2x}{dt^2} + 4t\frac{dx}{dt} + t^2x = 0$ by an equivalent system of first order equations. **[2]**

Q4) a) Find the general solution of $\left(1+x^2\right)y'' + 2xy' - 2y = 0$ in terms of power series in x. **[5]**

b) Determine the nature of the point $x = \infty$ for the equation $x^2y'' + xy' + \left(x^2 - 4\right)y = 0.$ **[3]**

c) Locate and classify the singular point on the X - axis of $x^2\left(x^2-1\right)^2 y'' - x(1-x)y' + 2y = 0$ **[2]**

Q5) a) Find two independent Frobenius series solutions of the differential equation $4xy'' + 2y' + y = 0$. **[5]**

b) Prove that the function $E(x, y) = ax^2 + bxy + cy^2$ is positive definite if and only if $a > 0$ and $b^2 - 4ac < 0$. **[3]**

c) Find the critical point of the system

$$\frac{dx}{dt} = 2x - 2y + 10$$

$$\frac{dy}{dt} = 11x - 8y + 49$$ **[2]**

Q6) a) Solve the system

$$\frac{dx}{dt} = x + y$$

$$\frac{dy}{dt} = 4x - 2y.$$ **[5]**

b) Prove that $\log(1 + x) = x\,F(1, 1, 2, -x)$. **[3]**

c) State Picard's existence and uniqueness theorem. **[2]**

Q7) a) Find the general solution near x = 0 of the hypergeometric equation $x(1-x)y'' + [c-(a+b+1)x]y' - aby = 0$ where a, b and c are constants. **[5]**

b) If m_1 and m_2 are roots of the auxiliary equation of the system

$$\frac{dx}{dt} = a_1x + b_1y$$

$$\frac{dy}{dt} = a_2x + b_2y$$

which are real, distinct and of same sign then prove that the critical point (0, 0) is a node. **[5]**

Q8) a) Let F(x, y) be a continuous function that satisfies a Lipschitz condition $|f(x, y_1) - f(x, y_2)| < K|y_1 - y_2|$ on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y), y(x_0) = y_0$ has one and only one solution on the interval $a \le x \le b$. **[5]**

b) Solve the following initial value problem

$$\frac{dy}{dx} = z,\ \ y(0) = 1$$

$$\frac{dz}{dx} = -y,\ \ z(0) = 0.$$ **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1412

[Total No. of Pages : 3

[5221]-201
M.A./M.Sc.
MATHEMATICS
MT - 601:Complex Analysis
(2013 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt ANY FIVE questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If $f(z)=f(x+iy)=\sqrt{|x||y|}, x, y \in \mathbb{R}$ then show that the function f satisfy C.R. equations at origin but f is not holomorphic at origin. **[5]**

b) If f is holomorphic in *a* region Ω and $f'=0$ then prove that f is constant function. **[3]**

c) Find radius of convergence of the series $\sum_{n=0}^{\infty} \frac{n^2}{4^n+3n} z^n$. **[2]**

Q2) a) Show that the power series $\sum_{n=1}^{\infty} nz^n$ does not converge on any point of the unit circle. **[5]**

b) If γ be a smooth curve in $\mathbb{C}$ parametrized by $z(t)=[a,b] \to \mathbb{C}$ and γ^- denote the curve with same image as γ but with opposite orientation then prove that, $\int_{\gamma} f(z)dz = -\int_{\gamma^-} f(z)dz$. **[3]**

c) If f is continuous function in region Ω then prove that any two primitive of f differe by *a* constant. **[2]**

Q3) a) If f is holomorphic in an open set Ω that contains *a* rectangle R and it's interior then prove that $\int_{R} f(z)dz = 0$. **[5]**

b) If f is holomorphic function in Ω^+ that extend continuously to I and such that f is real valued on I then prove that there exists a function F holomorphic in all of Ω such that F $= f$ on Ω^+. **[3]**

c) State symmetric principle. **[2]**

P.T.O.

Q4) a) If f is holomorphic in an open set Ω. If D is a disc centered at z_0 and whose closure is contained in Ω then prove that f has a power series expansion at z_0, $f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n$ $\forall Z \in D$ where

$$a_n = \frac{f^{(n)}(Z_0)}{n!}, \forall n \geq 0.$$ **[5]**

b) Show that every non-constant polynomial $P(z)=a_n z^n + --- + a_n$ with complex coefficient has a root in $\mathbb{C}$. **[3]**

c) State Runge's approximation theorem. **[2]**

Q5) a) If $\{fn\}_{n=1}^{\infty}$ is a sequence of holomorphic function that converges uniformly to a function *f* in every compact subset of Ω then prove that *f* is holomorphic in Ω. **[3]**

b) Show that $\int_0^{\infty} \frac{1-\cos x}{x^2} dx = \frac{\pi}{2}$ **[5]**

c) Find the nature of isolated singularity of origin for the function $f(z)=\frac{\sin z}{z}$. **[2]**

Q6) a) If *f* has a pole of order *n* at z_0, then prove that

$$f(z)=\frac{a_{-n}}{(z-z_0)^n}+\frac{a_{-n+1}}{(z-z_0)^{n-1}}+---+\frac{a_{-1}}{(z-z_0)}+G(z)$$

Where G is a holomorphic function in a neighborhood of Z_0. **[5]**

b) If *f* and *g* are holomorphic in an open set containing a circle C and it's interior and $|f(z)| > |g(z)|$ for all Z in C then prove that *f* and *f* + *g* have the same number of zeros inside a circle C. **[3]**

c) State morera's theorem. **[2]**

Q7) a) Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$. **[5]**

b) If f is holomorphic in an open set containing a circle C and it's interior except for poles at the points Z_1, Z_2, ---- Z_N inside C then prove that

$\int_C f(z)dz = 2\pi i \sum_{k=1}^{N} res_{\Sigma} f$. **[5]**

Q8) a) Show that the complex zeros of sin πz are exactly at the integers and each of order one.

Also find residue of $\frac{1}{\sin \pi z}$ at $z = n \in \mathbb{Z}$. **[5]**

b) Let $D = \{z \in \mathbb{C} / |z| = 1\}$ and $f : D \to D$ be holomorphic function then prove that $|f'(z)| \leq \frac{1}{1-|z|}$ $\forall Z \in D$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1413

[Total No. of Pages : 3

[5221]-202
M.A./M.Sc.
MATHEMATICS
MT - 602 : General Topology
(2013 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let A be a set. If there exists an injective function $f : \mathbb{Z}_+ \to A$, then prove that there exists a bijection of A with a proper subset of itself. **[5]**

b) If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\cap\tau_\alpha$ is a topology on X. Is $\cup\tau_\alpha$ a topology on X? Justify your answer. **[3]**

c) Define order topology. Give an example of order topology. **[2]**

Q2) a) Let X be an ordered set in the order topology. Let Y be a subset of X that is convex in X. Then prove that the order topology on Y is the same as the topology on Y inherits as a subspace of Y. **[5]**

b) If $\mathbb{B}$ is a basis for the topology of X and $\mathscr{C}$ is a basis for the topology of Y, then prove that the collection $\mathscr{D} = \{B\times C \mid B\in \mathbb{B} \text{ and } C\in \mathscr{C}\}$ is a basis for the topology of X×Y. **[3]**

c) If τ and τ' are topologies on X and τ' is strictly finer than τ, what can you say about the corresponding subspace topologies on the subset Y of X? Justify your answer. **[2]**

P.T.O.

Q3) a) Let Y be a subspace of X and A be a subset of Y. Then prove that A is closed in Y if and only if it equals the intersection of a closed set of X with Y. **[5]**

b) If $A \subset X$ and $B \subset Y$, show that in the space X×Y, $\overline{A \times B} = \bar{A} \times \bar{B}$. **[3]**

c) Show that every order topology is Hausdorff. **[2]**

Q4) a) Let X and Y be topological spaces. Let $f : X \to Y$. Prove that f is continuous if and only if for each $x \in X$ and each neighbourhood V of f(x), there is a neighbourhood U of x such that $f(U) \subset V$. **[5]**

b) Show that the subspace (a,b) of $\mathbb{R}$ is homeomorphic with (0,1) and the subspace [a,b] of R is homeomorphic with [0,1]. **[3]**

c) Let Y be an ordered set in the order topology. Let $f, g : X \to Y$ be continuous. Show that the set $\{x \mid f(x) \leq g(x)\}$ is closed in X. **[2]**

Q5) a) Let $f : A \to X \times Y$ be given by the equation $f(a) = (f_1(a), f_2(a))$. Then prove that f is continuous if and only if the functions $f_1 : A \to X$ and $f_2 : A \to Y$ are continuous. **[5]**

b) Define the box topology and the product topology. State the comparison of the box and product topologies. **[3]**

c) Let $f : X \to Y$. If the function f is continuous, then prove that for every convergent sequence $x_n \to x$ in X, the sequence $f(x_n)$ converges to $f(x)$. **[2]**

Q6) a) Let $p : X \to Y$ be a quotient map. Let Z be a space and let $g : X \to Z$ be a map that is constant on each set $p^{-1}(\{y\})$, for $y \in Y$. Then g induces a map $f : Y \to Z$ such that $f o p = g$. Prove that the induced map f is continouous if and only if g is continuous. Also prove that f is a quotient map if and only if g is a quotient map. **[5]**

b) Let τ and τ' be two topologies on X. Let $\tau' \supset \tau$. What does connectedness of X in one topology imply about connectedness in the other? Justify. **[3]**

c) State an intermediate value theorem for connectedness. **[2]**

Q7) a) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X. **[5]**

b) Show that if X is compact Hausdorff under both τ and τ', then either τ and τ' are equal or they are not comparable. **[5]**

Q8) a) Let X be a topological space. Let one point sets in X be closed. Prove that X is regular if and only if given a point x of X and a neighbourhood U of x, there is a neighbourhood V of x such that $\bar{V} \subset U$. **[5]**

b) Let X be a set. Let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Then prove that: **[5]**

i) Any finite intersection of elements of $\mathcal{D}$ is an element of $\mathcal{D}$.

ii) If A is a subset of X that intersets every element of $\mathcal{D}$, then A is an element of $\mathcal{D}$.

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Total No. of Questions :8]

SEAT No. :

P1414

[Total No. of Pages : 3

[5221]-203
M.A./M.Sc.
MATHEMATICS
MT - 603 : Ring Theory
(2013 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If R is commutative ring with 1, then prove that $A \in M_n(R)$ is a unit if and only if its determinant, det(A) is a unit in R. **[5]**

b) Give an example of subring S of a ring R in which an element $a \in S$ may be zero divisor in R but not in S.Justify. **[3]**

c) Give an example of division ring which is not a field. Justify. **[2]**

Q2) a) Let R be a ring with 1 such that the non units in R form a subgroup of (R,+). Prove that char(R) is either zero or else a power of a prime. **[5]**

b) In any ring R, show that ab is nilpotent if and only if ba is nilpotent where a,b$\in$ R. **[3]**

c) Prove or disprove

The ring $\mathbb{Z}_{18}[x]$ is an integral domain **[2]**

Q3) a) If R is ring with 1 and I is left ideal in R such that $I \neq R$, then prove that there exist a maximal left ideal M of R such that $I \subseteq M$. **[5]**

b) Prove or disprove:

The ideal (o) is maximal in Q[i] **[2]**

c) Prove that in a commutative ring every maximal ideal is a prime ideal. **[3]**

P.T.O.

Q4) a) Let $f:\mathrm{R}\to\mathrm{S}$ and $g:\mathrm{S}\to\mathrm{T}$ are homomorphism of rings. Prove that $\mathrm{Ker}(f)\subseteq\mathrm{Ker}(gof)$, also show that equality need not hold **[4]**

b) Prove or disprove:

Homomorphic image of an idempotent element is an idempotent element. **[2]**

c) Let $f:\mathrm{R}\to\mathrm{S}$ be a homomorphism of rings. If R and S are commutative rings then prove that inverse image of a prime ideal is a prime ideal in R. **[4]**

Q5) a) Let R be a commutative integral domain with 1. Prove that $(\mathrm{Q(R)},+,\bullet)$ is a field containing R as a subring. **[5]**

b) Let R be a commutative ring with 1. a_0 is unit in R and $a_1, a_2, ..., a_r$ are nilpotent in R. Prove that $a(\mathrm{X})=a_0+a_1\mathrm{X}+a_2\mathrm{X}^2+......+a_r\mathrm{X}^r$ is a unit in R[X]. **[3]**

c) Define local ring. Give an example of non-local ring. **[2]**

Q6) a) If d is positive integer then prove that the ring $\mathbb{Z}\left[i\sqrt{d}\right]$ is a factorization domain. **[4]**

b) Prove or disprove:

Subring of a UFD is UFD **[3]**

c) Let $R=(\mathrm{C}[0,1],\mathbb{R})$ be a ring of all real valued continuous function defined on [0,1] under pointwise addition and multiplication of maps. Show that the set of all differentiable functions in R is not an ideal in R. **[3]**

Q7) a) State and prove Eisensteins Criterion. **[5]**

b) Show that the element $2+i\sqrt{5}$ is irreducible in $\mathbb{Z}\left[i\sqrt{5}\right]$ but not prime in $\mathbb{Z}\left[i\sqrt{5}\right]$. **[5]**

Q8) a) Give an example of free module in which a linearly independent subset cannot be extended to a basis. Justify. **[5]**

b) Prove that any unitary module over ring with unity is a quotient of a free module. **[5]**

Total No. of Questions :8]

SEAT No. :

P1415

[Total No. of Pages :4

[5221] - 204

M.A./M.Sc.

MATHEMATICS

MT-604: Linear Algebra

(2013 Pattern) (Semester - II) (Credit System)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Answer any five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non - programmable, scientific calculator is allowed.*

Q1) a) Let V be the vector space of all mappings from R to R and V_1, V_2 be the subsets of even and odd functions respectively that is,

$$V_1 = \{f \in V | f(-x) = f(x)\}$$

and $V_2 = \{f \in V | f(-x) = -f(x)\}$

then show that V is direct sum of V_1 and V_2. **[5]**

b) Find a basis of the subspace of R^4 generated by the vectors $V_1 = (1, 1, 2, 0)$; $V_2 = (1, 2, 3, 4)$; $V_3 = (0, 4, 5, 2)$. **[3]**

c) Find a basis of the vector space $\mathbb{C}$ over $\mathbb{R}$. **[2]**

Q2) a) If V and U are vector spaces over F and $f : V \to U$ is a linear mapping from V onto U, with Kernal K then show that $U \cong V/K$. Further, show that there is a one - to - one correspondence between the set of subspace of V containing K and the set of subspace of U. **[5]**

b) Let $f: R^2 \to R^2$ be a linear mapping, where $f(a, b) = (2a - b, 4a + 5b)$. Find a basis for a range of f and hence determine the rank of f. **[3]**

c) Let $f: R^3 \to R^3$ be defined by $f(x, y, z) = (2x, y - 3z, 1)$. Determine whether f is linear transformation. **[2]**

Q3) a) Let $f: F^{n \times n} \to F^{n \times n}$ be a mapping such that $f(A) = AB$, $A \in F^{n \times n}$ and B is fixed $n \times n$ matrix **[5]**

i) Prove that f is a linear mapping.

ii) Show that ker$f = (0)$ if and only if B is invertible.

b) Let $f: R^3 \to R^3$ be a linear mapping defined by $f(a, b, c) = (a, a + b, 0)$. Find the matrices A and B respectively of the linear mapping f with respect to the standard basis (e_1, e_2, e_3) and the basis (e_1', e_2', e_3') where $e_1' = (1, 1, 0)$, $e_2' = (0, 1, 1)$, $e_3' = (1, 1, 1)$. **[3]**

c) What is the dimension of the vector space V = {P_n – Polynomial of degree $\leq$ n, with real coefficients} **[2]**

Q4) a) If $A \in F^{n \times n}$ matrix has n distinct eigen values $\lambda_1, \lambda_2, ----, \lambda_n$ then show there exists an invertible matrix P such that $P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, ----, \lambda_n)$. **[5]**

b) The three eigen vectors $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, of a 3×3 matrix A are associated respectively with eigen values 1, –1 and 0. Find matrix A. **[3]**

c) Determine the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, if exist. **[2]**

Q5) a) Reduce the following matrix into triangular form, $A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -1 & 2 \\ 0 & -3 & 4 \end{bmatrix}$ **[5]**

b) Find the Jordan canonical form of $A = \begin{bmatrix} 1 & 5 & 7 \\ 0 & 4 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ **[3]**

c) Determine whether the given set of vectors are orthogonal, S = {(1, 0, 1), (1, 0, 0), (0, –1, 0)} **[2]**

Q6) a) Let V be a vector space of dimension n over F. Then show that there is a 1 – 1 correspondence between the set of bilinear form on V and the set of n×n matrices over F. **[5]**

b) If B is symmetric bilinear form on a vector space V over a field F and let char(F) ≠ 2 then prove that there exists an orthogonal basis of V relative to B. **[3]**

c) If the matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & 4 \\ -2 & 4 & -2 \end{bmatrix}$ then find quadratic form of the matrix A. **[2]**

Q7) a) Prove that, if T is a self - adjoint operator on a finite - dimensional Euclidean vector space E then there is an orthonormal basis E consisting of eigen vectors of T. **[5]**

b) Let V be the vector space of continuous real valued functions on the interval [0, 1]. Define $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. Show that < , > is a symmetric bilinear form. **[5]**

Q8) a) State and prove Sylvester's theorem. **[5]**

b) If matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ then find a matrix P such that

$$P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$ **[5]**

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Total No. of Questions :8]

SEAT No. :

P1416

[Total No. of Pages : 3

[5221] - 205

M. A/M.Sc.

MATHEMATICS

MT - 605: Partial Differential Equations

(2013 Pattern) (Credit System) (Semester - II)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) Eliminate the arbitrary function, 'F' from the equation $x + y + z = F(x^2 + y^2 + z^2)$. **[4]**

b) Find the general integral of: $x(y - z)\,p + y(z - x)\,q = z\,(x - y)$. **[4]**

c) State the conditions for equations $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ to be compatible on domain D. **[2]**

Q2) a) Prove that the pfaffian differential equation $\vec{X}.d\vec{r} = P(x, y, z)\,dx + Q(x, y, z)\,dy + R(x, y, z)\,dz = 0$ is integrable if and only if $\vec{X}$. curl $\vec{X} = 0$ **[4]**

b) Show that the equations $f = p^2 + q^2 - 1 = 0$ and $g = (p^2 + q^2)\,x - pz = 0$ are compatible and find one- parameter family of common solutions. **[4]**

c) Find a complete integral of the partial differential equation, $z = px + qy + pq$. **[2]**

Q3) a) Find a complete integral of the partial differential equation:

$z\,(p^2 + q^2) + px + qy = 0$, by Charpit's method. **[4]**

b) Find the integral surface passing through the initial data curve: C: $x_0 = -1$, $y_0 = 5$, $z_0 = \sqrt{5}$ of the equation $(x + 2)\,p + 2yq = 2z$ **[4]**

c) Verify that the equation: **[2]**

$z\,(z + 4)\,dx + z\,(z + x)\,dy - 2xy\,dz = 0$ is integrable.

P.T.O.

Q4) a) Reduce the equation: $u_{xx} + x\, u_{yy} = 0$ in the region $x < 0$ to canonical forms. **[4]**

b) Solve the equation: $xu_x + yu_y = u_{z^2}$ by Jacobi is method. **[4]**

c) State Harnack's theorem. **[2]**

Q5) a) Using D - Alembert is Solution of infinite string,

find solution of: $y_{xx} = \frac{1}{c^2} y_{tt},\ 0 < x < \infty,\ t > 0$

$$y(x,0) = u(x),\quad yt(x,0) = v(x),\ x \geq 0$$

$$y(0,t) = 0,\ t \geq 0.$$ **[4]**

b) State and prove Kelvin's Inversion Theorem. **[4]**

c) Find two initial strips for the equation:

$Z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through x -axis. **[2]**

Q6) a) State Dirichlet's problem for rectangle and find it's solution. **[5]**

b) Find the Solution of the Heat - equation in an infinite rod which is defined as: $u_t = Ku_{xx},\ -\infty < x < \infty,\ t > 0$

$$u(x,0) = f(x),\ -\infty < x < \infty$$ **[5]**

Q7) a) Prove that for the equation: $Lu = u_{xy} + \frac{1}{4}u = 0$

The Riemann function is: $V(x, y, \alpha, \beta) = J_0\left(\sqrt{(x-\alpha)(y-\beta)}\right)$, where Jo - denote the Bessel's function of the first kind of order zero **[5]**

b) Classify the following equations into Hyperbolic, Parabolic or Elliptic type.

i) $u_{xx} + 2(1 + \alpha y)\, u_{yz} = 0$

ii) $u_{xx} + 2\, u_{xy} + u_{yy} + 2u_{zz} - (1 + xy)\, u = 0$ **[5]**

Q8) a) Using Duhamel's principle find the solution of non - homogenous heat - equation.

$u_t + \mathrm{K}\, u_{xx} = \mathrm{F}\,(x, t), \; -\infty < x < \infty, \;\; t > 0$

$u(x, 0) = 0, \; -\infty < x < \infty$ **[5]**

b) If $u(x, y)$ is harmonic in bounded domain D and continuous in $\overline{\mathrm{D}} = \mathrm{D} \cup \mathrm{B}$, then u attains it's maximum on the boundary B of D. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P1417

[Total No. of Pages : 3

[5221]-301
M.A./M.Sc.
MATHEMATICS
MT - 701 : Combinatorics
(2013 Pattern) (Semester - III) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) How many arrangements of 'MISSISSIPPI' are there with no pair of consecutive S's? **[5]**

b) How many numbers between 0 and 10,000 have a sum of digits equal to 13? **[3]**

c) How many subsets of three different integers between 1 and 90 inclusive are there whose sum is an even number? **[2]**

Q2) a) How many ways are there to distribute 25 identical balls into six distinct boxes with at most 6 balls in any of the first three boxes? **[5]**

b) How many arrangements of the 26 letters of the alphabet in which **[3]**

i) a occurs before b?

ii) a occurs before b and c occurs before d?

c) Find the number of derangements of 1,2,3,4,5 using the associated chessboard of darkened squares. **[2]**

Q3) a) How many arrangements of letters in 'INSTITUTIONAL' have all of the following properties. **[5]**

i) No consecutive T's

ii) The two N's are consecutive

iii) Vowels in alphabetical order

b) Solve the recurrence relation: **[3]**

$a_n = 3a_{n-1} + n^2 - 3$, with $a_0 = 1$.

c) How many 10 letter words are there in which each of the letters e,n,r,s occur at least once? **[2]**

Q4) a) If 10 steaks and 15 lobsters are distributed among four people, how many ways are there to give each person at most 5 steaks and at most 5 lobsters? **[5]**

b) How many r-digit ternary sequences are there in which no digit occurs exactly twice? **[3]**

c) Prove by combinatorial argument **[2]**

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$

Q5) a) Find ordinary generating function whose coefficient a_r equals $r(r-1)(r-2)(r-3)$. Hence evaluate the sum: **[5]**

$4\times3\times2\times1 + 5\times4\times3\times2 + \ldots\ldots\ldots\ldots\ldots + n(n-1)(n-2)(n-3)$

b) Solve the recurrence relation $a_n = -na_{n-1} + n!$ where $a_0 = 1$ **[3]**

c) How many numbers greater than 30,00,000 can be formed by arrangements of 1,2,2,4,6,6,6? **[2]**

Q6) a) Using generating functions, solve the recurrence relation: **[5]**

$a_n = a_{n-1} + n(n-1), \quad a_0 = 1.$

b) How many arrangements are there of 'MURMUR' with no pair of consecutive letters the same? **[3]**

c) Find a generating function for the number of ways to write the integer r as a sum of positive integers in which no integer appears no more than three times. **[2]**

Q7) a) Find a recurrence relation for a_n, the number of n-digit ternary sequences without any occurrence of the subsequence "012". **[5]**

b) There are 100 students in a school and there are 40 students taking each language, French, Latin and German. Twenty students are taking only French, 20 only Latin and 15 only German. In additon, 10 students are taking French and Latin. How many students are taking all three languages? How many students are taking no Language? **[5]**

Q8) a) How many ways are there to send seven birthday cards denoted $C_1, C_2, C_3, C_4, C_5, C_6, C_7$ to seven friends $F_1, F_2, F_3, F_4, F_5, F_6, F_7$ if friend F_1 would not like cards C_1 or C_3; F_2 would not like cards C_1 or C_5; F_4 would not like C_3 or C_6; F_5 would not like C_2 or C_7; F_7 would not like C_4; Friends F_3 and F_6 would like all the cards. **[5]**

b) Solve the recurrence relation, assuming that n is a power of 2(leaving a constant A to be determined). **[5]**

$a_n = 16a_{n/2} + 5n$

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Total No. of Questions : 8]

SEAT No. :

P1418

[Total No. of Pages : 2

[5221] - 302

M.A./M.Sc.

MATHEMATICS

MT-702 : Field Theory

(2013 Pattern) (Credit System) (Semester - III)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that any extension K of a field F of degree 2 is of the form $F(\sqrt{D})$, where D is an element of F which is not a square in F and characteristics of F is not equal to 2. **[5]**

b) Show that $p(x) = x^3 + 9x + 6$ is irreducible over Q. Let θ be root of $p(x)$ then find inverse of $1 + \theta$ in $Q(\theta)$. **[3]**

c) Show that any automorphism of a field K fixes it's prime subfield. **[2]**

Q2) a) If L is algebraic over K and K is algebraic over a field F then prove that L is algebraic over F. **[5]**

b) State fundamental theorem of Galois theory. **[3]**

c) Show that there is no element of $Q(\sqrt{2})$ that is a zero of $x^3 - 2$. **[2]**

Q3) a) Show that a field K generated over F by a finite number of algebraic elements of degrees $n_1, n_2, ..., n_r$ is algebraic of degree $\leq n_1 n_2.....n_r$.**[5]**

b) If a field K is algebraically closed and F be a subfield of K then prove that the collection of elements of K that are algebraic over F is an algebraic closure of F. **[3]**

c) Find the minimal polynomial for $\sqrt{2}+\sqrt{3}$ over Q. **[2]**

P.T.O.

Q4) a) Show that Galois group of $x^3 - 2 \in Q[x]$ is a group of symmetric on three letters. **[5]**

b) Prove that a polynomial $f(x) \in F[x]$ is separable if and only if $f(x)$ and $D_x(f(x))$ are relatively prime. **[3]**

c) Is $x^n - 1$ separable polynomial over a field of characteristic p where $p|n$? Justify. **[2]**

Q5) a) Find the spliting field of $f(x) = x^n - 1 \in Q[x]$ over Q. **[5]**

b) Is the extension $Q(\sqrt{2})$ of Q Galois? If yes then find $Gal(Q(\sqrt{2})/Q)$. **[3]**

c) Prove that it is impossible to construct the regular 9-gon by using straightedge and compass. **[2]**

Q6) a) Define cyclic extension. Prove that the extension $F(\sqrt[n]{a})$, $a \in F$ is cyclic over F of degree dividing n. Where F be a field of characteristic not dividing n and which contains the n^{th} root of unity. **[5]**

b) Prove that $x^n - 1 = \prod_{d/n} \Phi_d(x)$. Where $\Phi_d(x)$ is the d^{th} cyclotomic polynomial. Hence find $\Phi_1(x)$, $\Phi_2(x)$ and $\Phi_3(x)$. **[3]**

c) If $\phi : F \to F'$ be a homomorphism of fields then prove that ϕ is either identically 0 or is injective. **[2]**

Q7) a) Let F be a field and $f(x) \in F[x]$ then prove that there exist and extension K of F which is a splitting for $f(x)$ over F. **[5]**

b) Prove that the Galois group of $f(x) \in F[x]$ is a subgroup of A_n if and only if the discriminant $D \in F$ is the square of an element of F. **[5]**

Q8) a) Show that $f(x) = x^4 + 1$ is irreducible polynomial over $\mathbb{Z}$ but reducible modulo every prime p. **[5]**

b) Let F be a finite field of characteristic p then prove that $|F| = p^n$ for some positive integer n. **[5]**

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Total No. of Questions :8]

SEAT No. :

P1419

[Total No. of Pages : 2

[5221]-303

M.A./M.Sc.

MATHEMATICS

MT - 703 : Functional Analysis

(2013 Pattern) (Credit System) (Semester - III)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If M $\leq$ H, then prove that $(M^{\perp})^{\perp} = M$. **[4]**

b) If $\{h_n\}$ is a sequence in a Hilbert space H such that $\sum_n \| h_n \| < \infty$, then show that $\sum_{n=1}^{\infty} h_n$ converges in H. **[3]**

c) Let I be any set . Prove that $l^2(I)$ is a Hilbert space. **[3]**

Q2) a) State and prove Bessel's Inequality. **[5]**

b) If H is a Hilbert space, then prove that any two bases have the same cardinality. **[5]**

Q3) a) If T is a positive compact operator, then prove that there is a unique positive compact operator A such that $A^2 = T$. **[5]**

b) If $T \in B(H)$, show that $T^*T \geq 0$. **[3]**

c) Give an example of linear map between Hilbert spaces, which is an isometry but not surjective. **[2]**

Q4) a) If T is a compact operator on $H, \lambda \neq 0, \lambda \notin \sigma_p(T)$, and $\bar{\lambda} \notin \sigma_p(T)$, then prove that $ran(T-\lambda) = H$ and $(T-\lambda)^{-1}$ is a bounded operator on H. **[5]**

P.T.O.

b) Show that every operator of finite rank is compact. **[3]**

c) If $A = A^*$ and $\lambda \in \sigma_p(A)$, then prove that λ is a real number. **[2]**

Q5) a) If H is a $\mathbb{C}-$Hilbert space and $A \in B(H)$, then prove that A is hermitian if and only if $\langle Ah, h \rangle \in \mathbb{R}$ for all $h \in H$. **[5]**

b) If $A \in B(H)$, then prove that $\ker A = (ran\, A^*)^{\perp}$. **[3]**

c) Let S be the unilateral shift and compute SS* and S*S. **[2]**

Q6) a) If $\|.\|_1$ and $\|.\|_2$ are two norms on a vector space X, then prove that these two norms are equivalent if and only if there are positive constants c and C such that $c\|x\|_1 \leq \|x\|_2 \leq C\|x\|_1$ for all $x \in X$. **[5]**

b) Show that the norm on $C([0,1]) = C_b([0,1])$ does not come from an inner product by showing that it does not satisfy the parallelogram law. **[3]**

c) If X and Y are Banach spaces and $A : X \rightarrow Y$ is a bounded linear transformation that is bijective, then prove that A^{-1} is bounded. **[2]**

Q7) a) State and prove the Closed Graph Theorem. **[5]**

b) Show that c* is isometrically isomorphic to l^1. **[3]**

c) Show that $(X^*)^{**} = (X^{**})^*$ are equal. **[2]**

Q8) a) If X is a finite dimensional vector space over $\mathbb{F}$, then prove that any two norms on X are equivalent. **[5]**

b) Show that c_0 is not reflexive. **[3]**

c) State Hahn-Banach Theorem. **[2]**

Total No. of Questions :8]

SEAT No. :

P1420

[Total No. of Pages :3

[5221] - 401

M.A/M.Sc.

MATHEMATICS

MT - 801: Number Theory

(2013 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks :50*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove the Wilson's Theorem. **[5]**

b) Prove that $n^7 - n$ is divisible by 42 for any integer n. **[3]**

c) Prove that 3 is a quadratic residue of 13. **[2]**

Q2) a) Let p be an odd prime then prove that $\left(\frac{a}{p}\right) \equiv a^{(p-1)/2}$ (mod p). **[5]**

b) Find all solutions to the congruence $353x \equiv 254$ (mod 400). **[3]**

c) Evaluate: $\left(\frac{10}{89}\right)$ **[2]**

Q3) a) Let p denote a prime then prove that the largest exponent e such that $p^e \mid n!$ is $e = \sum_{i=1}^{\infty} \left[\frac{n}{p^i}\right]$. **[5]**

b) Let $\mu(n)$ denote the Möbius μ function then evaluate $\sum_{j=1}^{\infty} \mu(j!)$ **[3]**

c) Find the minimal polynomial of $\left(1+\sqrt[3]{7}\right)/2$. **[2]**

P.T.O.

Q4) a) State and prove the Chinese Remainder Theorem. **[5]**

b) For what real number x is it true that **[3]**

i) $[x] + [x] = [2x]$?

ii) $[x + 3] = 3 + [x]$?

iii) $[x + 3] = 3 + x$?

c) If the norm of an integer α in $Q\left(\sqrt{m}\right)$ is $\pm$ p, where p is a rational prime then prove that α is a prime. **[2]**

Q5) a) Prove that the product of two primitive polynomials is primitive. **[5]**

b) Find all integers that satisfy simultaneously $x \equiv 2(\text{mod}\,3)$, $x \equiv 3(\text{mod}\,5)$, $x \equiv 5(\text{mod}\,2)$. **[5]**

Q6) a) If a monic polynomial $f(x)$ with integral coefficients factors into two monic polynomials with rational coefficients, say $f(x) = g(x)\, h(x)$ then prove that $g(x)$ & $h(x)$ have integral coefficients. **[5]**

b) Find all solutions in positive integers of $10x - 7y = 17$ **[5]**

Q7) a) If ξ is an algebraic number of degree n then prove that every number in Q (ξ) can be written uniquely in the form. **[5]**

$a_0 + a_1\xi + --- + a_{n-1}\,\xi^{n-1}$

where the a_i are rational numbers.

b) Prove that $ax + by = a + c$ is solvable if and only if $ax + by = c$ is solvable. **[3]**

c) Find the number of positive integers $\leq$ 7200 that are relatively prime to 3600. **[2]**

Q8) a) State and prove the Gaussion reciprocity law. **[5]**

b) Show that 1387 is a pseudoprime. **[3]**

c) What are the last two digits in the ordinary decimal representation of 3^{400}? **[2]**

Total No. of Questions : 8]

SEAT No. :

P1421

[Total No. of Pages : 3

[5221]-402
M.A./M.Sc.
MATHEMATICS
MT - 802 : Differential Geometry
(2013 Pattern) (Semester - IV) (Credit System)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that the graph of any function $f:\mathbb{R}^n \to \mathbb{R}$ is a level set for some function $F:\mathbb{R}^{n+1} \to \mathbb{R}$. **[4]**

b) Let X be a smooth vector field on an open set $U \subset \mathbb{R}^{n+1}$ and $p \in U$. Prove that there exist an open interval I containing zero and an integral curve $\alpha : I \to U$ of X such that $\alpha(0) = p$. **[4]**

c) Find and sketch the gradient field of the function $f(x_1, x_2) = x_1 + x_2$. **[2]**

Q2) a) Let U be an open set in $\mathbb{R}^{n+1}$ and let $f:U \to \mathbb{R}$ be a smooth function. Let $p \in U$ be a regular point of f, and let $C = f(p)$. Prove that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$. **[4]**

b) Let $f:U \to \mathbb{R}$ be a smooth function, where $U \subset \mathbb{R}^{n+1}$ is an open set, and let $\alpha : I \to U$ be a parametrized curve. Show that the image of α is contained in a level set of f if and only if α is everywhere orthogonal to the gradient of f. **[4]**

c) Show that if $\alpha : I \to \mathbb{R}^{n+1}$ is a parametrized curve with constant speed then $\ddot{\alpha}(t) \perp \dot{\alpha}(t)$, for all $t \in I$. **[2]**

Q3) a) State and prove Lagranges Multiplier theorem. **[5]**

b) Let $S \subset \mathbb{R}^{n+1}$ be a connected n-surface in $\mathbb{R}^{n+1}$. Prove that there exist on S exactly two smooth unit normal vector fields N_1 and N_2, and $N_2(p) = -N_1(p)$ for all $p \in S$. **[5]**

Q4) a) Define n-surface in $\mathbb{R}^{n+1}$. For what values of c is the level set $f^{-1}(c)$ on n-surface, where $f(x_1,x_2,x_3,..........,x_{n+1}) = x_1^2 + x_2^2 + ... + x_{n+1}^2$? **[3]**

b) Let S be an n-surface in $\mathbb{R}^{n+1}$. Let $\alpha : I \to S$ be a parameterized curve in S, let $t_0 \in I$, and $v \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field V tangent to S along α, which is parallel and has $V(t_0) = v$. **[5]**

c) Let X and Y be smooth vector fields. Prove that $\dot{(X+Y)} = \dot{X} + \dot{Y}$ **[2]**

Q5) a) Let S be an n-surface in $\mathbb{R}^{n+1}$, let $p, q \in S$ and let α be a piecewise smooth parameterized curve from p to q. Then prove that parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot products. **[5]**

b) Let U be an open set in $\mathbb{R}^{n+1}$ and let $f : U \to \mathbb{R}$ be a smooth function. Show that if $e_i = (p,0,.......,|,.......,0)$ where $p \in U$ and the | is the (i + 1)th spot (i spots after p), then $\nabla_{e_i} f = \left(\frac{\partial f}{\partial x_i}\right)(p)$. **[3]**

c) Find velocity, acceleration and speed of parameterized curve $\alpha(t) = (\cos t, \sin t, t)$. **[2]**

Q6) a) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parameterization of C. Prove that β is either one to one or periodic. Also prove that β is periodic if and only if C is compact. **[5]**

b) Let $\alpha : I \to C$ be a unit speed local parameterization of the oriented plane curve C. Suppose $\theta : I \to \mathbb{R}$ is smooth and is such that $\dot{\alpha}(t) = (\alpha(t), \cos\theta(t), \sin\theta(t))$ for all $t \in I$. Show that $\frac{d\theta}{dt} = K o \alpha$. **[5]**

Q7) a) Show that local parameterization of plane curves are unique up to reparameterization. **[5]**

b) Compute the line integral $\int_{\alpha}(x_2 dx_1 - x_1 dx_2)$, where

$\alpha(t) = (2\cos t, 2\sin t),\ 0 \le t \le 2\pi$. **[3]**

c) Define the terms : **[2]**

i) Weingarten Map

ii) Gauss Map

Q8) a) Let S be an compact oriented n-surface in $\mathbb{R}^{n+1}$. Prove that there exist a point p such that the second fundamental form at p is definite. **[5]**

b) State Inverse function theorem. **[2]**

c) Let $g : I \to \mathbb{R}$ be a smooth function and C denote the graph of g. Show that the curvature of C at point $(t, g(t))$ is $\dfrac{g''(t)}{\left(1+(g'(t))^2\right)^{3/2}}$, for an appropriate choice of orientation. **[3]**

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Total No. of Questions :8]

SEAT No. :

P1422

[Total No. of Pages : 4

[5221]-403
M.A./M.Sc.
MATHEMATICS
MT - 803 : Fourier Analysis and Boundary Value Problems
(2013 Pattern) (Credit System) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 50*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let f denote a function that is piecewise continuous on the interval $-\pi < x < \pi$ and periodic with period 2π on entire x-axis. Then prove that Fourier series converges to the mean value $\frac{f(x+)+f(x-)}{2}$ of the one sided limits of f at each point x $(-\infty < x < \infty)$ where both of the one-sided derivatives $f_R^1(x)$ and $f_L^1(x)$ exists. **[5]**

b) Find the Fourier cosine series for the function $f(x) = x^2 \quad (0 < x < \pi)$. **[3]**

c) Find the Fourier sine series on the interval $0 < x < \pi$ that correspondes to the function f defined by $f(x) = \pi - x \ (0 < x < \pi)$. **[2]**

Q2) a) Let f denote a function such that (i) f is continuous on the interval $-\pi < x < \pi$ (ii) $f(-\pi) = f(\pi)$ (iii) its derivative f' is piecewise continuous on the interval $-\pi < x < \pi$. If $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx.$ and $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$ are Fourier coefficients for f, then prove that the series $\sum_{n=1}^{\infty}\sqrt{a_n^2 + b_n^2}$ converges. **[5]**

b) Find the fourier series for the function $f(x)=\begin{cases} -\pi/2 & when -\pi < x < 0 \\ \pi/2 & when\ 0 < x < \pi \end{cases}$. **[3]**

c) Let f denote the continuous function defined by the equation $f(x)=\begin{cases} x^2 & when\ x \le 0 \\ \sin x & when\ x > 0 \end{cases}$. Then find $f_R^l(o)$ and $f_L^l(o)$. **[2]**

Q3) a) Solve the following boundary value problem

$y_{tt}(x,t)=a^2 y_{xx}(x,t)\ (0 < x < c,\ t > 0)$

$y(0,t)=0,\ y(c,t)=0,\ y_t(x,0)=0.$ **[5]**

b) Solve the following boundary value problem.

$u_t(x,t)=\mathrm{K}u_{xx}(x,t)+q(t)(0 < x < \pi, t>0)$

$u(0,t)=0, u(\pi,t)=0, u(x,0)=f(x).$ **[5]**

Q4) a) Solve the following boundary value problem.

$u_{xx}(x,y)+u_{yy}(x,y)=0(0 < x < a, 0 < y < b)$

$u(0,y)=0, u(a,y)=0(0 < y < b)$

$u(x,0)=f(x), u(x,b)=0(0 < x < a).$ **[5]**

b) Solve the following boundary value problem.

$\rho^2 u_{\rho\rho}(\rho,\phi)+\rho u_\rho(\rho,\phi)+u_{\varphi\phi}(\rho,\phi)=0\ \ (0 < \rho < a, 0 < \phi < \pi)$

$u_\phi(\rho,0)=0, u_\phi(\rho,\pi)=0\ (0 < \rho < a)$

$u(a,\phi)=f(\phi)\ (0 < \phi < \pi).$ **[5]**

Q5) a) If C_n (n=1, 2, 3......) are the fourier constants for a function *f* in $C_p(a,b)$ with respect to an orthonormal set in that space, then prove that $\lim_{n\to\infty} C_n = 0$. **[5]**

b) Prove that a necessary and sufficient condition for an orthonormal set $\{\phi_n(x)\}$ (n = 1, 2, 3....) to be complete is that for each function F in space considered, parseval equation $\sum_{n=1}^{\infty} C_n^2 = \| f \|^2$. Where C_n are the Fourier constant $C_n = (f, \phi_n)$ be satisfied. **[3]**

c) Show that $\psi_1(x) = x$ and $\psi_2(x) = 1 - 3x^2$ are orthogonal on the interval $-1 < x < 1$. **[2]**

Q6) a) If X and Y are eigenfunctions corresponding to the same eigenvalue of a regular sturm-Liouville problem, then prove that $Y(x) = cX(x)$ where c is non zero constant. **[5]**

b) Find the eigenvalues and normalized eigen-function of sturm-Liouville problem $X'' + \lambda X = 0$, $X(0) = 0, X'(0) = 0$. **[3]**

c) Show that each of the functions $y_1 = \frac{1}{x}$ and $y_2 = \frac{1}{1+x}$ satisfies non-linear differential equation $y^1 + y^2 = 0$. Also show that if C is constant where $c \neq 0$ and $c \neq 1$ neither cy_1 nor cy_2 satisfies the equation. **[2]**

Q7) a) Prove that

$$J_n(x) = \frac{1}{\pi} \int_0^{\pi} \cos(n\phi - x\sin\phi)\, d\phi \; (n = 0,1,2....)$$ **[5]**

b) Let n be any fixed non-negative integer (n = 0,1,2,...). Prove that the positive zeros of the function $J_n(x)$ or positive roots of the equation. $J_n(x) = 0$ form a non increasing sequence of numbers x_j (j = 1,2,3....) such that $xj \to \infty$ as $j \to \infty$. **[3]**

c) If $F(x,t)=(1-2xt+t^2)^{-1/2}$, then show that $(1-2xt+t^2)\frac{\partial F}{\partial t}=(x-t)F$. **[2]**

Q8) a) Derive the Rodrigues' formula

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$ **[5]**

b) Verify directly that the Legendre polynomials

$P_0(x)=1$, $P_1(x)=x$, $P_2(x)=\frac{1}{2}(3x^2-1)$, $P_3(x)=\frac{1}{2}(5x^3-3x)$ form an orthogonal set on the interval $-1<x<1$. **[3]**

c) Expand the function $f(x)=1$ $(0<x<1)$ in a series of Legendre polynomials of odd degree. **[2]**